Problem Set 3: Optional. Due: Dec 10th.

December 4, 2002

1 Problem 1:

We are interested in testing if the log of nominal GNP has a unit root. Let $y_t = \log(GNP_t)$. We set up the following regression equation,

$$y_t = \alpha + \delta t + \rho y_{t-1} + \pi_1 \Delta y_{t-1} + \pi_2 \Delta y_{t-2} + \pi_3 \Delta y_{t-3} + \pi_4 \Delta y_{t-4} + u_t. (5.1)$$

An OLS regression on a sample of 157 observations leads to the following parameter estimates,

Parameter	Estimate	Standard Error
$\hat{\alpha}$	0.0299	0.06357
$\hat{\delta}$	0.0000	0.00009
$\hat{\rho}$	0.9957	0.01142
$\hat{\pi}_1$	0.2366	0.08002
$\hat{\pi}_2$	0.0544	0.08110
$\hat{\pi}_3$	0.0177	0.08097
$\hat{\pi}_4$	0.0312	0.07801

(a) Test the hypothesis that y_t is generated by $y_t = \mu + y_{t-1} + \varepsilon_t$, where ε_t is a stationary process. Write down the test statistic and indicate which table you are using for the critical values.
(b) Would it be better to estimate the model,

$$y_t = \alpha + \delta t + \rho y_{t-1} + \pi_1 \Delta y_{t-1} + u_t,$$

or the model,

$$y_t = \alpha + \delta t + \rho y_{t-1} + u_t,$$

in order to carry out the tests? Comment on the consequences for the limit distribution of $\hat{\rho}$ in both cases.
(c) Assume now that based on the results of our regression (5.1) we decide to estimate the model,

$$y_t = \rho y_{t-1} + u_t. \ (5.2)$$

Assume that y_t is generated by,

$$y_t = \mu + y_{t-1} + \varepsilon_t, \; (5.3)$$

where ε_t is iid $N(0,1)$. Derive the asymptotic distribution of the OLS estimator,

$$\hat{\rho} = \frac{\sum y_t y_{t-1}}{\sum y_{t-1}^2},$$

when the data is generated by (5.3).

2 Problem 2:

You are given a time series of the logarithm of the monthly spot exchange rate for the "US Dollar/Deutsche Mark" rate. There are 210 observations in the sample. The following table shows the autocorrelations of the level and the first differences of the series.

Lag	y_t	Δy_t
Lag1	0.98164213	-0.010129092
Lag2	0.96379890	0.10573291
Lag3	0.94295569	0.038651850
Lag4	0.92085450	0.040649990
Lag5	0.8972454	0.041795142
Lag6	0.87618383	-0.026381941
Lag7	0.85501833	0.14540808
Lag8	0.83031148	-0.036096069
Lag9	0.80578753	0.016415322
Lag10	0.78183452	-0.018371523
Lag11	0.75836956	0.13415457
Lag12	0.73174636	-0.070189250

(a) Applying any statistical procedure you know, decide which statistical model would best describe the level y_t of the spot exchange rate.
(b) Your friend Helmut who is an economist as the "Deutsche Bundesbank" needs to determine the stationarity properties of the exchange rate. He shows you output from the following two regressions (calculated using the OLS routine in Gauss - a copy of the manual describing the output is attached). The output from the first regression is,

Valid Cases:	209				Dependent Variable:	y_t
Missing Cases:	1				Deletion Method:	Listwise
Total SS:	1.487				Degrees of Freedom:	207
R-squared:	0.963				Rbar-squared:	0.963
Residual SS:	0.055				Std error of est:	0.016
F(1,207):	5356.368				Probability of F:	0.000
Durbin-Watson:	2.017					
Variable:	Estimate	Error	t-value$>\|t\|$	Estimate	Dep Var	
CONSTANT	0.025984	0.016728	1.553285	0.122	-	-
y_{t-1}	0.980179	0.0143393	73.187212	0.000	0.981220	0.981220

Regression number two leads to the following Gauss output:

Valid Cases:	208				Dependent Variable:	y_t
Missing Cases:	2				Deletion Method:	Listwise
Total SS:	1.470				Degrees of Freedom:	205
R-squared:	0.962				Rbar-squared:	0.962
Residual SS:	0.055				Std error of est:	0.016
F(1,205):	2620.783				Probability of F:	0.000
Durbin-Watson:	2.010					
Variable:	Estimate	Error	t-value$>\|t\|$	Estimate	Dep Var	
CONSTANT	0.025928	0.017038	1.521842	0.130	-	-
y_{t-1}	0.980200	0.013639	71.868956	0.000	0.980660	0.980996
Δy_{t-1}	-0.003273	0.015637	-0.209286	0.834	-0.002856	-0.118452

Describe in detail how you would test $H_0 : y_t = y_{t-1} + u_t$, where u_t is a stationary process using the results of the two regressions. What are the advantages and disadvantages of using the first versus the second regression to test the hypothesis? Using your assesement of the stochastic process under (a) carry out the test that best fits your Null of the process y_t. Explain why your choice is justified. Use the correct significance levels from the attached tables.
(c) Assuming that $y_t = y_{t-1} + \varepsilon_t$, where ε_t is iid, $E\varepsilon_t = 0$, $E\varepsilon_t^2 = 1$, derive formally the asymptotic distribution of the parameter estimate for y_{t-1} in the first regression.
(d) Under the same assumptions as in part (c) find the asymptotic distribution of the parameter on Δy_{t-1} in the second regression.

3 Problem 3:

Let $x_t = \sum_{s=1}^{t} \varepsilon_s$ and $\varepsilon_s \sim iidN(0,1)$ and $y_t = \sum_{s=1}^{t} v_s$ with $v_s \sim iidN(0,1)$. Assume that $E\varepsilon_s v_t = 0, \forall t, s$.
(a) Are x_t and y_t stationary?

(b) Consider a regression of y_t on x_t. Let $\hat{\beta}$ be the OLS estimator of that regression defined by,

$$\hat{\beta} = \frac{\sum_t y_t x_t}{\sum_t x_t^2}.$$

Show that,

$$\hat{\beta} \to \frac{\int W_1(r) W_2(r) dr}{\int W_2(r)^2 dr},$$

where $W_1(r)$ and $W_2(r)$ are two independent standard Brownian motions on $C[0,1]$. How do you interpret this result?

(c) Let the R^2 of the regression be given as,

$$R^2 = \frac{\hat{\beta}^2 \sum_t (x_t - \bar{x})^2}{\sum_t (y_t - \bar{y})^2},$$

with $\bar{x} = \frac{\sum_t x_t}{n}$ and $\bar{y} = \frac{\sum_t y_t}{n}$. Find an expression for R^2 as $n \to \infty$.

(d) Discuss the implications of your result for a standard F test of the hypothesis $\beta = 0$.

(e) Now assume that $x_t = \sum_{s=1}^{2} \varepsilon_s$ and $\varepsilon_s \sim iidN(0,1)$, and that,

$$y_t = x_t + v_t,$$

where $v_s \sim iidN(0,1)$ and $E\varepsilon_s v_t = 0, \forall t, s$. Consider the OLS estimator $\hat{\beta}$ from the regression of y_t on x_t where $\hat{\beta} = \frac{\sum y_t x_t}{\sum x_t^2}$. Is $\hat{\beta}$ consistent? Can you find a limit distribution for β after appropriate centering and scaling?

(f) Continue to maintain the assumptions under (e). How would you test the hypothesis that $\beta = 1$? Construct a test and indicate how you would obtain critical values under $H_0 : \beta = 1$.