VII. MICROWAVE TUBE RESEARCH

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RESEARCH OBJECTIVES

Current research in the tube laboratory is concerned with the study of the propagation of signals and noise along electron beams at microwave frequencies. A better understanding of the process of the propagation phenomenon at low modulation levels will lead to the construction of improved low-noise microwave amplifiers. A greater knowledge of the bunching in a beam under large-signal excitation will be used as the basis for the design of high-power microwave tubes.

A simplified theory reported in this and preceding Quarterly Progress Reports has shown that two noise parameters determine the quality of an electron gun so far as its use in a low-noise microwave amplifier is concerned. Future experimental work will aim at determining the two noise parameters for several electron guns under varying operating conditions. These experiments will provide further data for a test of the validity of the theory. The results will be used to predict the noise figure of a low-noise traveling-wave tube which will be built in the laboratory. A Dicke radiometer has been used in the past to raise the sensitivity of the detecting apparatus. Another radiometer will be put into operation.

Measurements will be performed on strongly bunched beams to determine the large-signal behavior of electron beams. A high-power klystron will be designed and built.

The problem of signal propagation along beams under the influence of transverse fields will be investigated theoretically and experimentally. Transverse field tubes will be built to check the theoretical predictions.

H. A. Haus

A. MINIMUM NOISE FIGURE OF LONGITUDINAL BEAM AMPLIFIERS

In the Quarterly Progress Report, October 15, 1954, certain relations were proved concerning the minimum obtainable noise figure of longitudinal beam microwave amplifiers whose input and output structures have no direct rf connection. In this report, corresponding relations are proved about the minimum obtainable noise figure of all lossless longitudinal beam amplifiers with matched input and output by means of arbitrary noise-smoothing schemes.

First, we find a convenient matrix representation for the functioning of the amplifier and noise-smoothing scheme. We make the usual assumptions of small-signal, one-dimensional, single-velocity theory. Figure VII-1 shows the schematic of a traveling-wave tube with a noise-smoothing scheme of the feedback type. We assume that the beam
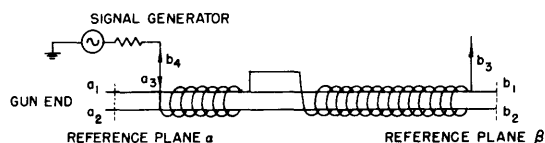

Fig. VII-1

Schematic of traveling-wave tube with
feedback for noise reduction.

enters the amplifier carrying a fast wave of complex amplitude a_1 and that the signal generator is short-circuited. This excitation produces in the output guide a traveling wave of complex amplitude b_3, a fast and a slow wave in the beam at the output end of the amplifier of amplitudes b_1 and b_2, respectively, and, in general, a wave b_4 traveling into the input circuit. We have

$$b_1 = A_{11}a_1, \quad b_2 = A_{21}a_1, \quad b_3 = A_{31}a_1, \quad b_4 = A_{41}a_1$$

Similar arguments show that a slow wave of complex amplitude a_2 excites, in general, all four waves b_1 to b_4. A wave of complex amplitude a_3 fed into the amplifier from the input generator excites only the waves b_1, b_2, and b_3, but not b_4 if the amplifier is matched to the line as assumed henceforth. The following matrix relation results from this reasoning[*]

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \\ A_{41} & A_{42} & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \tag{1}$$

The a's and b's are normalized so that their square gives the power (kinetic (1) or electromagnetic) carried in the respective waves. The arrows in Fig. VII-2 give the direction of the power flow associated with each wave. The amplification of the amplifier is $|A_{33}|^2$. Two beam waves of amplitude a_1 and a_2 fed into the amplifier cause a power output of

$$|b_3|^2 = |A_{31}|^2 |a_1|^2 + |A_{32}|^2 |a_2|^2 + 2 \, \text{Im}\left[A_{31}^* A_{32}\right] \text{Im}\left[a_1 a_2^*\right]$$

$$+ 2 \, \text{Re}\left[A_{31}^* A_{32}\right] \text{Re}\left[a_1 a_2^*\right] \tag{2}$$

A noise process can be analyzed as the limit of a periodic process of infinite period T. If we divide Eq. 2 by $\Delta\omega = 2\pi/T$, go to the limit $\Delta\omega \rightarrow 0$, and take an average over an ensemble of noise processes with identical statistical properties we get terms that can be related to quantities previously defined (2).

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[*]This matrix formalism is an extension of one suggested to the author by F. N. H. Robinson, of the Bell Telephone Laboratories.

$$\lim_{\Delta\omega\to 0} \overline{\frac{1}{\Delta\omega} |a_{1,2}|^2} = \frac{1}{4}\left(|Y_o| \, \Phi_a + \frac{1}{|Y_o|} \, \Psi_a\right) \pm \frac{1}{2} \, \Pi_a$$

(3)

$$\lim_{\Delta\omega\to 0} \overline{\frac{1}{\Delta\omega} a_1 a_2^*} = \frac{1}{4}\left(|Y_o| \, \Phi_a - \frac{1}{|Y_o|} \, \Psi_a\right) + j\frac{1}{2} \, \Lambda_a$$

where Φ_a and Ψ_a are the self power density spectra of the noise voltage and current in the beam at the reference cross section a; Π_a and Λ_a are the real and imaginary parts, respectively, of the cross power density spectrum between the voltage and current at the reference cross section a. The quantity Y_o is the characteristic admittance of the beam defined by

$$Y_o = -\frac{I_o}{2V_o}\frac{\omega}{\omega_q}$$

where I_o is the direct current in the beam, V_o is the beam voltage, ω is the operating frequency, and ω_q is the reduced plasma frequency of the beam.

The noise figure of the amplifier is

$$F = 1 + \frac{\lim\limits_{\Delta\omega\to 0} \frac{1}{\Delta\omega}\left[|b_3|^2\right]_{a_3=0}}{\lim\limits_{\Delta\omega\to 0} \frac{1}{\Delta\omega}\left[|b_3|^2\right]_{a_1=a_2=0}}$$

(4)

The denominator of Eq. 4 is simply $|A_{33}|^2 (kT)/\pi$, since the density spectrum of the power fed into the amplifier from the circuit is $(kT)/\pi$.

According to Eqs. 2 and 3, the numerator of Eq. 4 is a function of Φ_a, Ψ_a, Λ_a, and Π_a. If a lossless beam transducer is put in front of the amplifier, Φ_a, Ψ_a, and Λ_a can be changed, subject to the condition that $\Phi_a\Psi_a - \Lambda_a^2 = S_o^2$, where $S_o^2 = \Phi_o\Psi_o - \Lambda_o^2$ is a characteristic constant of the noise process in the electron gun (2). A lossless transducer leaves the kinetic power in the beam unchanged; thus we have $\Pi_a = \Pi_o$, where Π_o is the
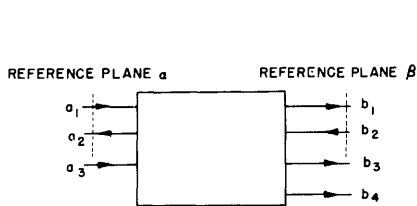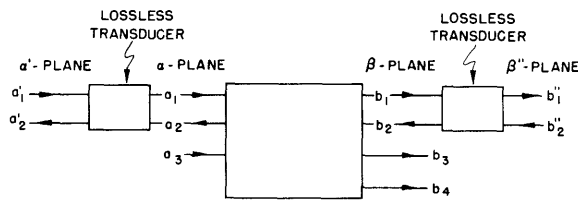

Fig. VII-2

Directions of power flow.

Fig. VII-3

Modified amplifier.

real part of the cross power spectrum density at a reference cross section in the electron gun. Carrying out the minimization of Eq. 4 according to the transformation laws of a lossless transducer given above, one finds

$$F_{min} = 1 + \frac{2\pi}{kT} |K| \left[S_o - \frac{K}{|K|} \Pi_o \right] \qquad (5)$$

where

$$K = \frac{1}{|A_{33}|^2} \left[|A_{32}|^2 - |A_{31}|^2 \right] \qquad (6)$$

It is of great interest to know what the minimum value of K can be. It is known (3) that a lossless traveling-wave tube in conjunction with a noise-smoothing scheme of velocity jumps acting upon the beam noise has a value of K = 1. If we find, as we indeed do, that the constant K can in no case be appreciably less than unity, we know that no lossless amplifier with matched input in conjunction with any lossless noise-smoothing scheme has a noise figure appreciably lower than that of a lossless traveling-wave tube which uses conventional velocity jumping.

In order to find the range of values that K can assume we note first of all the K could be negative if $|A_{32}| < |A_{31}|$ and is positive if $|A_{32}| > |A_{31}|$. All possible microwave structures can be divided into two classes.

1. First Class of Microwave Structures, $|A_{32}| > |A_{31}|$

The inequality $|A_{32}| > |A_{31}|$ has the following meaning: With a_3 equal to zero, it follows from the matrix Eq. 1 that b_3 can be made equal to zero with

$$\frac{a_1}{a_2} = - \frac{A_{32}}{A_{31}} \qquad (7)$$

A modulation of the beam that satisfies Eq. 7 with $|A_{32}| > |A_{31}|$ carries positive kinetic power. Equation 7 corresponds to a certain standing wave of current in the beam in front of the amplifier. Such a standing wave could have been produced out of a pure traveling wave a_1' with positive kinetic power through a lossless transducer, as shown in Fig. VII-3. The transducer can be considered a part of a new amplifier characterized by a new matrix A' with the element $A_{31}' = 0$. The amplifier has the same minimum noise figure as the original one.

Next, we consider an excitation of the amplifier by an input signal a_3 with $a_1' = a_2' = 0$. If the structure is an amplifier we must have $b_3 > a_3$. The balance of power requires that the kinetic power in the beam leaving the structure must be negative, and, accordingly, $b_1 < b_2$. Such a modulation on the beam can be transformed by a lossless transducer at the exit of the amplifier into a signal modulation characterized by $b_1'' = 0$. The

lossless transducer can be included in a revised amplifier structure with a matrix A'' in which the elements A''_{31} and A''_{13} are equal to zero. The revised amplifier has the same minimum noise figure as the original one.

These considerations simplify the ensuing mathematics considerably. We shall now assume that the amplifier structure is lossless. Then the equation

$$|b''_1|^2 - |b''_2|^2 + |b_3|^2 + |b_4|^2 = |a'_1|^2 - |a'_2|^2 + |a_3|^2 \tag{8}$$

must be satisfied for arbitrary values of the a variables.

Equation 8 imposes six conditions upon the magnitudes of the matrix coefficients A''_{ij}. After some simple manipulations, the use of the six conditions leads to the relation

$$\left|\frac{A''_{32}}{A''_{33}}\right|^2 = \frac{|A_{32}|^2 - |A_{31}|^2}{|A_{33}|^2} \geqslant 1 - \frac{1}{|A_{33}|^2} \tag{9}$$

Now, $|A_{33}|^2$ is the available power gain G of the amplifier. Thus, we have for the first class of structures

$$K \geqslant 1 - \frac{1}{G} \tag{10}$$

2. Second Class of Microwave Structures, $|A_{31}| > |A_{32}|$

Reasoning analogous to that used above leads to the addition of a lossless beam transducer at each end of the amplifier. The transducers can be included as an integral part of the amplifier and adjusted so that the new matrix of the structure has the elements A''_{32} and A''_{13} equal to zero, under the assumption that the structure is an amplifier, that is, has $|A_{33}| > 1$. However, with the six conditions imposed upon the matrix elements by Eq. 8, one can easily show that they lead to the requirement $A_{33} = 0$. Accordingly, it has been proved that there is no lossless amplifier structure with matrix elements $|A_{31}| > |A_{32}|$.

Thus, we have completed the proof that any lossless amplifier matched at its input and output end and using an arbitrary lossless noise-smoothing scheme must have a minimum noise figure given by Eq. 5 in which the constant K is limited by the inequality of Eq. 10.

H. A. Haus

References

1. B. Agdur and H. A. Haus, Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., April 15, 1954, p. 30.

2. H. A. Haus, Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., April 15, 1954, p. 32.

3. H. A. Haus, Paper to be published in the Journal of Applied Physics.

B. LOW-NOISE ELECTRON BEAMS

The electron beam noise from a new RCA low-noise gun (Fig. VII-4) with a Phillips impregnated cathode was investigated under certain conditions of interest. The noise along the beam was probed with a cavity resonant at about 2750 Mc/sec and was detected by a radiometer system. The mounting of the gun was provided with an adjustment for its position in the vacuum along the axis of the magnetic focusing field.
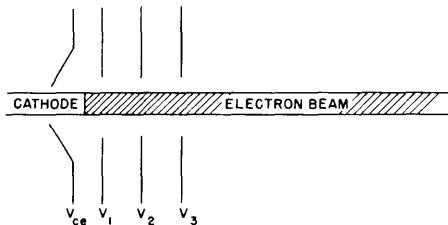

Fig. VII-4
RCA low-noise electron gun.

1. Cathode Activity Effects

The electron gun was placed well inside the magnetic focusing field (position A, Fig. VII-7) in a manner to insure normal magnetic field lines on the cathode. Three days after the activation of the cathode (during this time the cathode was kept at a temperature of 900°C and no current was drawn) noise standing wave curves were taken under the conditions of Fig. VII-5(a). The temperature of the cathode was then raised to 1210°C, and for 48 hours current was continuously drawn from the cathode. The standing wave of noise along the beam, taken after this period, showed an average that was approximately 4 db lower (Fig. VII-5(b)) than that of the first curves. This indicated a reduction in the noise content of the beam. This phenomenon was not found to be reversible: After reducing the cathode temperature to the original value of 1155°C, the average of the standing wave was raised by only slightly more than 1 db (Fig. VII-5(c)). No further reduction in noise was achieved by raising the cathode temperature from 1210°C to 1260°C (Fig. VII-5(d)).

The measurements were also performed at various values of the second anode voltage V_2. For each particular temperature, as V_2 was varied between 75 volts and 125 volts the standing-wave ratio and the phase of the noise standing wave varied. However, the average of the standing wave remained constant to within less than ± 1 db. A typical set of curves for a particular cathode temperature is shown in Fig. VII-6. These support recent theories of Pierce (1), Haus (2), and Bloom and Peter (3) regarding the action of lossless beam transducers (for example, dc accelerating regions) upon the noise in the beam (4).

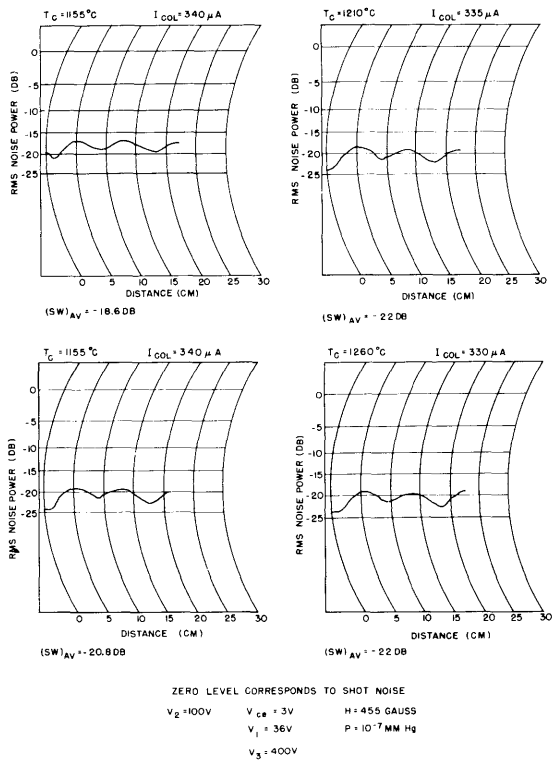An interesting result shown by Fig. VII-5 is that the temperature variations in the

Fig. VII-5

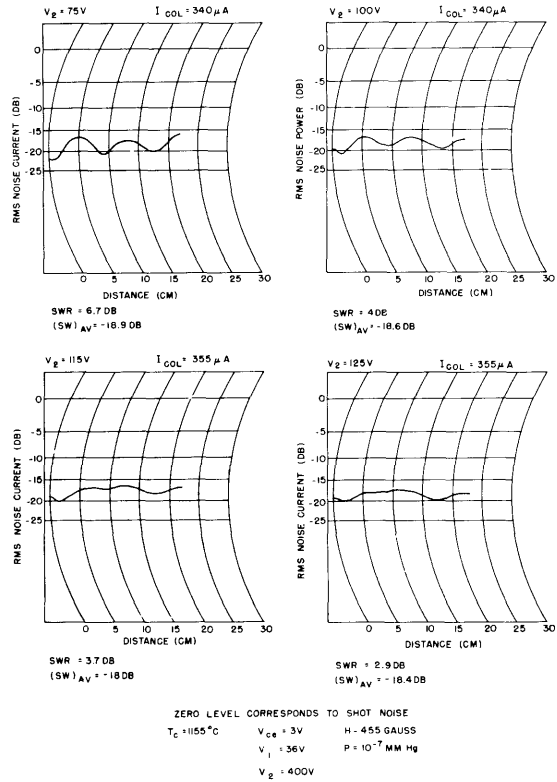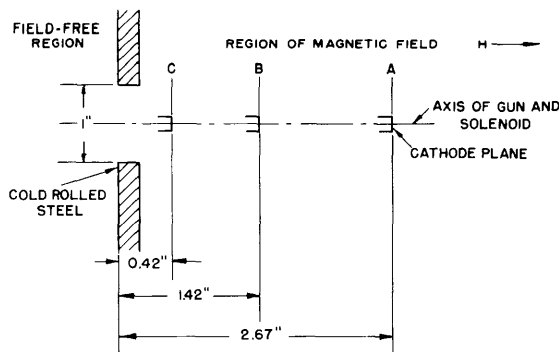Beam noise vs. cavity position (the cathode temperature is variable).

Fig. VII-6

Beam noise vs. cavity position (the cathode temperature is constant; the second anode voltage is varied).

Fig. VII-7

Cathode positions in magnetic focusing field.

cathode do not affect the standing-wave ratio or the phase of the noise standing wave but only the average of the standing wave.

2. Ion Trapping Effects

To determine the effect of ion bombardment of the cathode, a ring in front of the last anode was biased above the last anode voltage. This kept the ions in the beam, and prevented their draining off to the cathode. The noise standing wave curves taken with the ring at the third anode voltage, and those taken with the ring 50 volts above the third anode voltage were identical.

3. Magnetic Field Effects

The variation in the beam noise standing wave with the uniformity of the magnetic field in the gun region was tested by sliding the gun along the axis of the solenoid. The results obtained are shown in Fig. VII-8. The corresponding gun positions are given in Fig. VII-7. The standing-wave curves show that as the gun was moved out into the region of nonuniform (convergent) magnetic field the average of the standing wave was raised. This indicated an increase in the beam noise. At the same time the phase of the standing wave as well as the standing-wave ratio also varied drastically.
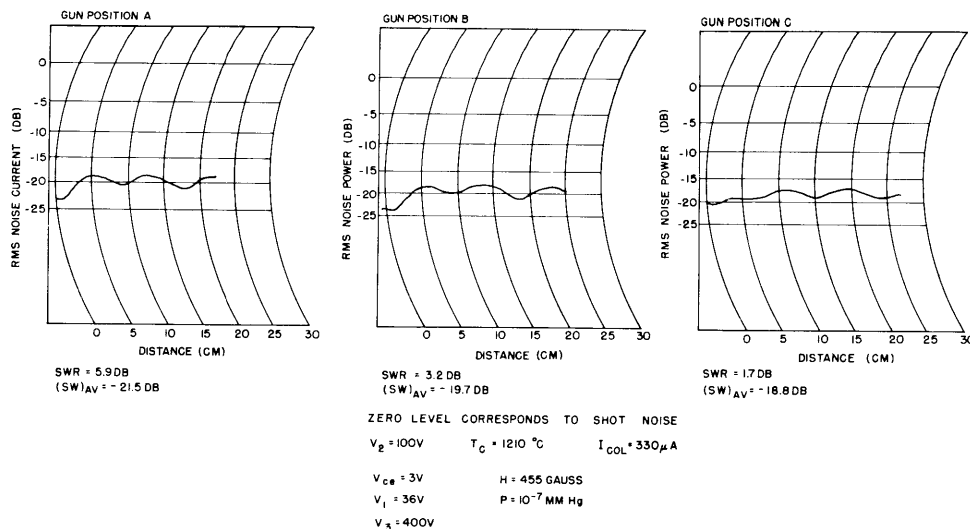

Fig. VII-8

Beam noise vs. cavity position. Gun position in
magnetic field as shown in Fig. VII-7.

It should be noted that all of these measurements were taken on the same cathode, and under the same vacuum. Throughout all of the experiments the saturation current was well above ten times the collector current; interception current was negligible at all times. All standing-wave curves were reproducible to within less than ±1 db.

A. Bers

References

1. W. E. Danielson and J. R. Pierce, J. Appl. Phys. 25, 9 (1954).
2. H. A. Haus, D. Sc. Thesis, Department of Electrical Engineering, M.I.T., June 1954.
3. S. Bloom and R. W. Peter, R.C.A. Rev. 15, 252 (1954).
4. C. Fried, Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., July 15, 1954, p. 40.

C. BEAM NOISE MEASUREMENTS

The measurements described in the Quarterly Progress Report of October 15, 1954, were repeated under more carefully controlled conditions. The convergent-flow gun used in these measurements had a perveance of 0.076×10^{-6} amp/volt$^{3/2}$. An auxiliary solenoid was placed around the gun for obtaining a controllable magnetic field in the gun region. As previously reported, strong growing noise waves appeared in the latter part of the drift space. With a relatively weak (8-15 gauss in the axial direction) auxiliary
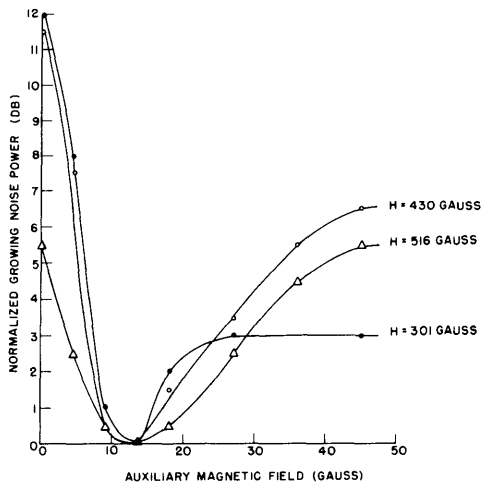

Fig. VII-9

Noise reduction curves of the growing noise wave with the main magnetic field as parameter.

magnetic field in the gun region, the growing noise waves could be eliminated at will. While an auxiliary field of either polarity reduced the growing noise wave, smaller field-strength was required if the axial components of the auxiliary and main focusing fields were similarly directed. The value of leakage field of the main solenoid was approximately 2-4 gauss at the cathode. Typical noise reduction curves for a fixed cavity position as functions of the auxiliary field are shown in Fig. VII-9.

C. Fried

D. MILLION-ELECTRON-VOLT TUBE

The designs of cathode and anode metal-to-ceramic seals were completed. With the cooperation of the Raytheon Manufacturing Company, the first tube envelope was brazed. Some leakage developed, however, from the porosity of a molybdenum cup. When this defect has been repaired work will proceed toward final assembly of the tube.

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