

## XX. SWITCHING CONTROL SYSTEMS

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Usually, switching circuits are analyzed and synthesized by representing each switch contact as a transmission that equals zero when the contact is open and equals one when the contact is closed (1). It is conventional to represent the transmission of each make contact (2) of relay A by the symbol A and of each break contact by the symbol A', where  $A = 0$ ,  $A' = 1$  when relay A is unoperated; and  $A = 1$ ,  $A' = 0$  when relay A is operated. The over-all transmission of a network of switch contacts may then be found by combining the individual switch transmissions in the same manner in which transmissions are combined in signal flow graphs (3). However, in the case of switch transmissions it is sometimes possible to simplify a network transmission by making use of the theorems of switching algebra (the algebra of logic). See reference 1.

In an IRE article (4), Shekel has shown how it is possible to combine this transmission notation with the impedance notation usually used for networks consisting of RLC elements, in order to analyze RLC networks containing switches.

It is also possible to study switch networks with an impedance type of notation, which thus permits an immediate extension to networks containing both RLC elements and switches. Other advantages of the impedance notation are: (a) ease of initial understanding for people already familiar with conventional network analysis, but not with switching circuit analysis; (b) replacement of the  $1 + 1 = 1$  postulate of switching algebra with the initially more acceptable  $\infty + \infty = \infty$ ; and (c) demonstration of the direct application to switch networks of the theorems ( $\Delta$  - Y transformations, duality, and the like) of lumped parameter networks.

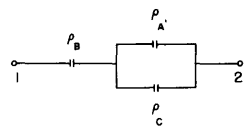
The impedance notation is presented here merely as an alternative to the transmission (switching algebra) notation with the hope that it provides some additional insight into switching circuits for both experts and beginners in this field.

With each switch will be associated a resistance  $\rho$  (conductance  $\gamma$ ) representing the resistance (conductance) between the terminals of the switch. The element that controls the switch (relay, button, etc.) will be denoted by a capital English letter; the resistance (conductance) of the switch will be written as  $\rho_A$  ( $\gamma_A$ ) if the switch is an open circuit when the element A is unenergized, and as  $\rho_{A'}$  ( $\gamma_{A'}$ ) if the switch is a short circuit when the element A is unenergized. Thus

$$\left. \begin{array}{l} \rho_A = \infty; \gamma_A = 0 \\ \rho_{A'} = 0; \gamma_{A'} = \infty \end{array} \right\} \text{element A unenergized} \quad (1)$$

$$\left. \begin{array}{l} \rho_A = 0; \gamma_A = \infty \\ \rho_{A'} = \infty; \gamma_{A'} = 0 \end{array} \right\} \text{element A energized} \quad (2)$$

$$\rho_A = 1/\gamma_A; \rho_{A'} = 1/\gamma_{A'}$$



$$R_{12} = \rho_B + \frac{1}{\gamma_{A'} + \gamma_C}$$

NOTE:  $\text{--|}$  REPRESENTS A SWITCH CONTACT.

(a)

RELAYS

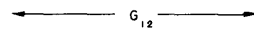
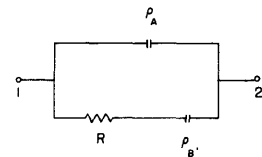
A	B	C	$R_{12}$
U	U	U	$\infty$
U	U	E	$\infty$
U	E	U	0
U	E	E	0
E	U	U	$\infty$
E	U	E	$\infty$
E	E	U	$\infty$
E	E	E	0

U = UNENERGIZED  
E = ENERGIZED

(b)

Fig. XX-1

Example of a simple contact network: (a) circuit diagram;  
(b) circuit condition table.



$$G_{12} = \gamma_A + \frac{1}{R + \rho_{B'}}$$

(a)

RELAYS

A	B	$G_{12}$
U	U	$1/R$
U	E	0
E	U	$\infty$
E	E	$\infty$

(b)

Fig. XX-2

Circuit consisting of one resistance and two switch contacts:  
(a) circuit diagram; (b) circuit condition table.

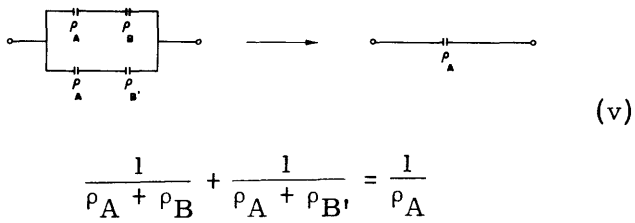
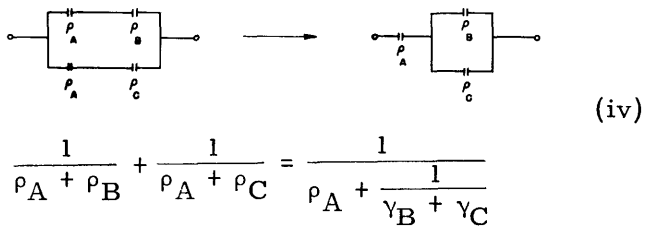
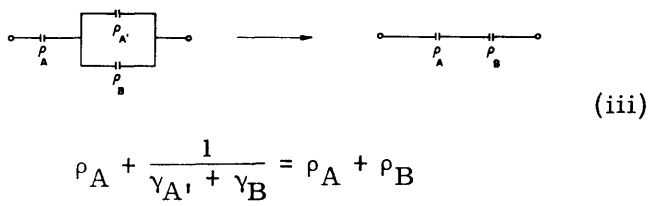
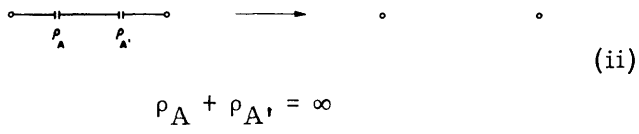
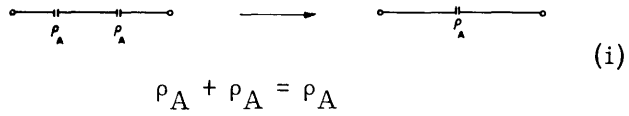


Fig. XX-3

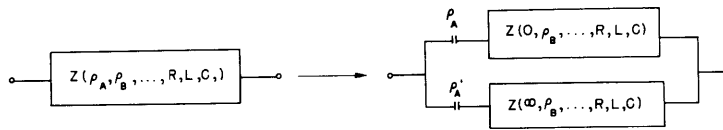
Some simplifying relations involving switch resistances only.



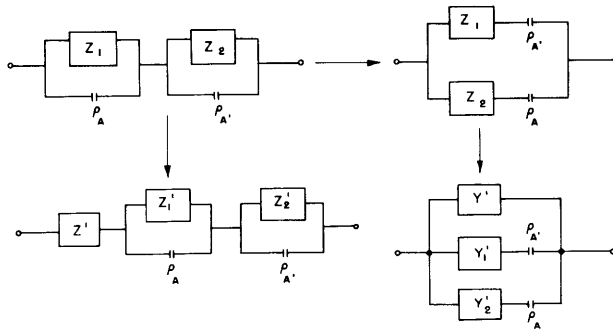
$$\rho_A + \frac{1}{\gamma_A + Y} = \rho_A \quad (i)$$



$$\rho_A + \frac{1}{\gamma_{A'} + Y} = \rho_A + \frac{1}{Y} \quad (ii)$$



$$Z(\rho_A, \rho_B, \dots, R, L, C) = \frac{1}{\frac{1}{\rho_A + Z(0, \rho_B, \dots, R, L, C)} + \frac{1}{\rho_{A'} + Z(\infty, \rho_B, \dots, R, L, C)}} \quad (iii)$$



$$\frac{1}{Y_1 + \gamma_A} + \frac{1}{Y_2 + \gamma_{A'}} = \frac{1}{\frac{1}{Z_1 + \rho_{A'}} + \frac{1}{Z_2 + \rho_A}} \quad Z_1 = 1/Y_1, \quad Z_2 = 1/Y_2 \quad (iv)$$

$$\frac{1}{Y_1 + \gamma_A} + \frac{1}{Y_2 + \gamma_{A'}} = Z' + \frac{1}{Y_1' + \gamma_A} + \frac{1}{Y_2' + \gamma_{A'}} \quad Z_1' = Z_1 - Z', \quad Z_2' = Z_2 - Z', \quad (v)$$

$$Z_1' = 1/Y_1'; \quad Z_2' = 1/Y_2'$$

$$\frac{1}{Z_1 + \rho_A} + \frac{1}{Z_2 + \rho_{A'}} = Y' + \frac{1}{Z_1' + \rho_A} + \frac{1}{Z_2' + \rho_{A'}} \quad Y_1' = Y_1 - Y', \quad Y_2' = Y_2 - Y', \quad (vi)$$

$$Y_1' = 1/Z_1'; \quad Y_2' = 1/Z_2'$$

Fig. XX-4

Some simplifying relations involving RLC impedances and switch resistances.

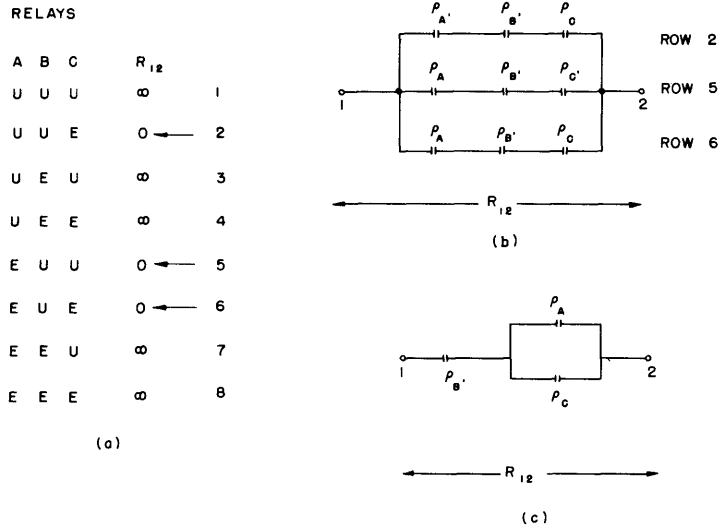


Fig. XX-5

An example of the derivation of a switching circuit from a circuit condition table: (a) circuit condition table; rows for which  $R_{12} = 0$  are indicated by arrows; (b) canonical circuit realization of circuit condition table; (c) simplified circuit.

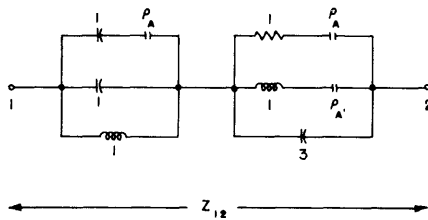


Fig. XX-6

Circuit for example 1.

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The resistance between two terminals of a switch network can be written in terms of these contact resistances just as in ordinary resistance networks. An example is shown in Fig. XX-1(a). Suppose that for the circuit of this figure relays A, B, and C are all unoperated; then

$$\rho_B = \infty, \quad \gamma_{A'} = \infty, \quad \gamma_C = 0 \quad (\text{see Eq. 1})$$

and

$$R_{12} = \infty + \frac{1}{\infty + 0} = \infty + \frac{1}{\infty} = \infty + 0 = \infty$$

By either a repetition of this process or careful inspection of the circuit diagram,  $R_{12}$  can be determined for each possible combination of conditions of the three relays A, B, and C, and listed in a table such as is shown in Fig. XX-1(b). (This table, here called the circuit condition table, corresponds to the truth table of switching algebra.) It is also possible to write, in the same fashion, the over-all resistance of a circuit containing both resistors and switch contacts. Such a circuit is shown in Fig. XX-2.

For each theorem of switching algebra there is a corresponding expression relating these switch resistances. Some of these expressions are illustrated in Fig. XX-3. There are also expressions that relate "impedances" consisting of both switch resistances and RLC impedances. These are shown in Fig. XX-4.

One general method for synthesizing a two-terminal switching circuit is to connect a parallel branch between the two external terminals (1 and 2) for each combination of relay conditions for which  $R_{12} = 0$ . Each branch is a series connection of switches so chosen that each of the associated switch resistances equals zero for the corresponding combination of relay conditions. This method for determining a switching circuit specified by a circuit condition table is illustrated in Fig. XX-5. It is possible to simplify the circuit of Fig. XX-5(b) to that of Fig. XX-5(c) by using the relations of Fig. XX-3. Thus

$$G_{12} = \frac{1}{\rho_{A'} + \rho_{B'} + \rho_C} + \frac{1}{\rho_A + \rho_{B'} + \rho_{C'}} + \frac{1}{\rho_A + \rho_{B'} + \rho_{C'}}$$

By Fig. XX-3(v)

$$\frac{1}{\rho_A + \rho_{B'} + \rho_C} + \frac{1}{\rho_A + \rho_{B'} + \rho_{C'}} = \frac{1}{\rho_A + \rho_{B'}}$$

so that

$$G_{12} = \frac{1}{\rho_{A'} + \rho_{B'} + \rho_C} + \frac{1}{\rho_A + \rho_{B'}}$$

and by Fig. XX-3(iv) this combines to

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$$G_{12} = \frac{1}{\rho_{B'} + \frac{1}{\gamma_A + \frac{1}{\rho_{A'} + \rho_C}}}$$

By Fig. XX-3(iii)

$$\gamma_A + \frac{1}{\rho_{A'} + \rho_C} = \gamma_A + \gamma_C$$

so that

$$G_{12} = \frac{1}{\rho_{B'} + \frac{1}{\gamma_A + \gamma_C}}$$

Similarly the relations of Fig. XX-4 can be used to simplify a circuit consisting of RLC elements and switches. Example 1 shows how the addition of one switch contact can be used to reduce the number of RLC elements required for a given network. Since relays are often furnished with a specified number of contacts, such a procedure may allow the use of an idle contact to eliminate the need for several RLC elements.

Example 1.

It is desired that

$$Z_{12} = \frac{1}{1/s + 2s} + \frac{1}{3s + 1}$$

when relay A is operated and that

$$Z_{12} = \frac{1}{1/s + s} + \frac{1}{3s + 1/s}$$

when relay A is unoperated. This may also be stated by requiring that

$$Z_{12} = \frac{1}{1/s + 2s + \gamma_{A'}} + \frac{1}{3s + 1 + \gamma_{A'}} + \frac{1}{1/s + s + \gamma_A} + \frac{1}{3s + 1/s + \gamma_A}$$

This expression is then simplified, by use of the relations of Fig. XX-4, in order to reduce the number of elements required. Let

$$Z_a = \frac{1}{1/s + 2s + \gamma_{A'}} + \frac{1}{1/s + s + \gamma_A}$$

$$Z_b = \frac{1}{3s + 1 + \gamma_{A'}} + \frac{1}{3s + 1/s + \gamma_A}$$

where  $Z_{12} = Z_a + Z_b$ . Let  $Y_1 = 1/s + 2s$ ,  $Y_2 = 1/s + s$ . Then by Fig. XX-4(iv)

$$Y_a = \frac{1}{Z_a} = \frac{1}{\frac{1}{1/s + 2s} + \rho_A} + \frac{1}{\frac{1}{1/s + s} + \rho_A}$$

If we let  $Y' = 1/s + s$ , then  $Y'_1 = Y_1 - (1/s + s) = s$  and  $Y'_2 = Y_2 - (1/s + s) = 0$ . Therefore, by Fig. XX-4(vi),

$$Y_a = \frac{1}{s} + s + \frac{1}{1/s + \rho_A} + \frac{1}{\infty + \rho_A}$$

$$Y_a = \frac{1}{s} + s + \frac{1}{\rho_A + 1/s} \quad (3)$$

Now let  $Y_1 = 3s + 1$ ,  $Y_2 = 3s + 1/s$ . Then by Fig. XX-4(iv)

$$Y_b = \frac{1}{Z_b} = \frac{1}{\frac{1}{3s + 1} + \rho_A} + \frac{1}{\frac{1}{3s + 1/s} + \rho_A}$$

Letting  $Y' = 3s$ , then  $Y'_1 = Y_1 - 3s = 1$  and  $Y'_2 = Y_2 - 3s = 1/s$ . Therefore, by Fig. XX-4(vi),

$$Y_b = 3s + \frac{1}{1 + \rho_A} + \frac{1}{s + \rho_A} \quad (4)$$

and combining Eqs. 3 and 4, we obtain

$$Z_{12} = \frac{1}{\frac{1}{s} + s + \frac{1}{\rho_A + 1/s}} + \frac{1}{3s + \frac{1}{1 + \rho_A} + \frac{1}{s + \rho_A}}$$

The corresponding circuit is shown in Fig. XX-6.

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#### References

1. S. H. Caldwell, Teaching Notes for use in M.I.T. Subjects 6.567T and 6.568, Chapter II.
2. A make contact is one which is open when its relay is unenergized. A break contact is one which is closed when its relay is unenergized.
3. S. J. Mason, Proc. I.R.E. 41, 9 (1953).
4. Jacob Shekel, Proc. I.R.E. 41, 7 (1953).