Massachusetts Institute of Technology
Department of Economics
Time Series 14.384

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Final Exam-Fall 2000

Instructions: This is a 180-minute open-book, open-notes exam.

1. Assume that ε_t and v_t are iid sequences of random variables, independent of one another, with $E\varepsilon_t^4 < \infty, Ev_t^4 < \infty$, $E\varepsilon_t = Ev_t = 0$ and $E\varepsilon_t^2 = Ev_t^2 = 1$. Let y_t be the stationary solution to the stochastic difference equation for all t

$$y_t = \phi y_{t-1} + v_t - \theta v_{t-1} \text{ for } |\phi| < 1, |\theta| < 1 \tag{1}$$

and let x_t be given as

$$x_t = y_t + \alpha \cos(\lambda t) + \beta \sin(\lambda t) + \varepsilon_t \tag{2}$$

where α and β are random variables independent of ε_t and v_s for all t and s. Also assume that α is independent of β and that $E\alpha = E\beta = 0; E\alpha^2 = E\beta^2 = 1$.

   a) Show that x_t is a weakly stationary process. Does this still hold if $E\alpha \neq 0$? Is x_t ergodic?
   b) Find the autocovariance function of x_t (Hint: note the attached page with trigonometric identities).
   c) Find the spectral distribution function of x_t. Does this distribution function have a density?
   d) Find the Wold decomposition for x_t.
   e) Find an estimator of the random variables α and β assuming that y_t is observable. In what sense, if at all, does your estimator converge to α and β. Now assume that y_t is unobservable. How could you proceed now. How would your estimator be affected by the unobservability of y_t.

2. Assume that $\varepsilon_t \in \mathbb{R}^2$ is iid $N(0, I)$ where I is the 2×2 identity matrix. Assume that $z_t' = (y_t, x_t)$ is generated by the model

$$C_0 z_t = C(L)\varepsilon_t \tag{3}$$

where $C(L) = I + \sum_{j=1}^{\infty} C^j L^j$ with the coefficient matrix C given by

$$C = \begin{bmatrix} \beta & \gamma \\ 0 & \phi \end{bmatrix}$$

and C_0 some 2×2 matrix.

   a) Let $\theta = [\beta, \gamma, \phi] \in \mathbb{R}^3$. Find the subset $\Theta \subset \mathbb{R}^3$ such that $C(L)^{-1}$ exists. Find the reduced form (i.e. the VAR representation) of model (3), i.e. find the polynomial $\Pi(L) = I - \sum_{i=1}^{p} \Pi_i L^i$ such that

$$\Pi(L) z_t = u_t.$$

   Show how $\Pi(L)$ depends on the matrices C_0 and C. How is u_t related to the structural innovations ε_t.

   b) Describe in detail how you would estimate the reduced form parameters Π. Derive the limiting distribution of your estimator.
   c) Using your reduced form parameters, can you identify the structural form parameters C_0 and C? Do you need to impose additional restrictions on C_0?
   d) Assuming that you have identified the structural model, find the impulse response function of a shock ε_t onto z_t.

3.

Assume that $x_t = x_{t-1} + \varepsilon_t$ and $\varepsilon_t \sim$iid$N(0,1)$ and $x_0 = 0$. Consider a regression of x_t onto x_{t-2}, ie. an estimator

$$\hat{\alpha} = \frac{\sum_{t=3}^{n} x_t x_{t-2}}{\sum_{t=3}^{n} x_{t-2}^2}.$$

a) Is $\hat{\alpha}$ consistent for the parameter value $\alpha = 1$? Prove your answer.

b) Find the limiting distribution of a properly scaled and centered version of $\hat{\alpha}$. Could you use your result to test the hypothesis $H : x_t \sim I(1)$? If so, how would you construct the test?

c) Now assume that

$$y_t = \beta x_t + u_t + c \sum_{s=1}^{t-1} u_s$$

where u_t is an iid(0,1) sequence indpendent of ε_t and c is a constant. You run a regression of y_t on x_t. The residuals of this regression are denoted by $\hat{u}_t$. Find the limiting distribution of the OLS estimator of regressing $\hat{u}_t$ onto $\hat{u}_{t-1}$ assuming that $c \neq 0$. How can you use this estimator to test $H : c \neq 0$.

d) Discuss the behavior of your test under the alternative when $c = 0$ (derive the limiting distribution of the estimator under the alternative).