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RECENT DEVELOPMENTS IN FREQUENCY STABILIZATION OF MICROWAVE OSCILLATORS

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RECENT DEVELOPMENTS IN FREQUENCY STABILIZATION OF MICROWAVE OSCILLATORS

by

W. G. Tuller, W. C. Galloway, and F. P. Zaffarano

Abstract

A system developed by R. V. Pound for frequency stabilizing a microwave oscillator is discussed. The frequency range of the Pound stabilizer is considered and a change in the microwave circuit is described which increases the usable frequency range of the stabilizing circuit. This new circuit is discussed in detail, and a graphical method of obtaining the frequency versus voltage characteristic is presented. The effect of harmonic distortion in the output of the stabilizing circuit when the microwave oscillator is frequency modulated is discussed and the amount of the distortion is evaluated.

RECENT DEVELOPMENTS IN FREQUENCY STABILIZATION OF MICROWAVE OSCILLATORS

It is the purpose of this paper to present some recent developments in the field of microwave frequency stabilization. Work has centered on a stabilization system developed by R. V. Pound[1], and in particular on the microwave frequency-discriminator which it incorporates. Emphasis has been placed not on improving the already high stability of this system but on developing the stabilizer into an engineering system, free of involved tune-up procedure and dependence on the uncontrolled characteristics of microwave tubes and crystals.

1. The Pound Stabilizer

A block diagram of the original Pound system is given in Fig. 1. Also indicated in this figure is a convenient way of plotting the detected discriminator output voltage as a function of frequency on an oscilloscope. Power from the microwave oscillator divides at the magic T between the two adjacent arms. The useful energy component reflected from the reference frequency cavity goes to the modulating crystal, which reflects the "carrier frequency" energy incident on it in the form of two sidebands at carrier frequency plus and minus the modulating frequency, suppressing the carrier.

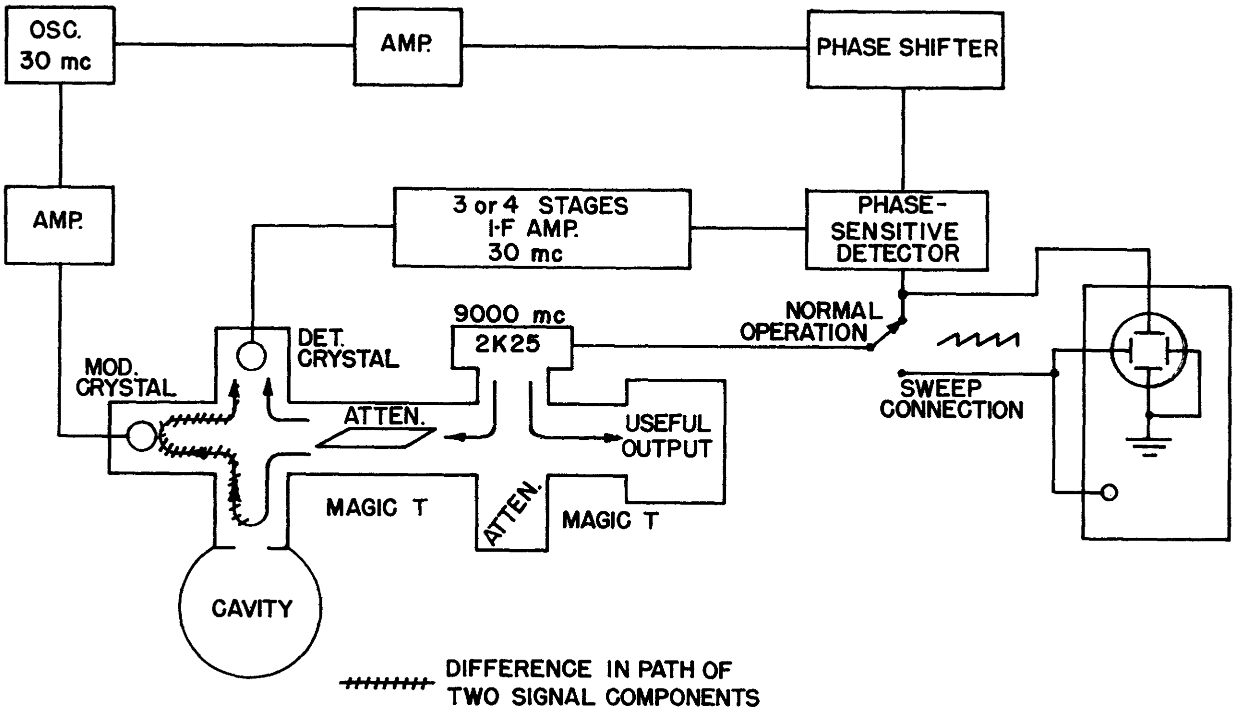

Figure 1. Block diagram - Pound stabilizer

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[1] R. V. Pound - "Electronic Frequency Stabilization of Microwave Oscillators", Rev. Sci. Inst., Vol. 17, pp. 490 - 505 (1946).

These sidebands, combining at the detector crystal with the carrier frequency signal entering directly from the source arm of the magic T, give rise to a modulation frequency signal at the detector crystal output. It is convenient to consider the detector crystal output signal as having a magnitude which is a function of the imaginary component of the reference cavity reflection coefficient, and a phase which shifts by 180° as the frequency of the microwave source goes through the resonant frequency of the reference cavity. After amplification in the modulation frequency (30 Mc) amplifier, the discriminator output is compared in phase with the signal applied to the modulating crystal. The phase-sensitive detector employed for this purpose produces a voltage output which, it has been shown, closely approximates the following relation:

$$E = \frac{\Delta}{1 + \Delta^2} \qquad (1)$$

for a matched cavity, where

Δ = Q df/F
Q = unloaded Q of cavity
F = resonant frequency of cavity
df = deviation from frequency F.

Deviations from the theoretical waveform make Fig. 2 typical of the discriminator curves obtained with the Pound circuit. Figure 2 is a photograph of an oscillograph produced with the circuit of Fig. 1 by sweeping the frequency of a 9000-Mc oscillator over a 60-Mc frequency range (cavity Q = 9000). These deviations are undesirable if, when the stabilizer is switched on, it is to lock the oscillator on the correct frequency without further attention. Their major cause is mismatch of the detector crystal at the edges of the oscillation mode of the klystron, where there is low power output and hence low crystal current. A d-c bias of 0.25 volt across the crystal in such a direction as to increase crystal current will stabilize crystal impedance and will permit the improved curve of Fig. 3 to be realized.

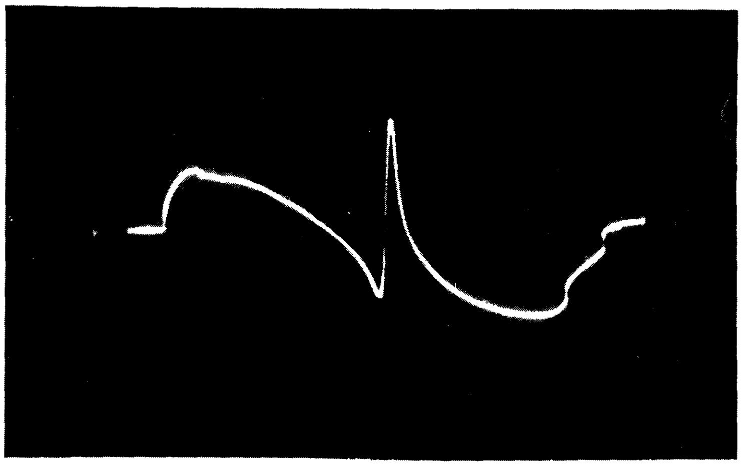

Figure 2. Typical discriminator curve - Pound system, no crystal bias.

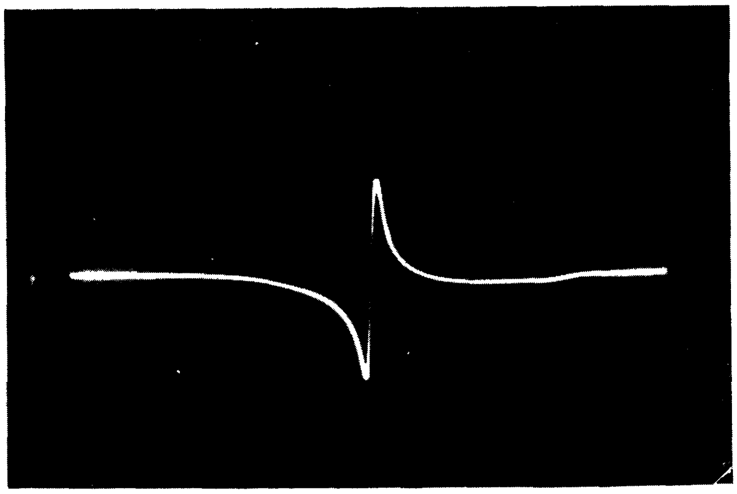

Figure 3. Typical discriminator curve - Pound system, biased crystal.

Pound's system, while representing a marked advancement in stabilizing technique, has an inherent limitation in that critical phase relations within the waveguide discriminator depend on the difference in electrical length between two waveguide paths of considerably different physical length. The effect of this path length difference may be resolved into three separate phase shifts, two of which are functions of the microwave frequency. The most important of the factors will be evaluated first.

A carrier frequency component of guide wavelength ℓ will have a new guide wavelength, $\ell \pm d\ell$, when the frequency is changed. After traversing n wavelengths the total apparent phase shift in this component will be $nd\ell$. In terms of more tangible factors, if F is the microwave frequency in cycles per second, dF is the total change in F, L is the length of the difference path in centimeters, c is the velocity of light, and f_c is the cut-off frequency for the waveguide used, the phase shift of the carrier in degrees, ϕ_1, caused by a frequency change, dF , is:

$$\phi_1 = \frac{360 \cdot F \cdot dF \cdot L}{c\sqrt{F^2 - f_c^2}} \qquad (2)$$

or, for 1" by ½" waveguide (.050" wall) at 9000 Mc, ϕ_1 is 0.0175 degrees per centimeter per megacycle.

Stated in other terms, the theoretical maximum range of frequencies over which the Pound circuit can operate, even with the waveguide arm lengths reduced to 4 cm each, is less than 640 Mc at 9000 Mc. (Note from Fig. 1 that the total path length differential is 16 cm in this case).

The two other phase shifts, which are of negligible importance in any practical discriminator of this type, can be evaluated by considering the waveguide section through which sidebands must travel. The phase shift between the sidebands in this section, ϕ_2, also determines whether the signal at the detector crystal is a phase or amplitude modulated wave, and is a function of the modulating frequency.

The rotating vector representation of a pair of sidebands, B_h and B_ℓ, and a carrier, A, is shown in Fig. 4. Assume that, as shown in Fig. 4, the phase relations are proper for amplitude modulation sidebands when the wave enters a short section of waveguide. In traversing one wavelength of the guide (for the carrier) the sidebands will be shifted in phase relative to the carrier, and an exaggerated vector diagram will appear as in Fig. 5. Note that the lower sideband, B_ℓ, has shifted through a smaller angle than the carrier, while the upper sideband, B_h, has shifted through a larger angle than the carrier.

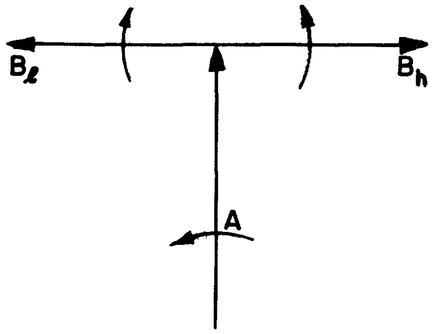

Figure 4. Relation of vectors entering waveguide.

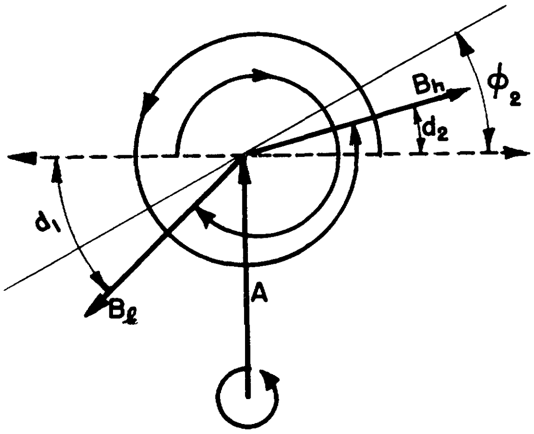

Figure 5. Exaggerated vector relations after one guide wavelength.

Also note that the angles d_1 and d_2 are not exactly the same because the wavelength in the guide changes more rapidly as the cut-off frequency is approached. The angle, ϕ_2, then, represents the amount by which the phase relation of carrier to sidebands has varied in one wavelength of guide. The angle ϕ_2 can be evaluated as

$$\phi_2 = \left[\sqrt{f_h^2 - f_c^2} - \sqrt{f_\ell^2 - f_c^2}\right] \times \frac{180 \times L}{c} . \qquad (3)$$

For a modulating frequency of 30 Mc, a center frequency of 9000 Mc, and with 1" by ½" waveguide,

f_h = 9030×10^6 cycles/sec
f_ℓ = 8970×10^6 cycles/sec
f_c = 6561.7×10^6 cycles/sec = guide cut-off frequency
c = 2.998×10^{10} cm/sec
ϕ_2 = 0.526 degrees per cm of waveguide.

An error of this type may be introducted by variations of the modulating frequency. For constant modulation frequency the total phase shift may be adjusted to the proper value by adjustment of the cavity arm length.

A third phase error, ϕ_3, is found from the derivative of ϕ_2 with respect to the microwave frequency, which is given by

$$\frac{d\phi_2}{df} = \left[\frac{f_\ell}{\sqrt{f_\ell^2 - f_c^2}} - \frac{f_h}{\sqrt{f_h^2 - f_c^2}}\right] \times \frac{180 \times L}{c} . \qquad (4)$$

With a 30-Mc modulating frequency, $f_c = 6561.7 \times 10^6$, and the carrier frequency = 9000×10^6, ϕ_3 = 0.0666 degrees per cm per 1000 Mc. This ϕ_3 is so small as to be completely negligible in any discriminator circuit investigated.

2. The Equal - Arm Discriminator

The equal-arm discriminator shown in the block diagram of Fig. 6 was first used by one of the authors (FPZ) in January, 1946. It was not until this research was well along that it was discovered that there were beneficial effects achieved in addition to the broadbanding of the device by the equalization of the internal signal paths.

The physical similarity of the equal-arm discriminator to the Pound discriminator is deceiving. The only external indication of a change is that the modulating crystal and detector crystal positions have been interchanged. There are, however, six fundamental differences in operation which bore investigation and will be discussed in sequence. These are:

(1) The frequency range has been extended because of path equalization.
(2) Reflections which can cause spurious signals are eliminated.
(3) Sideband energy is transmitted to the detector crystal with greater efficiency giving increased over-all discriminator sensitivity with the same input power.
(4) Power input may be several db greater for the same detector crystal noise.

(5) Large signals at twice the intermediate frequency are greater when the microwave source frequency is near the resonant frequency of the cavity.

(6) The carrier energy rather than the sideband energy is a function of frequency; thus the detector output varies as a function of the carrier-to-sideband ratio.

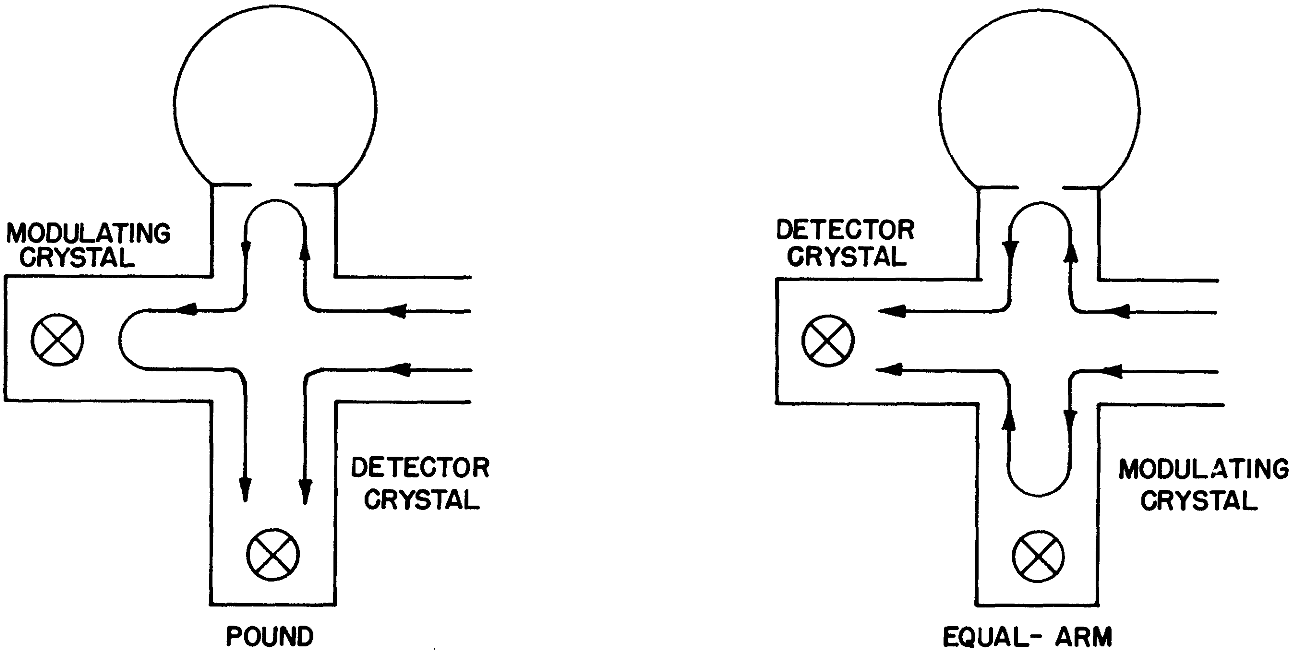

Figure 6. Comparison of discriminator circuits.

<u>2.1 Increased Frequency Range.</u> By reference to Fig. 6 it may be noted that, in the equal-arm discriminator, the waveguide path via the modulating crystal (which produces the sideband energy) can be made equal to the path of the carrier energy reflected from the cavity. This means that the relative phases of the two energy paths may be made constant over wide frequency ranges -- that the discriminator is no longer limited to the narrow band of operation of a single-control Pound system. In the experimental work it was found possible to make a fixed tuned discriminator whose range was greater than that of the available microwave source (1100 Mc in 9000). By comparison a fixed-tuned (i.e. fixed waveguide lengths) Pound discriminator, with all path lengths minimized, covered less than 500 Mc range, 350 being a typical value.

At 9000 Mc the minimum path length difference obtainable in the Pound arrangement is about 16 cm which allows 4 cm for the modulating crystal termination and a similar length for the cavity arm.

These measurements, in the case of a TE_{011}-mode cavity, are made between the center post of the magic T and the cavity side of the coupling iris. The effective modulator crystal arm length has not been measured under operating conditions. A few d-c measurements indicate that the length should be measured from center post to some point between the crystal and the end plate, but their validity under operating conditions is questionable.

The theoretical frequency range calculations check to within 20 per cent of measured values when the path length difference is increased to 60 cm. Since they do not check as well for small path length differences, one is inclined to suspect fringing effects at the magic T junction as well as an uncertainty in the exact location of the effective position of the modulating crystal.

2.2 Elimination of Reflections. In the new circuit the power reflected from the modulating crystal divides between two matched loads, the detector crystal and source arm. This is in distinct contrast to the Pound arrangement where the sideband power divides between the detector crystal and cavity. The cavity is far from being matched at the sideband frequencies, and in Pound's system will reflect energy back into the modulating crystal for remodulation. The equal-arm system will reflect more sideband energy into the attenuator in the source arm than the Pound system, but if the source is connected into the discriminator through a magic T, the sideband energy getting into the useful output will be at least 43 db below the carrier level.

2.3 Increased Sensitivity. A very important feature of the new configuration is indicated by comparing the modulation frequency output with that of its predecessor. The same power input to each discriminator is assumed, of course.

The sideband energy suffers the same modulator crystal loss in each case, but in the equal-arm circuit traverses the magic T one time less and does not encounter the 6-db loss at the cavity. This is a total increase of 9 db over the Pound circuit. As will be seen, however, only about a 5-db improvement can be realized because of the influence of the carrier-to-sideband ratio.

2.4 Increased Power-Handling Capacity. In the new discriminator the power reaching the detector crystal via the modulating crystal loses 6 db from two traverses of the magic T. Three to ten additional db are lost in the modulating crystal, depending on the efficiency of sideband production. The total loss is therefore 9 to 16 db. If the cavity is well matched, the detector crystal current caused by the power reflected from the cavity in the vicinity of resonance is very small. By comparing this result with the total of 3-db loss in the Pound circuit (between source and detector crystal), it may be seen that an input increase of 6 to 13 db is possible before the same crystal current and, therefore, noise is produced in the equal-arm unit. Thus, if the microwave source can supply the power, even greater discriminator output than the 5 db increase previously mentioned can be realized. It must be remembered, of course, that the isolation between source and cavity must be great enough to prevent the reference cavity from "pulling" the oscillator.

2.5 Variable Carrier Amplitude. Considerable importance should be attached to the fact that it is the carrier rather than the sideband energy which is attenuated as the cavity of the equal-arm discriminator is varied through resonance. With normal modulating crystal efficiencies, the carrier-frequency energy arriving at the detector crystal is usually of smaller amplitude than the sideband energy. This is analagous to overmodulation at the detector crystal.

Two effects of this overmodulation are important: the production of large double frequency components of the modulation frequency when source and cavity frequency are nearly identical and, more important, a decrease in the effectiveness of the sideband energy in producing output at the intermediate frequency. The double frequency signal is of small concern unless it overloads the first tube of the modulation frequency amplifier. In this case it may be reduced easily by a filter network in the input to the amplifier. The second effect, however, produces a very definite change in the crystal output voltage as a function of the carrier-to-sideband ratio. The discriminator characteristic is therefore modified somewhat.

A Fourier analysis may be performed on the envelope of the signal impressed on the detector crystal to find the useful signal component. The result of such an analysis gives:

$$\frac{a_1}{2B} = \frac{2}{\pi}\left[\frac{A}{2B}\sqrt{1-\left(\frac{A}{2B}\right)^2} + \sin^{-1}\left(\frac{A}{2B}\right)\right] \qquad (5)$$

where B is sideband voltage, A carrier voltage, and a_1 crystal output voltage at modulation frequency. The quantity $a_1/2B$ is plotted in Fig. 7 as a function of A/2B, the carrier-to-total-sideband voltage ratio.

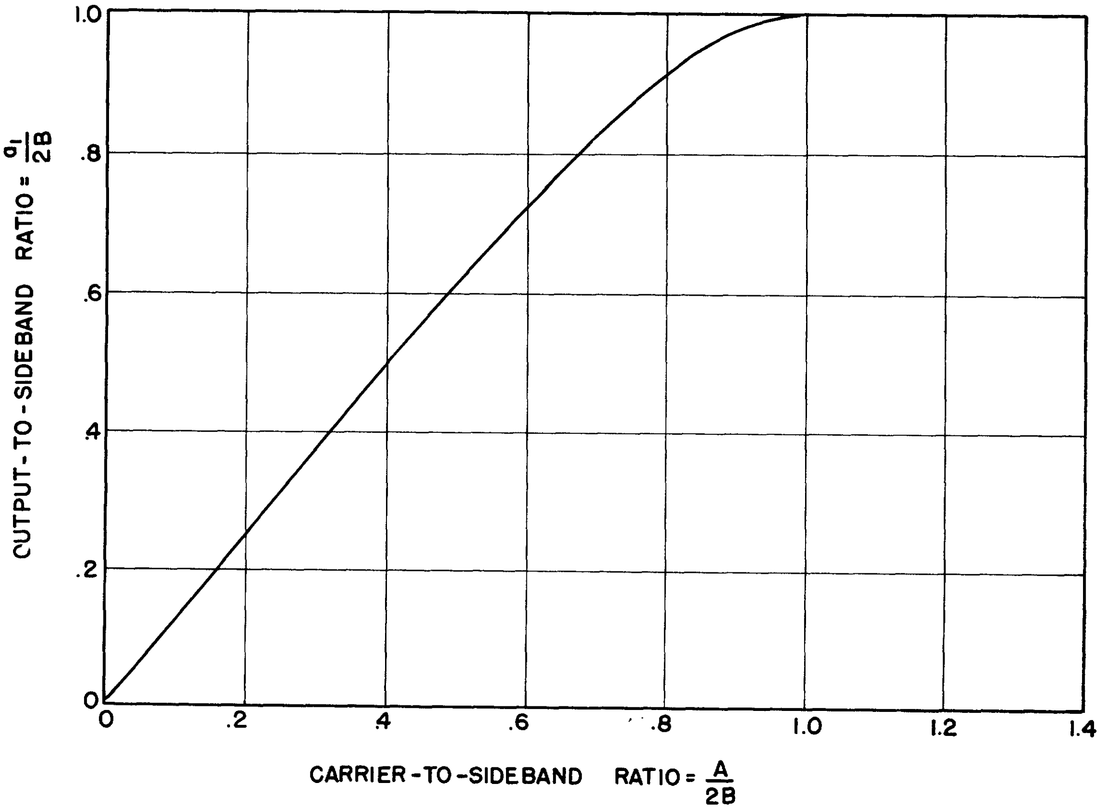

Figure 7. Result of Fourier analysis.

2.6 Calculation of Discriminator Characteristics. It has been found convenient to use a graphical method to calculate the characteristics of the equal-arm discriminator. The multiple graph shown in Fig. 8 allows rapid plotting of the discriminator curve for any modulating crystal efficiency.

At any desired value of Δ (the normalized frequency scale of Fig. 8a), the value of A, (the useful part of the cavity reflection coefficient), can be read from the dashed thin line curve. Figure 8b is a set of curves correcting for the fact that the ratio of carrier-to-sideband voltage A/2B depends on both the carrier-frequency component reflected by the cavity and the efficiency with which first-order sideband energy is generated. By entering the value of A obtained from Fig. 8a on the proper efficiency curve in Fig. 8b, it is possible to read off the value of A/2B which has resulted from the value of frequency error, Δ , chosen initially. To find $a_1/2B$, the efficiency with which a lossless detector crystal converts the modulated signal impressed on it to an output signal at modulation frequency, the value of A/2B found from Fig. 8b is transferred to Fig. 8c. Having determined $a_1/2B$, it is only necessary to multiply by 2B to get the output, a_1. The straight lines graphed in Fig. 8d determine the value of a_1 for any modulating crystal efficiency and ratio $a_1/2B$. To permit the value of a_1 to be transferred easily to Fig. 8a where it is plotted against the value of Δ to give the desired discriminator curve, a one-to-one transfer curve (shown as a double-dashed line) has been located on Fig. 8a.

The value of a_1 is found graphically for a crystal efficiency of 12.5 per cent at the point where Δ equals 0.5 in Fig. 8 to show the graphical construction involved. For this case a_1 is 0.457. Choosing several other values of Δ allows the entire curve for a modulating crystal efficiency of 12.5 per cent to be drawn. Other curves for different efficiencies may be drawn in a similar manner.

The slopes of the lines in Fig. 8b were obtained by the following reasoning:

The maximum value to which the ratio A/2B can rise is determined by the efficiency of the modulating crystal. The quantity A is limited by the reflection coefficient of the cavity and two traverses through the magic T (see Fig. 6) to the value

$$A = V_{input} \times 0.707 \times 0.5 \times 0.707 \qquad (6)$$

where V_{input} is the voltage into the discriminator.

Similarly the value of 2B is limited by the input voltage, the modulating crystal efficiency (Eff), and two traverses of the magic T. Note that the voltage of each sideband reflected from the crystal bears the following relation to the efficiency (which is defined on a power basis as P_r/P_i)

$$\frac{\text{Reflected Voltage of Each Sideband}}{\text{Incident Voltage}} = \sqrt{\frac{P_r/2}{P_i}} = \sqrt{\frac{Eff}{2}} \qquad (7)$$

where $P_r/2$ is the power in each of the two first-order sidebands reflected from the crystal and P_i is the total carrier frequency power incident on the crystal.

Accounting for the losses in the sideband path, therefore,

$$B = V_{input} \times 0.707 \times 0.707 \sqrt{Eff} \times 0.707 \qquad (8)$$

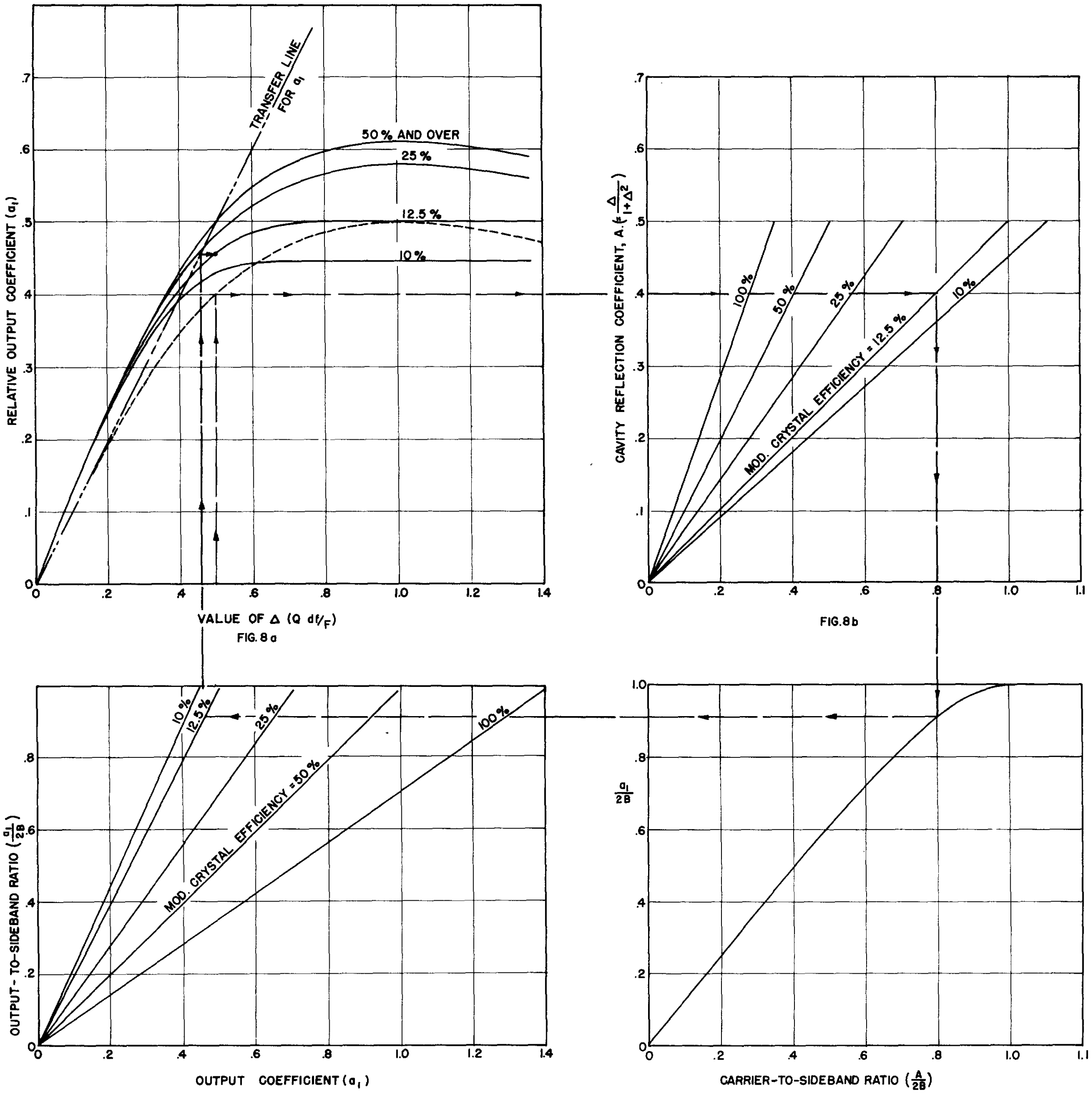

FIG. 8a

FIG. 8b

FIG. 8d

FIG. 8c

Figure 8. Graphical calculation of discriminator waveforms.

and the ratio

$$\left(\frac{A}{2B}\right)_{max} = \frac{.3535}{\sqrt{Eff}} . \qquad (9)$$

Since in Fig. 8a the maximum value of A has been chosen as 0.5, the slopes of the efficiency lines in Fig. 8b are found from

$$\frac{A}{\left(\frac{A}{2B}\right)_{max}} = \frac{.5}{.3535\sqrt{Eff}} = \sqrt{2} \times \sqrt{Eff} . \qquad (10)$$

The slopes of the efficiency lines in Fig. 8d are chosen so as to multiply the $a_1/2B$ scale of Fig. 8c by the proper value of 2B. From Eq. (6) and the fact that A_{max} has been defined as 0.5, V_{input} is equal to two. Using this result in Eq. (8)

$$2B = \sqrt{2} \times \sqrt{Eff} \qquad (11)$$

which could have been obtained directly from Eq. (10). Therefore, the slopes of the lines in Fig. 8d are

$$\frac{\left(\frac{a_1}{2B}\right)}{a_1} = \frac{1}{2B} = \frac{0.707}{\sqrt{Eff}} . \qquad (12)$$

3. Distortion

It has been mentioned earlier that the output of the phase-sensitive detector in Pound's system closely approximates the expression

$$E = \frac{\Delta}{1 + \Delta^2} . \qquad (13)$$

As a matter of fact this expression is exact for high efficiency crystals and operation between the peaks of the demodulator curve. This equation has been subjected to harmonic analysis to determine the harmonics present in the output of the phase-sensitive detector when the oscillator frequency is sinusoidally modulated. The result of this analysis is shown in Fig. 9, a plot of the harmonic distortion (in db below fundamental) as a function of the normalized frequency deviation (normalized with respect to the frequency deviation giving maximum peak-to-peak output). As is shown in this figure, distortion at low deviations is low enough to permit operation of multichannel frequency division multiplex systems without appreciable crosstalk from nonlinearity. At high deviations distortion is slightly greater than that present in the Foster-Seeley discriminator, but is still reasonably low for single channel operation. If the bandwidth of the feedback loop comprising the stabilizing system is wide enough to pass all components of the modulating wave, modulation may be applied in series with the output of the phase-sensitive detector. The stabilizer system will act to reduce the frequency modulation caused by the modulating wave, thus giving negative feedback. In the limit, the linearity of such a modulation system is the same as that of the microwave discriminator and the distortion will be as shown in Fig. 9.

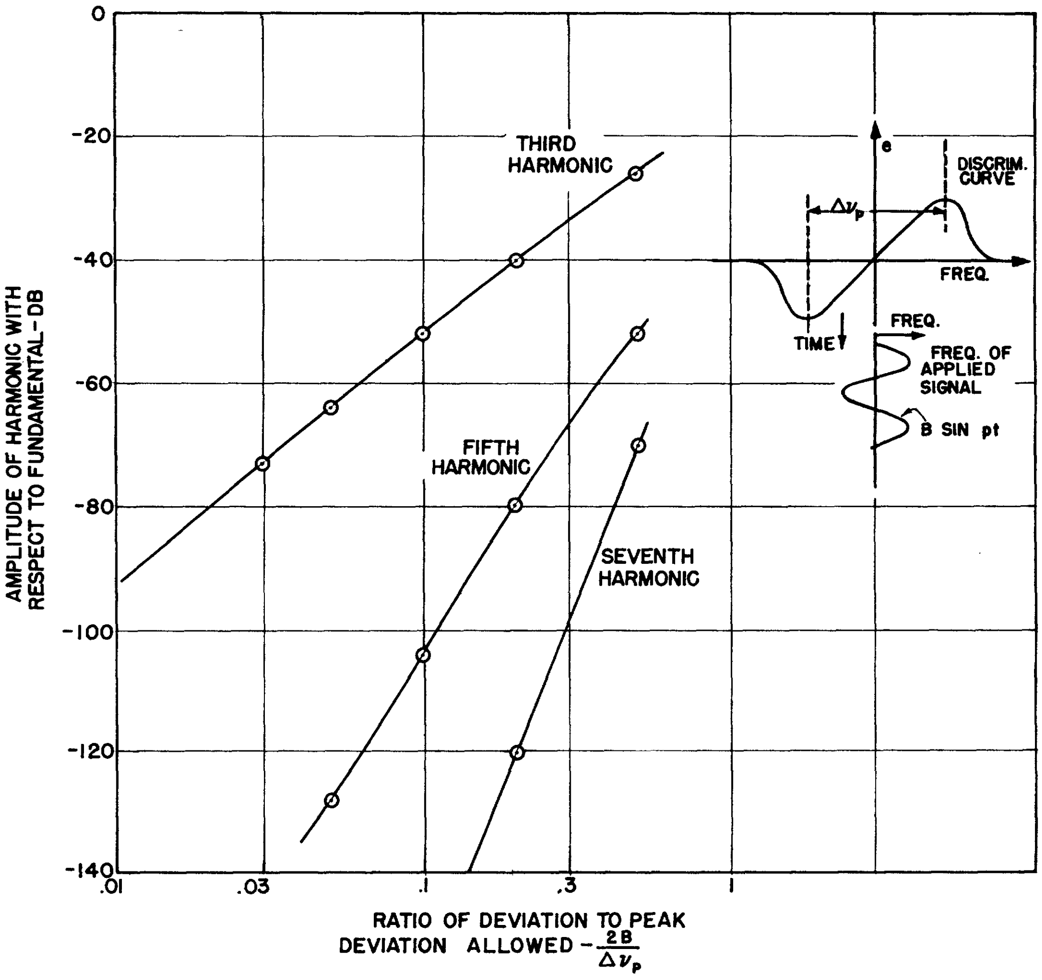

Figure 9. Harmonic distortion in balanced discriminator.

It is interesting to note that the frequency vs. modulating voltage characteristic obtained in this manner is identical to the voltage vs. frequency characteristic of the cathode-driven discriminator,[1] and hence a transmission system using a modulated stabilized microwave oscillator and a properly matched cathode-driven discriminator may be made highly linear, independent of the characteristics of the microwave transmitter tube employed.

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[1] W. G. Tuller, T. P. Cheatham, Jr., "Adjustable - Bandwidth FM Discriminator", Electronics, 20 pp. 117-119, September, 1947.

4. Conclusions

The equal-arm discriminator described above is a microwave discriminator with no spurious locking frequencies anywhere in a 12 per cent band. When this circuit is used with an oscillator tube such as the 2K25, tune-up procedure is reduced to two adjustments: the reference cavity is set to the desired operating frequency, and the oscillator tube cavity is tuned for maximum output. This procedure holds anywhere within the tuning range of the 2K25. If a thermally tuned tube is used, even the oscillator cavity tuning adjustment may be dispensed with. The circuit has the capability of being frequency modulated with inherent negative feedback and low distortion. It has proved very reliable in use in laboratory microwave power sources for the past eighteen months, giving sure lock-in of oscillator frequency reliably from a cold start each time it was turned on.