Problem Set 4

Ec2390 / 14.771 Fall, 2002

General Instructions

PLEASE KEEP ALL ANSWERS BRIEF AND CIRCLE FINAL EQUATIONS

Problem 1 *Consider a simplified version of the Tirole model discussed in class. There are a large number of agents. Agents manufacture either a simple or complex item (e.g., white or colored T-shirts) for a large number of principals. Principals and agents are randomly matched. Agents come in three types: those who are always honest (fraction α), those who are always dishonest or always cheat (fraction β), and those who are opportunistic (fraction γ). Where $\alpha + \beta + \gamma = 1$. The total population of agents is stationary, but each period $1 - \lambda$ agents die and are replaced with new agents. Principals only imperfectly observe each agent's history. In particular let x be the probability that the principal will "catch" or learn that an agent has cheated in the past. Payoffs to the principal depend on whether the agent cheats as well as project type. The payoff matrix for the principal is*

	Agent Cheats	*Agent Does Not Cheat*
Simple Project	d	h
Complex Project	D	H

where $H > h > d > D$. This implies that ***cheating is very costly to the principal when the assigned project is complex****. Payoffs to the agent are defined by the matrix*

	Agent Cheats	*Agent Does Not Cheat*
Simple Project	$b + G$	b
Complex Project	$B + G$	B

where G is the gains that accrue from cheating and b and B are the payments which vary with project type.

1. What is the probability that a cheater is *not* detected by the principal in any given period/match? .

2. Consider a high reputation equilibria where all opportunistic agents do not cheat and principals assign the complex project to all individuals who have not been caught cheating (or remain "undiscovered").

   (a) What fraction of the total population of agents have either never cheated or are not discovered as cheating during the current period and hence will receive the complex project in the high reputation equilibrium?

   (b) What fraction of those individuals in your answer in part (a) will not cheat if given the complex project, what fraction will cheat?

   (c) Assume that the principal's outside option is 0. Write the principal's IC constraint for the high reputation equilibria.

3. We would now like to write the IC constraint for opportunistic agents to not cheat in the high reputation equilibrium. Assume that agents discount the future at rate δ_0. Remember that if an agent cheats and is discovered she will not be assigned the complex project. The probability that she is discovered is x each period and from period-to-period she can either be discovered, in which case she will get the simple project, or she can be undiscovered, in which case she will get the complex project. (Recall that cheaters always cheat, and the honest are always honest so incentives don't matter for these groups).

   (a) Conditional on an opportunistic agent having cheated in the past, what is the probability that she will cheat in the future? Why? (**one sentence**)

   (b) Write the present discounted value of always remaining honest for an opportunistic agent.

   (c) What is the probability that an opportunistic agent who has cheated in the past will live to the next period and be undiscovered? And discovered (as the rotten cheater she is)?

   (d) Write the present discounted value of cheating (conditional on not having cheated in the past).

(e) Use your answer in (c) and (d) to write the opportunistic agent's IC constraint. Show that this constraint is equivalent to the condition that $G \leq \lambda\delta_0 x (B - b)$

(f) Briefly comment on the role of x, $B - b$ and $\lambda\delta_0$ in determining whether the IC constraint is satisfied (**three sentences max**).

4. In order to ensure that the high reputation equilibrium developed in (2) and (3) above is indeed an equilibrium we need to check that the IC constraints for the principal and agent corresponding to the low reputation equilibrium do not bind. In the low reputation equilibrium opportunistic agents always cheat and principal's always give the simple project, regardless of whether the agent is a known cheater or not.

   (a) Write the IC constraint for the principal in the low reputation equilibrium.

   (b) Write the IC constraint for the agent in the low reputation equilibrium.

5. **The Challenge of Inter-Equilibrium Transitions**. In light of the model briefly comment on the following scenario. Consider a version of the model where the agents are public servants, and principals are citizens of country X. Assume that the high reputation equilibrium prevails initially. Consider the case where the IMF enters country X and implements an austerity package that includes budget cuts which are financed by cutting the wages of public servants. How might this affect the sustainability of the high reputation equilibrium? If country X switches to the low reputation equilibrium will it be hard to move back to the high reputation equilibrium once the budget crunch is over? (**two paragraph maximum**).

Problem 2 *This problem explores how credit market imperfections and perceived or actual differentials in the rates of return to investing in human capital across individuals in a household interact. The returns to investing H units of wealth in education are given by the return functions $R_f = R_f(H)$ and $R_m = R_m(H)$ for female and male children respectively (where $R_j' > 0, R_j'' < 0$ for $j = f, m$). Assume that $R_m(H) > R_f(H)$ at all investment levels, H. To impose this condition we will use the parametric returns function $R_m(H) = aH - bH^2$ and $R_f(H) = \alpha R_m(H)$ where $\alpha < 1$. Assume that a and b are positive numbers sucht that $R_m(H)$ is indeed concave.*

1. Sketch $R_m(H)$ and $R_f(H)$ and also sketch their derivatives (on two separate graphs).

2. Assume housholds can borrow freely at a gross interest rate of $1 + r$, what will be the equilibrium investment level for boys and girls in the household?

3. How will the "gender gap" in education be related to the gross interest rate (hint: use your two figures from (1) above)?

4. Assume that a household's gross cost of investment is a decreasing function of household wealth, W. That the cost of investment fund is $1 + r(W)$ where $r'(W) < 0, r''(W) > 0$. How should the gender gap in education be related to household wealth? Graphically *derive* a relationship between W and H^f and H^m using your plot of the derivative of the returns functions and of $1 + r(W)$.

5. Assume all families have two children. Provide yes/no answers to the following questions and then provide a **three sentence** explanation of your results. Assume that credit constraints are in effect and hence funds need to be rationed across different investment oppportunities.

   (a) If you are a boy is it better to have a sister or a brother?

   (b) If you are a girl is it better to have a sister or a brother?

6. Assume you are an econometrician. You observe two types of households, those that are poor, and those that are rich.

   (a) Assume that preferences, ability etc. do not vary systematically with household wealth. Should the gender gap vary across the two types of households if credit markets are perfect?

(b) Use your data to derive an empirical test for credit market imperfections under the maintained assumptions of (a) above. You observe $i = 1...N$ households with $j = 1...J^i$ children in the i^{th} household. Let $W_i = 1$ if the household is poor and zero otherwise and let M_{ij} be a dummy indicating that the observed child is male.

(c) Is our test valid if household preferences for gender equity are positively correlated with wealth? Formally relate your answer to the standard "common trend" assumption of the differences-in-differences estimator. Is our test biased toward finding that credit market imperfections are important or against this hypothesis? **Some helpful background:** Assume that our observed education outcomes are generated by the following latent variable process:

$$H_{ij} = H_{ij}^1 W_i + H_{ij}^0 (1 - W_i)$$

where H_{ij}^1 is the education outcome if you live in a poor household and H_{ij}^0 if you live in a rich household. *Note that these are latent education outcomes – we don't observe the relevant counterfactual for each individual.* Ideally we would like to randomly assign individuals (or families) to be poor or rich in order to "test" our model's empirical implications. That is we'd like to compare gender gaps in the *same* family when it is poor as well as when it is rich, that is we are interested in:

$$\underbrace{E\left[H^1|W=1, M=1\right] - E[H^1|W=1, M=0\right]}_{\text{gender gap for observed poor households}} \\ - \underbrace{\left\{E\left[H^0|W=1, M=1\right] - E[H^0|W=1, M=0\right]\right\}}_{\text{gender gap if same poor households were observed rich}}.$$

The problem is that the second term in $\{\cdot\}$ is not observed. This question is about what conditions need to be satisfied in order to replace $E\left[H^0|W=1, M=1\right] - E[H^0|W=1, M=0\right]$ with the observed $E\left[H^0|W=0, M=1\right] - E[H^0|W=0, M=0\right]$.

Problem Set 4
Development Economics 14.771

Question 1: Sibling Rivalry [empirical exercise]
Download the file "Tanz93.dta" from the course web site
Please use Explorer for downloading, NOT Netscape.

In Garg and Morduch (1998), "Sibling Rivalry" , we learned a model of sibling rivalry in which if parents prefer sons, then children prefer sisters. In this exercise, you will use a data set from Tanzania to test whether this prediction holds. This data set, named tanz93.dta, is from a nationally representative household survey completed in 1993, and was part of the World Bank's Living Standards Measurement Survey program. Before you begin, restrict your data set to children aged 13 to 16.

a. Form a new variable named educ6 which is a dummy variable indicating whether the individual has received 6 or more years of education

b. Consider educ as the outcome of interest. You want to know how the number of sisters affects the number of years of education.

i. Run a regression of educ on the number of sisters. What is the estimated regression coefficient for the number of sisters? Is the coefficient biased?

ii. What is the best regression you can run to answer the above question? Run it and report the results.

c. Consider educ6 as the outcome of interest. You want to know the effect of number of sisters on the percent of children who have completed grade 6.

i. Fill in the table 1 below with the mean proportion of children who have completed grade 6. To make things clear, as an example, A in the following table would be the mean of educ6 for children with 3 siblings, of whom 2 are sisters.

Stata hints: First, use table y x, c(mean educ6 n educ6) to create a two-way table that gives you the mean of educ6 as well as the number of observations for each cell of the table. This is a great time saver.

ii. Consider what happens as the number of siblings increases. What does the sibling rivalry model predict will happen to the mean of educ6? Is this what you observe in the data?

iii. Consider what happens as the number of sisters increases. What does the sibling rivalry model predict will happen to the mean of educ6? Is this what you observe in the data?

d. How does the analysis in (b) compare to the analysis in (c)? Do they both lead to similar conclusions about the relevance of the sibling rivalry model for Tanzania?

Table 1

		Number of siblings (other than self)					
		0	**1**	**2**	**3**	**4**	**5 or more**
	0						
	1						
number	**2**				*A*		
of sisters	**3**						
	4						
	5 or more						