Problem Set 2

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1 Problem 1: Modeling Trend

Consider the following time series model:

$$x_t = \beta_0 + \beta_1 t + \varepsilon_t$$
$$\varepsilon_t \sim \text{White Noise}(0, \sigma^2).$$

(a) Use least squares to detrend x_t. What is the autocovariance function for the cyclical component.
(b) Use first differences to detrend x_t. What is the autocovariance function for the cyclical component.
(c) Use first differences to detrend x_t assuming that $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$. What is the autocovariance function for the cyclical component.
Now assume that the true model is given by a random walk with drift:

$$x_t = \mu + x_{t-1} + \varepsilon_t, \forall t = -n, ..., 0, ..., n.$$

(d) Use first differences to detrend x_t. What is the autocovariance function of the cyclical component?
(e) Use least squares to detrend x_t. Use $x_{-n-1} = 0$. What is the autocovariance function of the cyclical component?
(f) Simulate the results obtained for $n = 100, \sigma^2 = 1$, and plot the autocorrelation function for the 2 models considered using the two detrending methods. Comment on the results.

2 Problem 2: Forecasting

Assume that ε_t is iid with $E(\varepsilon_t^4) < \infty$, $E(\varepsilon_t) = 0$, and $E(\varepsilon_t^2) = 1$. Let ϕ_1 and ϕ_2 be such that $\phi(L) = 1 - \phi_1 L - \phi_2 L^2$ has all its roots outside the unit circle. Assume that x_t is the stationary solution to,

$$\phi(L)X_t = \varepsilon_t.$$

1. You are given a sample $X_1, ..., X_T$ of observations. Find the best linear predictor in the mean square sense of X_{T+1} and X_{T+2}.
2. Calculate the MSE of your forecast conditional on $X_1, ..., X_T$, i.e. calculate $E((X_{T+h} - \hat{X}_{T+h})^2 | X_T, X_{T-1}, ...)$ where $\hat{X}_{T+h}$ is the h-step ahead forecast.
3. Find the unconditional MSE of the forecast.

3 Problem 3: Simulation

Consider the time series model given by the formula,

$$x_{t+1} = 0.5 x_{t-1} + \varepsilon_t,$$

where ε_t are iid $N(0,1)$.
(a) Compute the theoretical ACF for the process.
(b) Simulate a series of length 1000 from this process. Calculate the sample autocovariance. Does the theoretical autocovariance look close to the sample one? (c) Simulate 100 different paths of size 1000 from this process. Estimate the autocovariance. Then plot a histogram of the sample estimated autocovariances at lag 1 from this process.
(d) Find the asymptotic distribution of the sample autocovariance.

4 Problem 4: Specification Testing

Consider an ARMA model given by the formula,

$$X_{t+1} = \sum_{j=1}^{p} \phi_j X_{t-j+1} + \varepsilon_{t+1} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j+1}.$$

(a) Find a test for the hypothesis that $\phi_1 = 0$.
(b) Estimate the model using GMM and design a similar test.
(c) Take $q = 1$ and $p = 1$. Estimate the model on S&P data.
(d) Apply these two tests on S&P data. Comment of their differences.