VIII. COMMUNICATIONS RESEARCH

As mentioned in the January 15, 1948 Progress Report, work in the Communication Group may be considered in two parts, the first containing a group of independent researches on technique and equipment, as for example the work on FM multipath phenomena, and the second, a somewhat coordinated series of investigations encompassing noise theory, information theory, and statistical design of systems.

A. MULTIPATH TRANSMISSION

Staff: Professor L. B. Arguimbau
L. Dolansky
J. Granlund
W. L. Hatton
E. C. Ho
C. F. Hobbs

1. Speech and Music

The July 15, 1947 Progress Report contained a brief summary of the difficulties encountered in transmitting speech or music over an experimental two-path link by means of frequency modulation. These difficulties are mentioned below for continuity of this report.

In general, the problem may be stated to be one of constructing a transmission link that is essentially free from distortion as long as the ratio of the voltage of the weaker signal to the voltage of the stronger signal is less than a. This research has been concerned with the most difficult case of deep fading where a is very nearly unity. Since both paths carry the same information, it does not matter which is stronger; only values of a less than unity need be considered.

The receiver and associated equipment mentioned in previous reports have been completed. From preliminary listening tests, it appears that the equipment is essentially free from distortion as long as the ratio a is less than about 0.95, corresponding to a range of distortion of $\pm \frac{1}{2}$ db about the point where the signals from the two paths are equal. The receiver uses the standard deemphasis time constant, RC = 75 μsec, and frequency deviation, Δf = 75 kc/sec.

In order that the peak distortion be essentially only that inherent in two-path transmission (the distortion obtaining in an "ideal" receiver), the following rough criteria, for a = 0.95, should be met:

(1) The transmission through one path, from transmitter up to the limiter in the receiver, should have a total variation much less than (1-a) = 5 per cent over the band occupied by the signal, $2\Delta f$ = 150 kc/sec.

(2) The transmission through the limiter and following circuits should not vary more than a few per cent over a bandwidth of $2\Delta f(1+a)/(1-a) = 5.8$ Mc/sec for a constant amplitude input. Also, the output of the limiter should not change more than a few per cent for a ratio of input amplitudes of $(1+a/1-a) = 39:1$.

(3) The discriminator should give an output that is linear with applied frequency to within a few per cent over a frequency range of $2\Delta(1+a)/(1-a) = 5.8$ Mc/sec. The associated detector circuits should not be subject to diagonal clipping when detecting the type of spike train pictured in the July 15, 1947 Progress Report. The spike train has a maximum repetition rate of $2\Delta f = 150$ kc/sec.

The first condition is perhaps the most critical since a slight non-uniformity of transmission through the linear portion of the link may have an effect similar to full-wave rectification of the audio signal, whereas misadjustment of the limiter or discriminator results only in a distortion that is of the order of magnitude of the misadjustment, or less. In aligning the linear amplifiers, it was found helpful to use an FM signal generator tuned to the center of the band and sweeping over the desired bandwidth of the amplifiers. If the output is detected with a peak detector, the transmission through the amplifier may be observed on an oscilloscope. For the actual alignment, the detected output was observed with an audio voltmeter, and the procedure was to produce a minimum voltmeter reading.

The new receiver employs one stage of amplification at 28 Mc/sec, and two stages of amplification at 13 Mc/sec before the limiter. The gain of the latter stages is adjusted so that the signal level at the input to the limiter is approximately one volt.

The limiter consists of four stages of biased crystals of the type shown in Fig. VIII-1. Since the crystals are driven to conduction

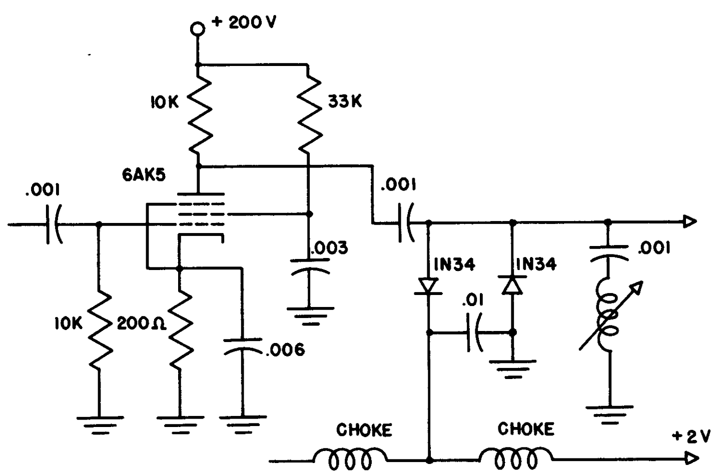

Fig. VIII-1. Limiter stage.

each cycle, the tuning of these stages is not critical. However, the dynamic conducting resistance of each crystal is very low, about 70 ohms. One must therefore be careful to hold variations of all impedances in series with the crystal to within a small fraction of 70 ohms over the large bandwidth of the limiter, or large variations in the transmission through the limiter may result.

A simplified schematic of the discriminator and detector circuits is given in Fig. VIII-2. The discriminator is the cathode-

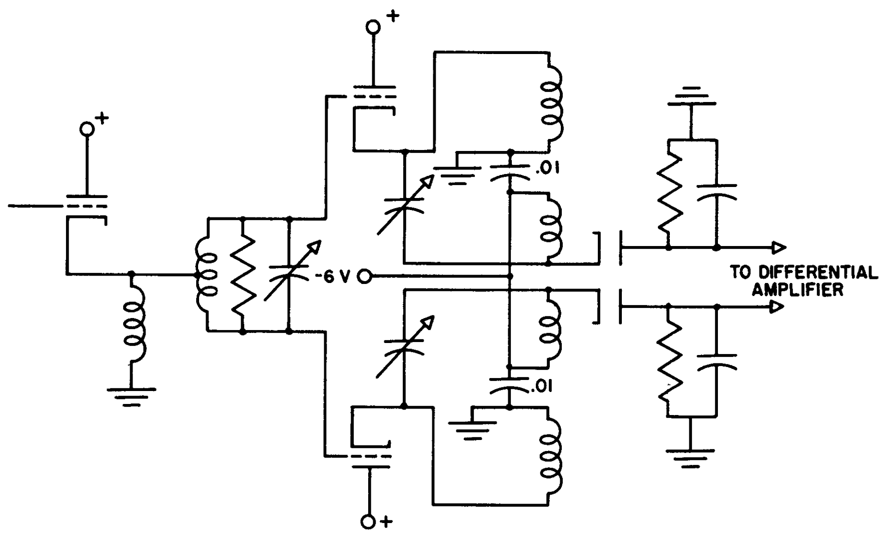

Fig. VIII-2. Simplified schematic of discriminator and detectors.

driven type developed by Cheatham and Tuller.[1] The diode detectors are not driven directly from the discriminator secondary, as it was felt that the loading of the wide-band detectors on the discriminator would be undesirable. The schematic of Fig. VIII-2 shows a method of effecting this decoupling and at the same time biasing the detectors to reduce the possibility of diagonal clipping. The output of the detectors is subtracted and amplified in the following audio amplifiers.

Since the present receiver appears to agree quite closely with theoretical results, the next step is to construct a tuner which will be used for field test under actual conditions of multipath transmission.

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1. W. G. Tuller and T. P. Cheatham, Jr., "An Adjustable Band-width F.M. Discriminator", RLE Technical Report No. 6, June 30, 1946.

2. <u>Broadbanding of the Bradley Detector</u>

In addition to the work on the main problem of setting up a high-grade trans-Atlantic link, attention has been paid to the design of a reasonably economical receiver embodying the same principles. In particular, an effort is being made by graduate students to develop a receiver similar to that described by W. E. Bradley of Philco but capable of linearity over several megacycles.

3. <u>Television</u>

The work since the last report has been devoted mainly to techniques. In particular, work has been done on AM and FM transmitters and on measurement techniques.

A crystal-controlled, AM transmitter for 200-Mc/sec operation has been built. Crystal control is used to insure freedom from any residual frequency modulation, as this has a strong influence on the multipath effects.

In this transmitter, a harmonic crystal oscillator operates at 25 Mc/sec. The frequency is doubled three times to 200 Mc/sec and this frequency-multiplying unit drives a push-pull triode output stage. A 6J6 triode is used here because of its common cathode unit and consequent freedom from cathode-lead inductance effects.

For checking amplifiers and test pulses used in television work, a fast sweep was developed as part of undergraduate thesis work carried out by H. D. Field at the Laboratory. The sweep gives speeds of a little better than 10 inches per microsecond for 5 inches. An improved model using miniature tubes was also built and is in use. Miniature tubes can be used because the sweep is formed directly on the plates of the cathode-ray tube, and no amplifiers are required.

An attempt is being made to build a 200-Mc/sec FM transmitter using reactance tubes. It is difficult to obtain the large deviation needed, at this frequency. Preliminary work is being done on a model basis at 20 Mc/sec.

To obtain a wide percentage change of frequency it is necessary to make the electronically variable portion of the capacitance a large portion of the whole. At the same time some linearity must be achieved. Further, the reactance tube and associated network must not present a large and variable resistive load to the oscillator. The use of an 829B high-current tube in the circuit has been tried but even with this the percentage frequency change is not large enough.

VIII. B. MICROWAVE MODULATION TECHNIQUES

Staff: L. D. Smullin
W. N. Coffey

The program of investigating silicon crystals as modulating devices is nearly complete and will be reported soon. The results may be summarized briefly in terms of the maximum power available in each sideband (without regard to linearity), the sideband power at the limit of the region of linear dependence upon i-f drive, and the conversion loss from i-f to sideband power. Measurements were made on 1N21B crystals drawn from stock, some special 1N28 crystals of the type used in the B.T.L. Boston-New York relay link,[1] and some special high-voltage silicon crystals.[2] In the following tables power is given in milliwatts, and the i-f modulating voltage (rms) is given as a subscript. The maximum sideband power is given for the optimum r-f and i-f driving levels.

TABLE VIII-1. Modulation Characteristics of Various Crystals

Crystal	R-f drive	$(\text{Sideband Power})_{\text{mod. volt.}}$		Conversion loss
		Max. power	Max. linear power	
1N21B	214	11_{3}	$3\text{-}6_{1.5\text{-}1.9}$	
	85	$8_{2.2}$	$4\text{-}5.7_{1.5}$	11 db
Special 1N28	214	$4.6_{3.2}$	$2.6\text{-}3.3_{1.9\text{-}2.5}$	
High-voltage silicon	800	26_{18}	$15_{10.8}$	~ 14 db

C. STATISTICAL THEORY OF COMMUNICATION

Introduction. The theory of communication and control which Wiener presents in his work entitled "Cybernetics" (in press) and his NDRC Report of 1942 entitled "The Extrapolation, Interpolation and Smoothing of Stationary Time Series" embraces a very wide field of application extending well beyond the bounds of communication engineering.

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1. These were kindly furnished by H. T. Friis of the Bell Telephone Laboratories, Holmdel, New Jersey.
2. These were made for us at Sylvania Electric Products from a slab of silicon furnished by Professor W. E. Stevens of the University of Pennsylvania.

In communication engineering, a basic difference between the Wiener theory and the generally accepted one lies in the statistical concept of messages of Wiener and the steady-state and transient concepts of the current theory. In the attempt to apply the statistical theory, new equipment and new techniques are being developed for the study of the statistical properties of messages and noise. Practical applications will depend largely upon the results of this development. For instance, correlation functions are the basic design data for optimum linear systems in addition to being useful in the theory of information and in the study and comparison of modulation systems of various types. Again, Wiener has shown that the amount of information is equivalent to negative entropy in thermodynamics or other similar situations. The application of this result for the improvement of communication systems demands a thorough knowledge of the conditional probability distribution densities of the messages.

1. Autocorrelation Function: Electronic Correlator

Staff: Professor Y. W. Lee
Professor J. B. Wiesner
T. P. Cheatham, Jr.
W. B. Davenport, Jr.
E. R. Kretzmer
D. F. Winter

The design and construction of an electronic correlator has continued along the general line described in the last progress report, namely, that of taking advantage of the relative simplicity of pulse-sampling and multiplication of amplitudes at discrete points in time-amplitude coordinates.

The general problems and techniques involved are outlined in the block diagram and waveform description of Figs. VIII-3 and VIII-4.

Waveform (1) of Fig. VIII-4 shows a section of a random time function f(t), assumed to be a stationary time series. The autocorrelation function of f(t) is mathematically defined by Wiener as:

$$\varphi_{11}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_1(t)\, f_1(t \pm \tau)dt. \qquad (1)$$

In other words, the autocorrelation function is the mean or average relationship of the product of all points of f(t) separated in time by an amount $\pm \tau$. Theoretically, it is observed from Eq. (1) that the time interval over which this mean relationship is to be determined is infinite. If a random function of time f(t) actually required an infinite period of time to approach statistical equilibrium, an impossible obstacle

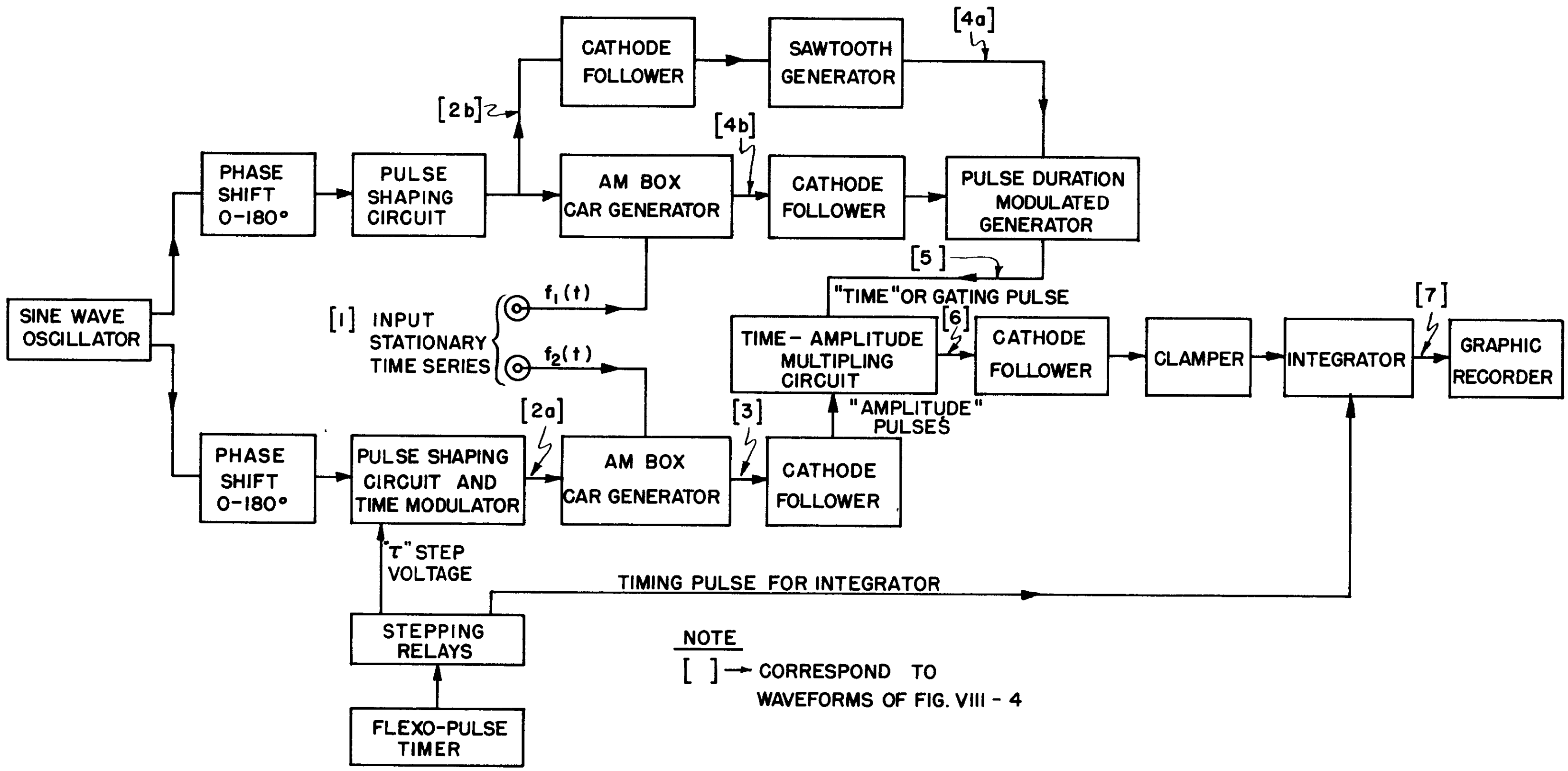
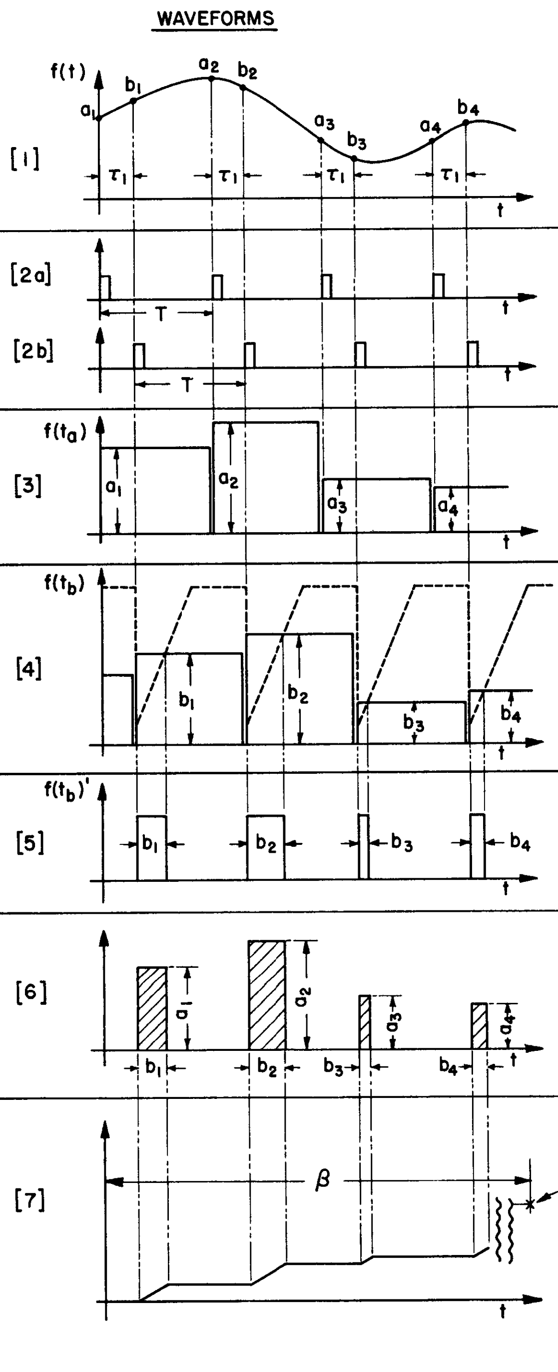


Fig. VIII-3. Electronic correlator.



DESCRIPTION

INPUT TIME FUNCTION, f(t), (STATIONARY TIME SERIES) WHICH IS TO BE CORRELATED.

TIMING PULSES FOR SAMPLING THE a_n POINTS OF f(t).

TIMING PULSES FOR SAMPLING THE b_n POINTS OF f(t).

BOTH SERIES OF PULSES DERIVED FROM A MASTER OSCILLATOR. τ IS OBTAINED BY LINEAR PHASE DELAY NETWORK FOR ONE FREQUENCY.

"BOXCAR WAVEFORM," $f(t_a)$ DETERMINED BY SAMPLED VALUES, a_n, IS THUS REPRESENTED AS AN AMPLITUDE PARAMETER.

[4a] SAWTOOTH SAMPLING WAVEFORM TO CONVERT BOXCAR WAVEFORM FOR POINTS b_n OF f(t) INTO WIDTH MODULATED PULSES.

[4b] "BOXCAR WAVEFORM," $f(t_b)$ DETERMINED BY SAMPLED VALUES, b_n OF f(t).

WIDTH MODULATED PULSES, TRAILING EDGES ARE COINCIDENT WITH t_{b_n} POINTS OF f(t), AND WIDTHS ARE PROPORTIONAL TO AMPLITUDES OF f(t) AT EACH b_n RESPECTIVELY. THE CLASS OF POINTS, b_n, IS THUS REPRESENTED AS A TIME PARAMETER.

OUTPUT WAVEFORM OF GATING CIRCUIT WHICH MULTIPLIES [3] AND [5]. AREA OF n PULSES IS PROPORTIONAL TO $\sum_n a_n b_n$.

$= \int_0^\beta [6]\, dt$. THIS IS A MEASURE OF $\phi_{11}(\tau_1)$, PROVIDING β IS TAKEN SUFFICIENTLY LONG TO APPROACH THE STATISTICAL EQUILIBRIUM OF f(t).

Fig. VIII-4.

In deciding on the design of a specific electronic correlator, one must fix within some limits, the following constants of the sytem:

$T \longrightarrow$ The period of sampling

$\beta \longrightarrow$ The period of each τ_n

$\alpha \longrightarrow$ The number of discrete "τ" points.

In the design of the present correlator, attention has been given to fixing the constants of the system so that speech may be included in the time functions possible of examination. One desired result of this research will be to fix the range of these constants for classes of stationary time series; that is, we should like to answer such questions as : How long is an "approximately" stationary time series for speech?

The autocorrelation function of a stationary time series, in addition to being of utmost importance in the synthesis of the optimum linear network for the various operations of filtering, predicting, differentiating, etc., has also an important relation to the general theory of information, its efficient coding, and rate of transmission.

Experimentally, work on the correlator has progressed through time-amplitude multiplying circuit, waveform (6). The integration and graphic recording operations are still to be completed.

2. Probability Distributions:[1]

a. Random Noise

Staff: Professor Y. W. Lee
Professor J. B. Wiesner
N. H. Knudtzon
T. B. Cheatham, Jr.
W. B. Davenport, Jr.
D. F. Winter

The statistical property of random noise which is to be investigated is the distribution of peaks in the instantaneous voltage, i.e., the probability distribution of the envelope.

Data already obtained from provisional electronic apparatus show interesting departure from the Rayleigh distribution. These results will now be checked and extended by means of improved equipment.

The block diagram of this new equipment is shown in Fig. VIII-5. The noise source is an 884 gas tube, which is supposed to give mainly

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1. Considerable assistance has been given by K. Boyer, Laboratory for Nuclear Science and Engineering, in the general solution to the circuit problems encountered in this section.

would exist in the experimental determination of $\varphi_{11}(\tau)$. Fortunately, it is expected that most stationary time series encountered in normal communications will approach "statistical equilibrium" in reasonable finite periods of time. Our first approximation in the laboratory is therefore to replace the infinite limits of the defining equation of $\varphi_{11}(\tau)$ with finite limits, the assumption being that statistical equilibrium is approached sufficiently close to introduce small or negligible error. A second approximation is to replace the integral sign with a summation sign, that is, instead of multiplying $f(t)\ f(t+\tau)$ simultaneously for all t (continuously) as a function of τ, we sample, determine, and multiply n discrete points of $f(t)_n\ f(t+\tau)_n$ for each discrete value of z. $\varphi_{11}(\tau)$ is then approximated by the following expression:

$$\varphi_{11}(\tau_k) = \frac{1}{n_\beta} \sum_n a_n b_n \Big|_{\tau_k}$$

where a_n and b_n are two corresponding points of $f(t)$ separated by τ_k, and β is the approximated period of the stationary time series; n is therefore a function of the repetition rate of the sampling pulses and of the period β.

A sine wave oscillator is used to derive the timing and sampling pulses (2a) and (2b). It is passed through a phase shift network to give an initial or coarse adjustment of τ. The (2a) timing pulses are time-modulated in discrete steps in order to vary τ. It would appear at first glance that one could time-modulate either (2a) or (2b); however, it can be shown that time-modulation of (2b) results in a decrease in the allowable range of τ to something less than one half the sampling period, T. The timing pulses and the random time function f(t), to be examined, are then fed into a box-car generator to derive the waveforms (3) and (4b). In this operation the amplitude of the a_n and b_n points are stored or held over a sampling period (T). The amplitude of (3) is therefore solely a function of the amplitude of f(t) at a_n. We have therfore our amplitude coordinate. To convert the stored amplitude of (4b) to a time parameter, a linear sawtooth (4a) is used to sample the box-car waveform of (4b). The result is a linear transformation to the width (time) modulated pulses of (5). Waveforms (3) and (5) are then placed in coincidence in a gating circuit, the output being (6). The integral of (6) over a period β will therefore give us a measure of $\varphi_{11}(\tau)$ for a specific τ_1. The total length of time required to determine a complete autocorrelation function is therefore equal to the product of β times the number of discrete intervals of τ selected.

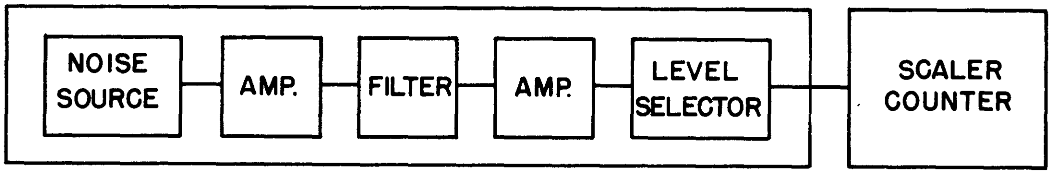


Fig. VIII-5. Block diagram of equipment for study of statistical property of random noise.

shot noise with a flat frequency spectrum over a relatively wide range (0 - 200 kc); sources for other types of random noise may, however, be inserted. The filter has a center frequency of 20 kc and a variable bandwidth. The scaler counts up to $2^{15} \times 10^{6}$ pulses at a rate of atleast 200 kc.

At the present time the counter is finished and works satisfactorily with a reasonable sensitivity both for pulse amplitude and width, A level selector to provide a large number of selectable levels over a sufficient dynamic range is being designed.

2. b. Instantaneous Amplitude Distributions of a Continuous Time Function in the Frequency Range between 50 and 10,000 cycles/sec

Staff: Professor J. B. Wiesner
Professor Y. W. Lee
D. F. Winter
T. P. Cheatham, Jr.
W. B. Davenport, Jr.

The circuit mentioned on page 52 (Sec. VIII.D.4.) of the last quarterly report was designed to measure (continuously) the time spent by an audio signal between V and V + ΔV volts. One version of this circuit has been constructed and tested. The circuit worked as expected. A second circuit has been designed, built, and is being tested. This circuit should provide more discrete levels for investigation than the first one.

2. c. Amplitude and Conditional Probability Distributions of a Quantized Time Function

Staff: W. B. Davenport, Jr.
T. P. Cheatham, Jr.
D. F. Winter

An attack differing from those discussed by Winter in the last progress report (see Sec. VIII.D.4.) has been made on the problem of experimentally determining the amplitude probability distribution of a stationary time series such as the human voice.

A block diagram of the present system is given by Fig. VIII-6. Basically the method used is as follows: the function of time being

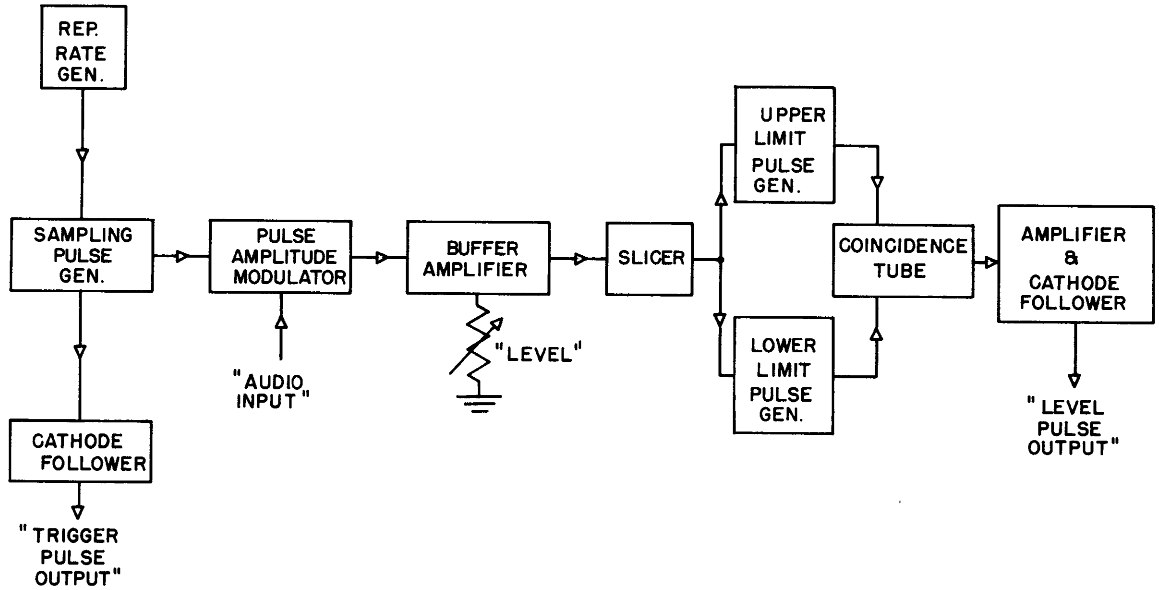


Fig. VIII-6. Amplitude distribution analyzer.

studied is converted into a series of pulses of varying amplitude but of constant duration and repetition period. The constancy of repetition period is a convenience rather than a necessity. The number of pulses in a given amplitude level are then counted. The probability of occurence of a level L_1 is:

$$P(L_1) = \frac{\text{number of } L_1 \text{ level pulses}}{\text{number of trigger pulses}} \tag{1}$$

where both numbers are determined over the same interval of time.

The maximum duration, δ, of the sampling pulse is determined by the worst possible case, where the signal momentarily consists of a maximum amplitude, E_{max}, sine wave whose frequency, $1/T_s$, is that of the highest frequency component of the signal being studied. This signal passes through a level, ΔE, in the shortest possible time when the sine wave passes through zero. At this time, if the level is small enough, the sine wave and its tangent are indistinguishable, and the following relation is valid:

$$\frac{\delta}{T_s/4} \leq \frac{\Delta E/E_{max}}{\pi/2} \tag{2}$$

In the present system, $\delta = 0.5$ μsec. Thus 50 levels may be obtained with a highest signal frequency component of 12.7 kc.

In reference again to Fig. VIII-1, the method of determining when a pulse, of amplitude E_p, is in a given level is as follows: Let E_1 be the triggering level of the lower limit pulse generator and E_2 be the triggering level of the upper limit pulse generator. There are three cases:

(a) $E_p < E_1$. Neither pulse generator operates and no output results.

(b) $E_1 < E_p < E_2$. The lower limit pulse generator forms a positive pulse occuring 2 μsec after the **sample** pulse reaches E_1. This pulse passes through the coincidence tube to the output.

(c) $E_2 < E_p$. When the sample pulse reaches E_1, a positive pulse 2 μsec later is again applied to the coincidence tube. However, when the sample pulse reaches E_2, a 4-μsec negative pulse is applied to the coincidence tube, preventing the lower limit pulse from being transmitted to the output. This process provides cancellation even though the sample pulse reaches E_2 at a later time than it reaches E_1.

Correct operation of the level discriminator requires that each of the limit pulse generators trigger at the same amplitude level every time and form the same duration pulse every time. As a rather short (7 to 10 μsecs) sample pulse repetition period is used, this places rather stringent requirements on the limit pulse generators. A circuit achieving this result is a modification of the conventional cathode-coupled multivibrator (see Fig. VIII-7). The grid of V2B is set so that the cathode

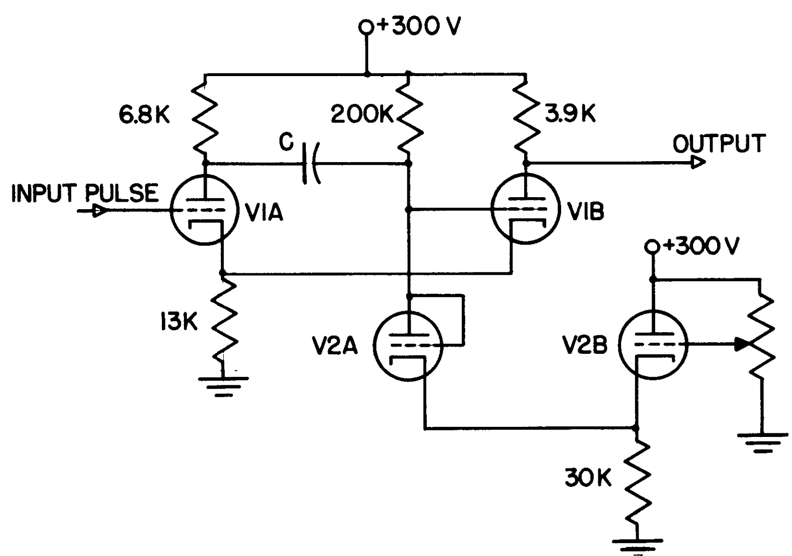


Fig. VIII-7. Limit pulse generator.

of V2A is at the same potential as the plate of V2A when V1 is in its quiescent condition. When a sample pulse triggers V1, the plate of V2A is driven negative with respect to its cathode and V2 is effectively out of the circuit. However, at the end of the V1 pulse, the plate of V2A is drawn positive with respect to its cathode, and the condenser, C, charges through the 6.8 K load of V1A, the resistance of V2A, and the output impedance of the cathode follower V2B. In a normal cathode-coupled MVBR circuit, C charges through the 6.8 K load of V1A and the input impedance of V1B when its grid is conducting – a much higher impedance than that of the compensated circuit. For example, with the values given and C = 50 μμfd, it was experimentally determined that C completely charges in about 1 μsec with the compensating circuit V2, and requires about 7 μsecs without V2. With such a circuit, the upper limit pulse generator (which generates a 4-μsec pulse) returns to its quiescent condition completely about 5 μsecs after being triggered.

A breadboard model of the circuit of Fig. VIII-6 has been built and tested, and a final model is under construction, and will be used to obtain data on the human voice. Also the pulse-sampling method of probability distribution is being extended to the problem of experimental determination of conditional probabilities.

3. Optimum Prediction

Staff: Professor Y. W. Lee
R. Cohen
T. P. Cheatham, Jr.

Studies of the fundamental requirement for prediction synthesis have been completed, and will soon be published in a technical report. Figure VIII-8 is a tabulation of the most significant results.

From a qualitative point of view, it may be said that: (a) a signal whose first derivative reaches infinite values is unpredictable, (b) prediction is possible if at least the first derivative of the signal remains finite, and (c) the quality of prediction performance that may be expected increases when derivatives of increasing orders of the signal are constrained to remain finite. These characteristics of the signal derivatives are interpreted in the central structure of the autocorrelation curve; careful reproduction of this structure in the analytical work is the fundamental condition for an accurate predictor design. A common feature of "predictable signals", whose nature precludes, as indicated, the existence of an infinite first derivative, is that their autocorrelation curves have a zero initial slope. This fundamental result leads to a flat, slowly rising Wiener-error curve, since the latter must be tangent at the origin to the mirror image of the autocorrelation[1]. Degree

1. see Quarterly Report of April 15, 1948.

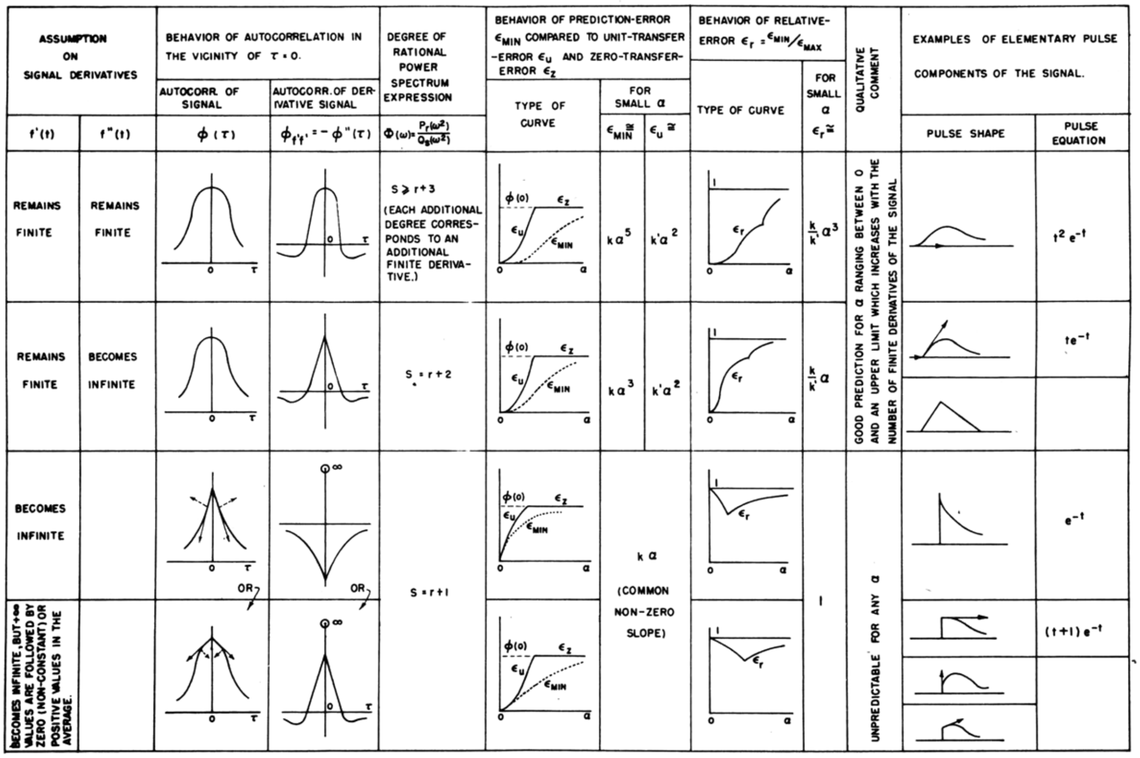


Fig. VIII-8.

of flatness depends upon the higher-order derivatives of the signal, whose behavior determines the shape of the central part of the autocorrelation: radius of curvature, slopes in the vicinity of the origin, etc. The severe requirements of this localized fitting of the autocorrelation curve, are interpreted by a Laplace Transform Theorem as mainly focussing the analytical study on the high-frequency components of the signal, which are therefore responsible for the prediction performance. It is therefore inadequate to take experimental data on power spectrum, since the solution of the prediction problem would require approximating with extreme accuracy the manner in which the power spectrum approaches zero for large frequency; moreover, however large the frequency range in which the experiment is performed, the most important data would still lie beyond that range. In practice, therefore, the power spectrum equation, representing the signal statistics in all of Wiener's analytical work, must be derived merely as the Fourier Transform of the autocorrelation curve.

VIII. C. 4. <u>Theory of Transmission of Information</u>

Staff: Professor R. M. Fano

Further progress has been made in the development of a theory of the transmission of information. A technical report (No. 65) on this subject is being prepared at this time and is expected to be published in the Fall.

5. <u>Pulse Modulation Studies</u>

Staff: E. R. Kretzmer

Progress is continuing on the experimental and theoretical study of interference between two pulsed carriers, such as are used in pulse-duration and pulse-position modulation (PDM and PPM). A technical paper on this subject was presented before the New England Radio Engineering Meeting (N.E.R.E.M.) on May 22, 1948. The following new items have been covered since the last progress report.

(1) Measurements of the various quantities encountered in common- and adjacent-channel interference of two PDM waves, as a function of all the variables involved, have been completed. Quantities measured include the random noise caused by the interference, the resulting signal-to-noise ratio, and the ratio of the two modulation signals at the output. Independent variables include the ratio of the two interfering signals at the input, the difference between their carrier frequencies, the pulse durations, and the system bandwidth.

(2) Construction of the PPM system (described in the Quarterly Progress Report of April 15, 1948) has been completed. The equipment has been put into working order and subjected to some preliminary qualitative interference tests.

(3) Preparations are now under way for an experimental study of two-path interference for PDM and PPM. A mercury delay line is being used to simulate the time lag in the second path (about $\frac{1}{2}$ millisecond). An additional variable which becomes important in multipath interference is the type of transmitter output, which can be either a pulsed r-f oscillator or a pulsed r-f amplifier. Only the latter allows the radio frequency in successive pulses to be coherent, so that the r-f phase difference between the two interfering signals is constant; when the pulses are generated by an oscillator, the phase difference is generally random. For this reason, the PPM transmitters (see above) and a new PDM transmitter now under construction, can be switched to either pulsed oscillator or amplifier operation.

(4) Theoretical work is continuing on the random noise which results from either common-channel, adjacent-channel, or multipath

interference. This is a rather involved function of many variables. It can be shown that the noise is due to two causes: (a) random jitter in the pulse edges and (b) occasional randomly missing pulses. The problem of finding the spectrum and power of the noise associated with either of these two phenomena has been solved through the use of the autocorrelation function (see Quarterly Progress Report, April 15, 1948, p. 48), which has proved to be a very useful tool in this work. It remains to find the functional relationships between the noise and all the variables involved, so that the measurements mentioned under (1) can be checked. This has already been done to a limited extent.

6. Effects of Transit Angle on Shot Noise in Vacuum Tubes

Staff: G. E. Duvall

Since the last progress report, stable low-noise amplifiers have been constructed and put into operation at frequencies of 30 and 60 Mc/sec. The noise figures obtained, with 6AK5 triode-connected input tubes, are within a few percent of theoretical values. The input circuits are loaded with about 10,000 ohms, the loading thus allowing noise of a level corresponding to a few microampers of diode current to be measured. The factor, Γ^2, by which the noise from a tube under test fails to equal $2eI\Delta f$ can be determined with a precision varying from $\pm$ 10 per cent at about 1 μamp of diode current to $\pm$ 1 per cent at diode currents greater than 100 μamps. The absolute accuracy is limited by the calibration of a step attenuator, which is known to within 2 or 3 per cent.

This equipment has been used to measure the noise produced by a saturated diode, the 2X2/879, at transit angles ranging from $\frac{1}{2}$ to 3 radians. In order to compare these measurements of Γ^2 with the theoretical values for zero space charge, Γ^2 was measured as a function of diode plate current and the resulting values were extrapolated to zero plate current. The resulting numbers, called Γ_o^2, are plotted as a function of transit angle in Fig. VIII-9. Theoretical curves for (A), a cylindrical diode with the same ratio of anode radius to cathode radius, and (B), parallel planes with the same transit time as the 2X2/879 are shown in the same figure. The discrepancy between the theoretical and measured values is attributed to secondary emission. The magnitude of the effect due to secondary emission is being evaluated, but computations have not yet been completed.

Values of Γ^2 for some negative grid triodes with space-charge-limited currents have also been measured and compared with the theory of

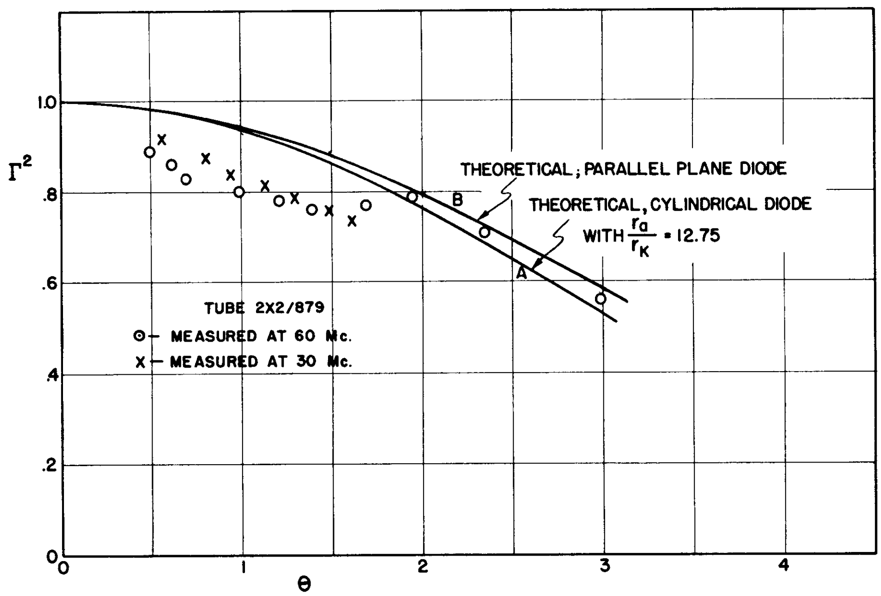


Fig. VIII-9. Transit time reduction of temperature-limited shot noise.

D. O. North.[1] The triodes that have been measured are the 6C4, the Eimac 15E, and a large triode especially designed for the purpose by H. J. McCarthy of this Laboratory. The 6C4 has been found to agree very closely with the theoretical values (Fig. VIII-10), whereas the 15E is

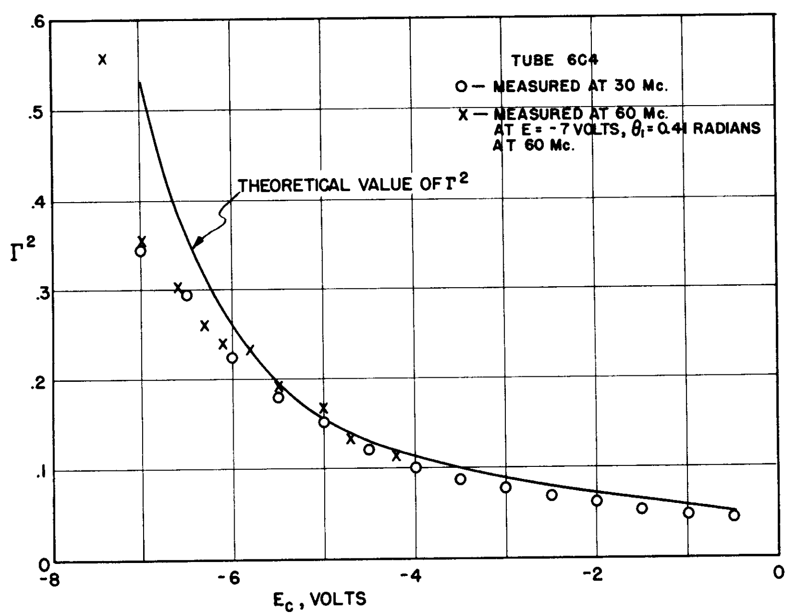


Fig. VIII- Space-charge reduction of shot noise.

1. B. J. Thompson, D. A. North, W. A. Harris, "Fluctuations in Space-Limited Currents at Moderately High Frequencies-Part II, RCA Review, 4, 441 (1940).

considerably more noisy than the theory would indicate (Fig. VIII-11). Measurements on the special triode have not yet been compared with the North theory.

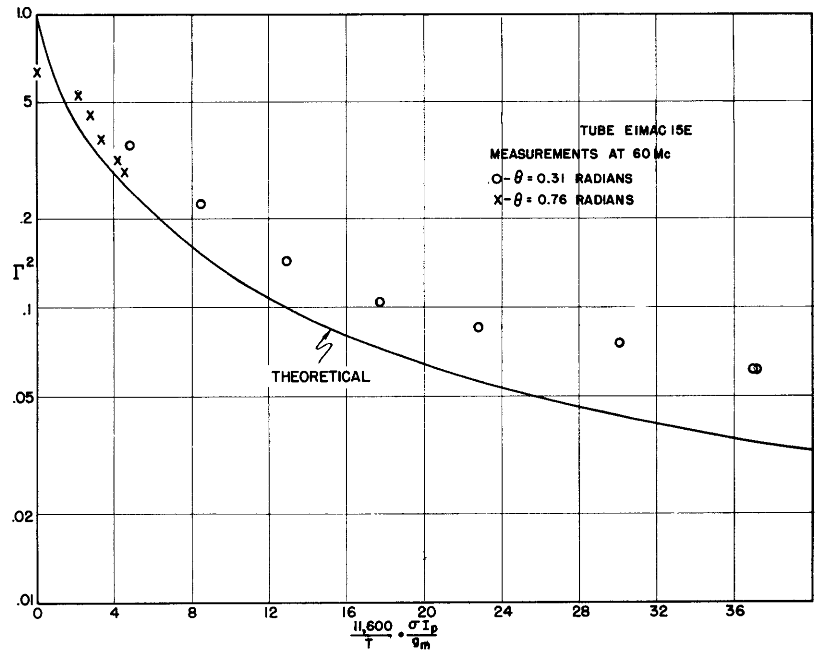


Fig. VIII-11. Space-charge reduction of shot noise.

Two CV172 diodes have recently been obtained through the cooperation of Captain R. E. Campbell, the Signal Corps Liaison Officer of this Laboratory. It is planned to use these tubes as standards when currents greater than 1 milliampere are needed to measure Γ^2.

D. TRANSIENT PROBLEMS

1. Envelope Studies

Staff: **Dr. M. V. Cerrillo**

Solutions for the amplitude and phase deviation functions of complete responses have already been obtained for the case in which the exponent $w(s,t)$ is not linear (see D.2. Transient Theories). These results will be given in RLE Technical Report No. 55. The case of the linear exponent is still under consideration. Solutions for the expressions of the rising slope of the main signal have already been obtained, and the investigation is looking toward a more complete mathematical solution which predicts the transient functions (amplitude and phase deviation) covering the different zones of formation.

VIII. D. 2. Transient Theories

Staff: Dr. M. V. Cerrillo

An extensive and detailed study of the theory of the aymptotic solution of integrals of the type

$$\int_{\gamma_s} F(s)e^{3(s,t)}ds$$

has been completed and the results are to be published in the near future as RLE Technical Report No. 55. This report will contain the mathematical aspect, the general procedures, and proper techniques to be used in connection with this method. A complete and detailed discussion on the subject will be included showing the classical method of attack as well as the new results, interpretations, and presentations obtained from the recent investigation.

3. Theory of the Synthesis of Networks for a Specific Transient Response

Staff: Dr. M. V. Cerrillo

A simple and powerful theory of network synthesis for a prescribed transient response has been partially developed by using asymptotic methods of solution of the integral of the type indicated in VIII. D.2. At present the method can be applied only to recurrent structures, but as soon as complete solutions of the functions mentioned in D.1. can be obtained, the method of attack can be extended to other circuit configurations. The progress in this connection will be reported on periodically. (See also RLE Technical Report No. 55.)

4. Response of Networks to Frequency Transients

Staff: Professor E. A. Guillemin
D. M. Powers

As described previously,[1] the method of saddle-point contour integration was first applied to the case of a linear frequency transient and a parallel tank circuit. This solution is nearly complete. The next function to be treated will be the sinusoidal frequency-modulation function.

The direct experimental attack, begun in the past, involving the setting up of equipment to picture the actual transient for a

1. RLE Quarterly Progress Report, April 15, 1948, p. 54.

particular network and applied function has been continued. In addition, work has begun on another approach, one whose purpose is to discover experimental ways of justifying the steps of the contour-integration process. It is hoped that this will lead to more simple and direct methods of examining a network for its response.

E. ACTIVE NETWORKS

1. General Theory

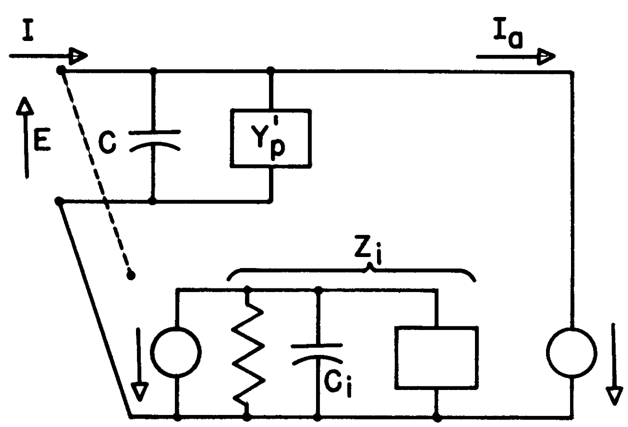
Staff: Dr. M. V. Cerrillo

All effort has been concentrated on the preparation of RLE Technical Report No. 55 and on work of Sec. D.1. For the moment, not much advance has been made in the general theory of active networks.

2. Broadbanding of Amplifiers by the Use of Active Elements in the Interstage

Staff: Professor E. A. Guillemin
J. G. Linvill

The last progress report suggested that the active network illustrated below be used as a two-terminal interstage, and that such a network can exhibit a larger constant magnitude of impedance over a specified frequency range than a passive network with its associated parasitic capacitance. Several limitations of the configuration of Fig. VIII-12



$$\frac{I}{E} = Y = Y_p + Y_a$$

$$Y_a = \frac{I_a}{E}$$

Fig. VIII-12. Equivalent circuit of active interstage.

were mentioned, among them the fact that the integral of $\mathrm{Re}Y_a$ over the whole frequency range is limited by C_i and the transconductance of the tubes. Since the last report, a more satisfactory method has been found for splitting a prescribed Y into Y_a (negative real) and Y_p(positive real), and its limitations evaluted. Unfortunately, it turns out that this network configuration becomes less effective in maintaining a large magnitude of input impedance over a continuous range of frequencies as the width of this range becomes greater. Consequently, attention was directed toward modifying the circuit configuration to overcome these limitations.

An appropriate modification of the network described in the previous progress report is to include in the active part more than two vacuum tubes in cascade. It is appreciated that with such a configuration Y_a (or I_a/E) is no longer a negative real function (its phase may shift more than 90°) and a different synthesis procedure must be applied. Necessary and sufficient conditions that Y_a be the admittance (I_a/E) of such a network have been worked out. Further, a method of splitting a prescribed Y into Y_p, a positive real function, and Y_a, a function satisfying the conditions mentioned, has been developed. By this method one can synthesize an impedance approximating any constant magnitude over an arbitrarily wide finite frequency range. Theoretically, there is no gain-bandwidth limitation for amplifiers using such interstages.

The process for synthesizing an interstage impedance of this nature may be summarized by the following three steps:

(1) Finding a ratio of Hurwitz polynomials which has the desired magnitude over the specified frequency range and approaches $1/C\lambda$ as $\lambda \longrightarrow \infty$.

(2) Splitting the reciprocal of this function (an admittance) into a positive real function which has a pole at infinity and a remainder which is realizable as I_a/E for a chain of amplifiers.

(3) Realizing the passive and active networks which have the admittances found in the second step.

Methods have been developed for carrying through each of these steps but more thought is needed to making them less cumbersome. The working out of several examples and the design, construction, and test of an experimental model are contemplated.

F. HIGHER MODE PROBLEMS

1. Steady-State Propagation of Electromagnetic Waves along Cylindrical Structures

Staff: Professor L. J. Chu
R. B. Adler

The work outlined in the last progress report has been continued along the lines indicated therein. In particular, the fields about an infinite lossless dielectric cylinder containing a dipole on its axis have been examined in some detail, especially for the case in which the dipole is polarized parallel to the cylinder axis.

The problem has been investigated by first expanding the localized dipole source into a set of infinite line sources, with relative amplitude and phasing given by $F(\beta)e^{-j\beta z}$. This expansion has been accomplished by using a Fourier Integral with respect to the longitudinal z-coordinate, and amounts to finding the transform of a narrow pulse located at $z=0$. The fields excited by each such distributed source can be found as a solution to the boundary-value problem involving the rod, with the proper discontinuity on the cylinder axis. Finally, the total field is represented as the superposition of all these component fields, by using the synthesis integral with respect to the phasing constant β. From the resulting integrals, it is possible to give further interpretation to the "guided cylindrical modes" which were referred to in the previous quarterly report. Waves of the type in question were mentioned there as solutions to the rod problem with no sources present; or more accurately as a result only of possible sources at $z = -\infty$. They will be called "free modes" in the ensuing discussion. Such free modes now appear in the dipole problem as poles of the integrand in the synthesis integral. The corresponding values of β, which are equal to the "natural propagation constants" of these free modes, are dictated only by the dielectric constant and radius of the rod, for the mechanism of propagation is one of successive internal critical reflections from the rod boundary. Once excited, these free modes are trapped within the lossless rod, and persist indefinitely, even for infinitely large values of z. In fact these may be looked upon as "resonance" modes, as far as propagation in the (longitudinal) z-direction is concerned, because they comprise the only part of the solution which does exist for large z. Since they die out exponentially with radius, however, these cylindrical waves do not contribute to the radiation field at large distances from the source in the radial direction. The radiation field actually comes about from a major part of the rest of the general integral describing the over-all fields about the cylinder. The latter contribution to the solution is confined to a much smaller portion of the z-axis, the exact extent of which requires a numerical analysis not yet completed. It represents essentially a "diffracted" dipole field, which therefore accounts for the radiation properties of the infinite rod as an antenna.

It is now clear that the free cylindrical modes, which originally prompted this investigation, form only a portion of the solution to the rod problem; in fact it is possible to construct the rod in such a way that none of these resonance modes is excited by the dipole, in which case the only important contribution to the total result is the diffracted dipole field. The situation is in some ways similar to the transient behavior of a network which is driven by a very narrow pulse. Any undamped "natural frequencies" which could exist in the network without sources (as a result of some initial condition, for example) will now appear as oscillations in the response long after the driving pulse has disappeared; but there is a considerable portion of the response a short time after the application of the impulse which may be of great importance in the applications of the network. The Fourier Integral Method of discussing such transient problems parallels closely the analysis applied above to the dipole field.

A similar investigation of the fields set up by a dipole polarized in a direction transverse to the cylinder axis has been attempted. The qualitative aspects of the resulting integrals appear very similar to those of the simpler case described above, but the added complication in detail has made a numerical discussion prohibitive for the time being.

Certain general conclusions may be drawn about "free modes" on cylindrical structures with an "open boundary", in which radiation becomes a significant part of the problem. It is shown by the above examples that the fields which can exist about the structure depend so heavily upon the particular source distribution considered that discussion which limits itself to free modes of the "guided" cylindrical type must necessarily be extremely incomplete, if not misleading. In this connection, for example, it has occasionally been suggested that a complex propagation constant may occur for such a free mode in a completely lossless system, and that it thereby accounts for a transfer of power from the flow in the z-direction into the form of outward radiation, much in the manner of a leaky water-pipe. It is not difficult to show however that in the absence of dissipation throughout the system, any free cylindrical mode which can exist must have an entirely imaginary propagation constant. That is, no free cylindrical mode which can exist without sources on the axis can account for <u>purely radiation losses</u>. If, on the other hand, there is dissipation in the outside medium, a complex propagation constant does result, and it then accounts for power leaving the lossless medium to supply the <u>dissipative losses</u> outside.

As a consequence of these general results, certain properties of the strictly "free modes" on a lossless dielectric rod should be pointed out. It has been shown in the course of this investigation that these modes can only have purely imaginary propagation constants in the z-direction. There are none which damp out exponentially in z, and hence none which behave like the modes beyond "cut-off" in an ordinary waveguide. A mode which propagates on a given rod at a certain frequency may cease to propagate when the frequency is made too low, but it simply ceases to exist then as a cylindrical wave. It does not become a "cut-off" mode in the usual sense. At first surprising, this result is easily visualized in terms of the plane waves which can make up any particular mode. When the mode propagates, these waves are critically reflected every time they strike the rod boundary. As the frequency is lowered, the component plane waves strike the boundary more nearly on the normal, and when the angle of incidence becomes less than the critical angle, the entire field structure must change to allow for the radiated power. In order to get "cut-off" modes in the usual sense, a closed boundary is necessary. Such modes would then be caused by waves which bounce back and forth in the transverse plane, because they cannot propagate longitudinally, and are unable to leave the system radially.

At present, further work is being conducted on the waveguide partially filled with dielectric. It is hoped that the results previously reported may be somewhat extended, and that they will then lead naturally into a discussion of the problem of the waveguide containing an electron beam.

2. Transmission Through Large Metallic Waveguides

Staff: Professor L. J. Chu
T. Moreno

No progress to report.

G. LOCKING PHENOMENA IN MICROWAVE OSCILLATORS

Staff: Professor J. B. Wiesner
E. E. David, Jr.

Studies of the behavior of synchronized oscillators when operated as pulsed generators have been completed. It was found that if the starting transient is short compared to the pulse duration, then the steady-state locking theory, as derived in Technical Report No. 63, is valid. In the case of the magnetrons under test the estimated starting time is of the order of 3×10^{-8} seconds.

Examination of the effect of a shift in the natural oscillator frequency on the locking action has been made. So long as this shift remains small, its effect is merely a change in the locking phase. Such frequency shifts are caused principally by power supply ripple.

A technical report will be published on the results of this work.