

*ProblemSet1*

## 1 ARCH(1) stochastic process

In conventional econometric models, the variance of the disturbance term is assumed to be constant. However many economic time series exhibit periods of unusually large volatility followed by periods of relative tranquility. In such circumstances, the assumption of a constant variance is inappropriate.

consider the following:

$$\begin{aligned}\epsilon_t &= u_t h_t^{1/2} \\ h_t &= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 \\ u_t &\sim N(0, 1), \text{ independent of } \epsilon_{t-1}\end{aligned}$$

a. Compute the unconditional expectation, the unconditional variance and the autocovariances of  $\epsilon_t$ . For which values of  $\alpha_0$  and  $\alpha_1$ , is  $\epsilon_t$  white noise? Now compute the conditional mean of  $\epsilon_t$  ( $E(\epsilon_t \mid \epsilon_{t-1}, \epsilon_{t-2}, \dots)$ ) and its conditional variance. Do we have to restrict further the values of  $\alpha_0$  and  $\alpha_1$ ?

b. Assume you are interested in the following stationary ARMA model:

$$y_t = a_0 + a_1 y_{t-1} + \epsilon_t, \text{ where } \epsilon_t \text{ is defined above}$$

Compute the unconditional mean and variance of  $y_t$ .

c. Go through the attached GAUSS program and generate the previous processes. (I give you this first program to allow you to get familiar with Gauss).

d. Now consider

$$x_t = \epsilon_t^2 - E(\epsilon_t^2)$$

Is  $x_t$  covariance stationary? If so, compute the autocovariance function.

## 2 Consider the following ARMA(2,2) model

$$x_t = 1.3x_{t-1} - 0.4x_{t-2} + \epsilon_t - 1.2\epsilon_{t-1} + 0.2\epsilon_{t-2}$$
$$\epsilon_t \text{ iid } (0,1)$$

- Is  $x_t$  weakly stationary? If so compute the autocovariance function.
- Is  $x_t$  a casual ARMA process? Is it invertible?
- If  $x_t$  is casual, find the infinite order MA representation.
- Using GAUSS (this time you have to write the routine), generate the previous process, and compute the sample autocorrelations and the sample partial autocorrelations.

## 3 Modeling Trend

Lecture Note 1 stresses the idea that stationarity is not typical in economic time series. Think about macroeconomic data: you often see a real series, such as GDP, decomposed into a secular part (the long run growth) and a cyclical part. If the secular movement is stochastic rather than deterministic, then a model based on time trend residuals is misspecified. Thus it appears important to determine if a time series is better characterized as stationary around a deterministic trend or as a non-stationary process. Let us define

$$y_t = \beta_0 + \beta_1 t + u_t$$
$$\phi(L)u_t = \theta(L)\epsilon_t; \epsilon_t \sim i.i.d.(0, \sigma_\epsilon^2)$$

a trend stationary process (TS), and

$$\Delta s_t = \mu + v_t$$
$$\delta(L)v_t = \lambda(L)\epsilon_t; \epsilon_t \sim i.i.d.(0, \sigma_\epsilon^2)$$

a difference stationary process (DS). Both  $u_t$  and  $v_t$  are stationary.

a. Using Gauss or Matlab, generate a TS and a DS assuming  $u_t$  and  $v_t$  are white noise, plot them. Compute their autocorrelations (the first 6 are more than enough).

b. Load the dataset and compute the sample autocorrelations of the natural logs of the data and of the first difference of the natural logs of the data. Compare these autocorrelations with those simulated in a. (you have to use the same number of observations for the observed and the simulated).

c. Now assume that the TS disturbance is  $(1 - \phi L)u_t = (1 - \theta L)\epsilon_t$ . Compute the first difference of  $y_t$ . How does it affect the autocorrelations, and how does it affect your answer in point b.

d. Using the dataset, compute the sample autocorrelations of the deviations from fitted trend lines. Compare them with the sample autocorrelations of the deviations of  $s_t$  from a fitted trend line.