Lecture 18 - p-n Junction *(cont.)*

October 17, 2001

Contents:

1. Ideal p-n junction out of equilibrium (cont.)

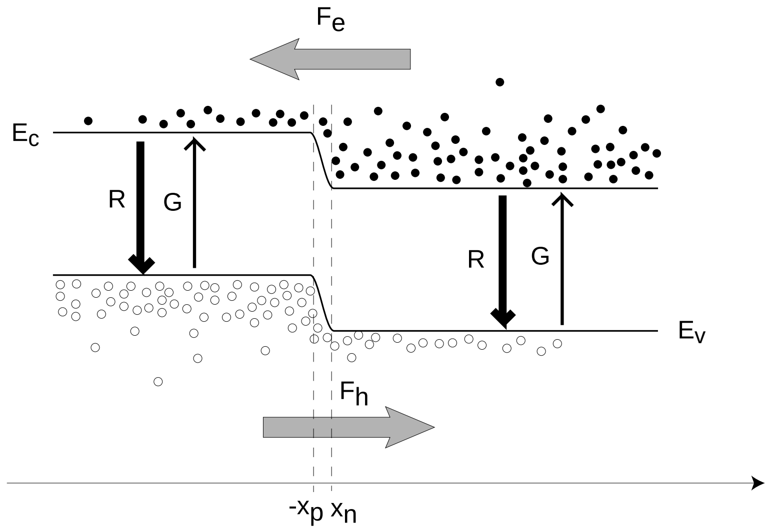
Reading assignment:

del Alamo, Ch. 7, §7.2 (7.2.3)

Key questions

- What are the key assumptions that allow the development of a simple model for the I-V characteristics of a p-n diode?
- What are the key dependencies of the current-voltage characteristics of the p-n diode?

□ I-V characteristics *(cont.)*



Strategy for deriving first-order model for I-V characteristics:

- Compute diode current density as follows:

$$J = J_e(-x_p) + J_h(x_n)$$

- Compute each minority carrier current contribution as follows:

$$J_e(-x_p) = -qn'(-x_p)v_e(-x_p)$$

$$J_h(x_n) = qp'(x_n)v_h(x_n)$$

- Use expressions of $v_e(-x_p)$ and $v_h(x_n)$ derived for similar minority carrier type problems in Ch. 5.
- Derive expressions for $n'(-x_p)$ and $p'(x_n)$ assuming *quasi-equilibrium* across the space-charge region.
- Unified result for forward and reverse bias.

⋆ Boundary conditions across SCR.

In thermal equilibrium, Boltzmann relations:

$$\phi(x_n) - \phi(-x_p) = \phi_{bi} = \frac{kT}{q} \ln \frac{n_o(x_n)}{n_o(-x_p)}$$
$$\phi(x_n) - \phi(-x_p) = \phi_{bi} = -\frac{kT}{q} \ln \frac{p_o(x_n)}{p_o(-x_p)}$$

If net current inside SCR is much smaller than drift and diffusion components, then *quasi-equilibrium* ⇒ Boltzmann relations apply:

$$\phi_{bi} - V \simeq \frac{kT}{q} \ln \frac{n(x_n)}{n(-x_p)}$$
$$\phi_{bi} - V \simeq -\frac{kT}{q} \ln \frac{p(x_n)}{p(-x_p)}$$

In LLI, $n(x_n) \simeq N_D$, and $p(-x_p) \simeq N_A$. Also $\phi_{bi} = \frac{kT}{q} \ln \frac{N_D N_A}{n_i^2}$. Then:

$$n(-x_p) \simeq \frac{n_i^2}{N_A} \exp \frac{qV}{kT}$$
$$p(x_n) \simeq \frac{n_i^2}{N_D} \exp \frac{qV}{kT}$$

In terms of excesses:

$$n'(-x_p) \simeq \frac{n_i^2}{N_A}(\exp\frac{qV}{kT} - 1)$$
$$p'(x_n) \simeq \frac{n_i^2}{N_D}(\exp\frac{qV}{kT} - 1)$$

Boundary conditions have all expected features. For electrons (for example):

- For $V = 0$:

$$n'(-x_p) = 0$$

- For $V \gg \frac{kT}{q}$:

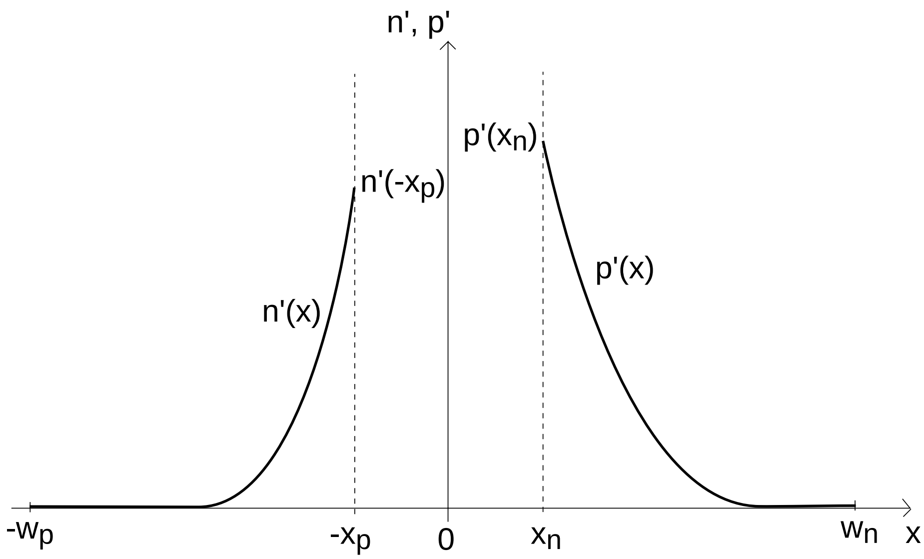
$$n'(-x_p) \simeq \frac{n_i^2}{N_A} \exp\frac{qV}{kT}$$

- For $V \ll -\frac{kT}{q}$:

$$n'(-x_p) \simeq -\frac{n_i^2}{N_A}$$

⋆ Minority carrier velocity at edges of SCR (”long” diode: $W_n \gg L_h, W_p \gg L_e$).

Problem identical to ”long bar” studied in Ch. 5: exponentially decaying excess minority carrier profiles.



$$n'(x) = n'(-x_p) \exp \frac{x + x_p}{L_e} \qquad \text{in p-QNR: } x \leq -x_p$$
$$p'(x) = p'(x_n) \exp \frac{-x + x_n}{L_h} \qquad \text{in n-QNR: } x \geq x_n$$

Carrier velocities:

$$v_e(-x_p) = v_e^{diff}(-x_p) = -\frac{D_e}{L_e}$$
$$v_h(x_n) = v_h^{diff}(x_n) = \frac{D_h}{L_h}$$

⋆ Excess minority carrier currents:

$$J_e(-x_p) \simeq -qv_e^{diff}(-x_p)\, n'(-x_p) = q\frac{D_e}{L_e}\frac{n_i^2}{N_A}(\exp\frac{qV}{kT}-1)$$

$$J_h(x_n) \simeq qv_h^{diff}(x_n)\, p'(x_n) = q\frac{D_h}{L_h}\frac{n_i^2}{N_D}(\exp\frac{qV}{kT}-1)$$

⋆ Total current: sum of electron and hole current:

$$J = J_e(-x_p) + J_h(x_n) = qn_i^2(\frac{1}{N_A}\frac{D_e}{L_e} + \frac{1}{N_D}\frac{D_h}{L_h})(\exp\frac{qV}{kT}-1)$$

Define $J_s \equiv$*saturation current density* (A/cm^2):

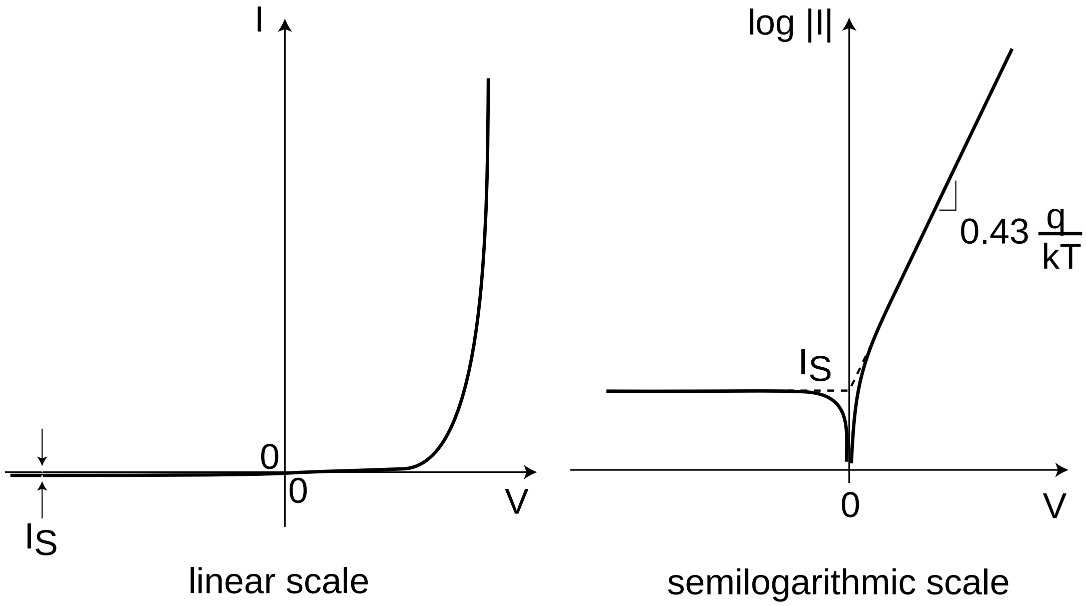
$$J = J_s(\exp\frac{qV}{kT}-1)$$

If diode area is A, current is:

$$I = I_s(\exp\frac{qV}{kT}-1)$$

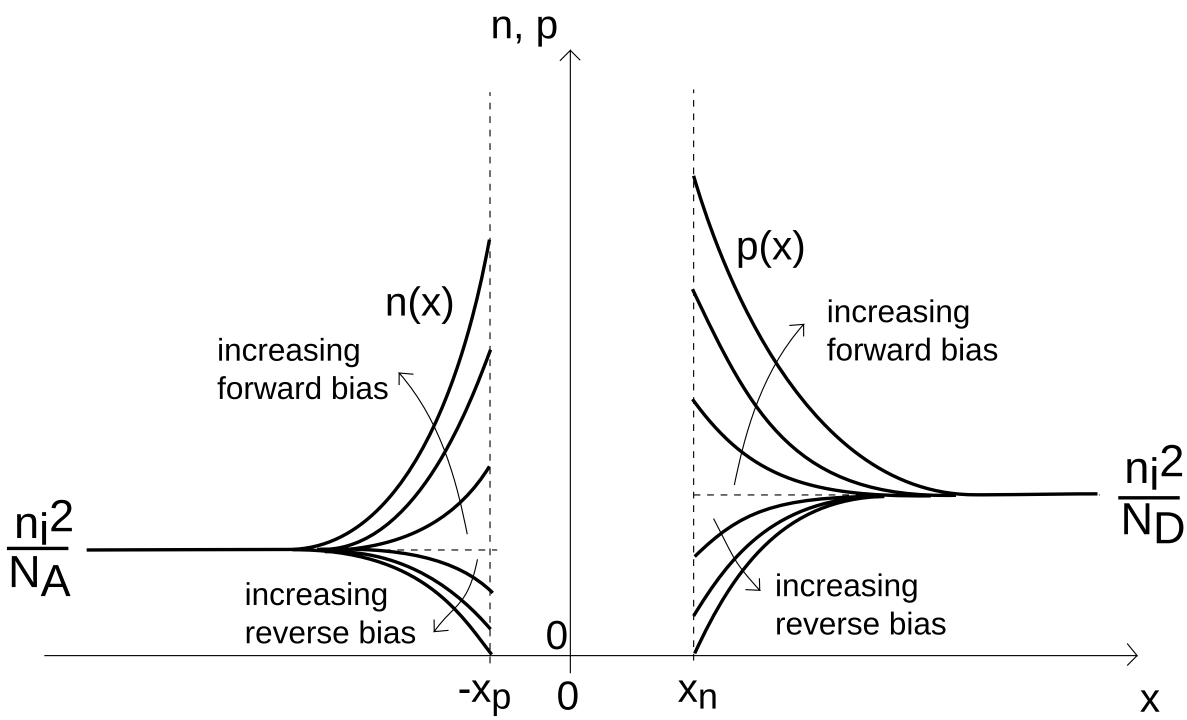
$$I = I_s(\exp\frac{qV}{kT} - 1)$$

Classic rectifying behavior



Should test quality of quasi-equilibrium approximation [problem #1 in homework # 6].

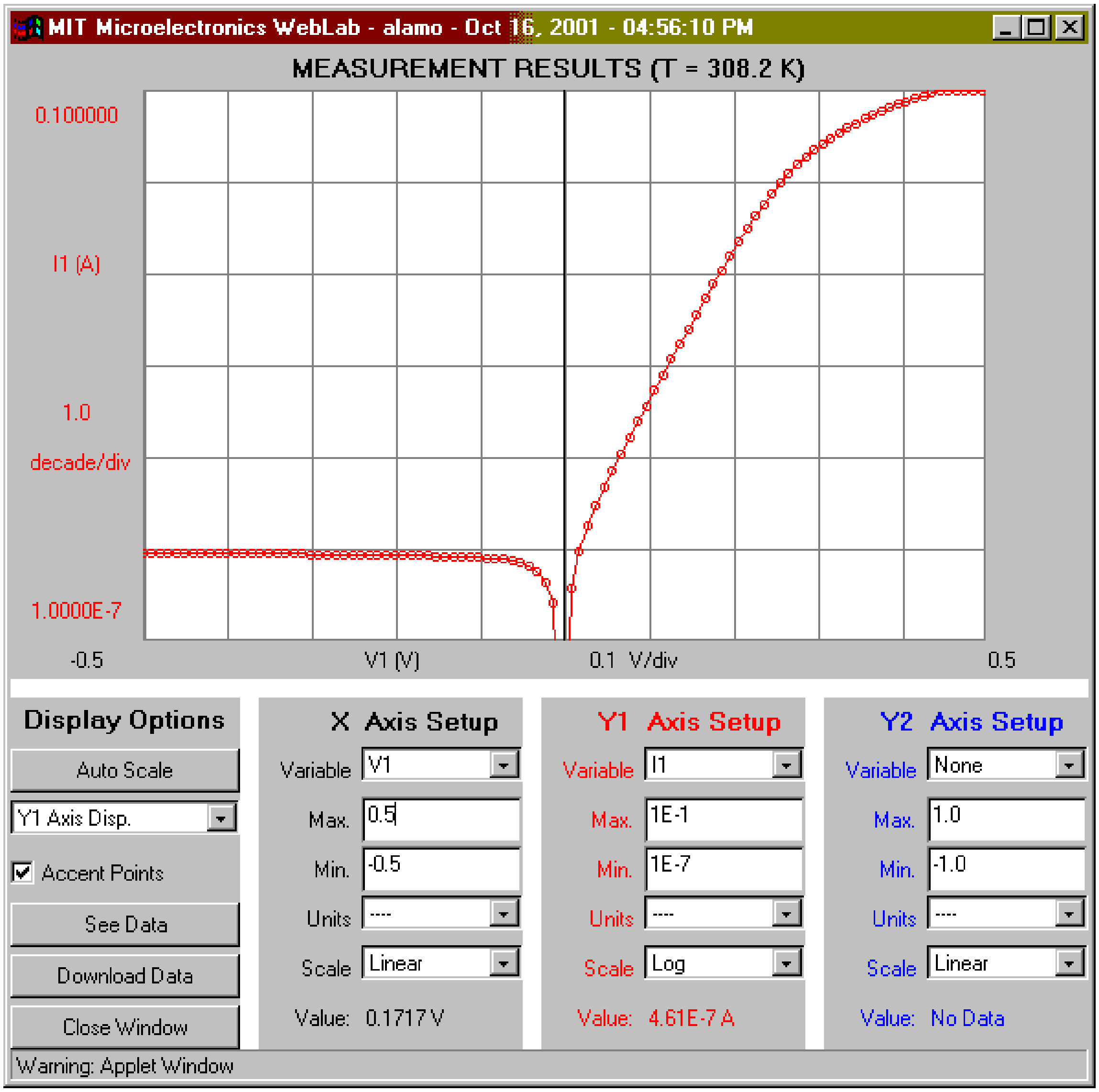
Rectifying behavior arises from boundary conditions across SCR:



-In forward bias: carrier concentrations at SCR edges grow up exponentially $\rightarrow I \sim e^{qV/kT}$

-In reverse bias: carrier concentrations at SCR edges reduced quickly to zero (can't go below!) $\rightarrow I$ saturates

Experimental verification:



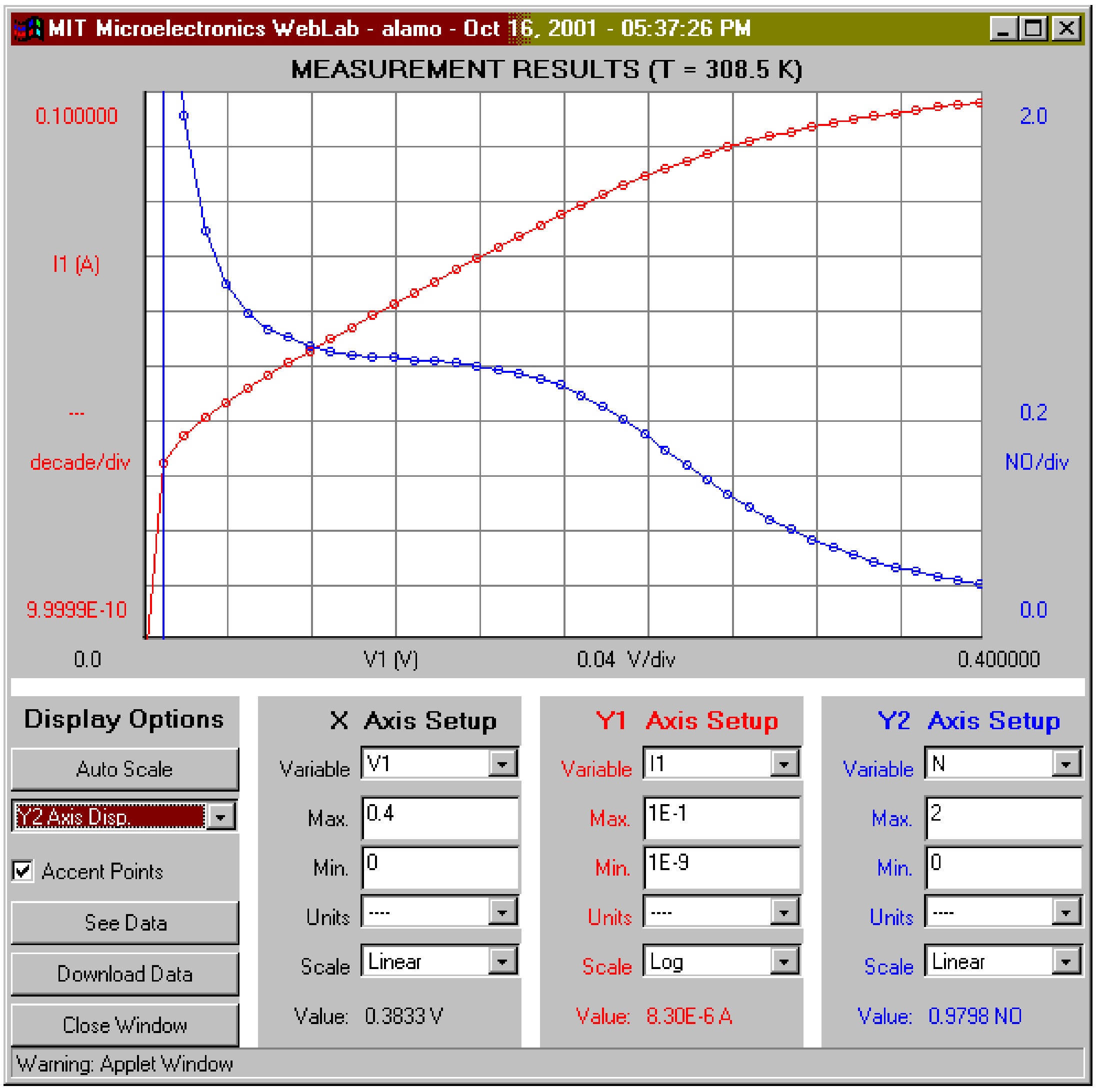
[currently featured in weblab: "6.012 diode"]

Ideality factor:

$$I = I_S(\exp \frac{qV}{\mathbf{N}kT} - 1)$$

Then, for sufficient forward bias ($V \gg kT/q$):

$$\mathbf{N} \simeq \frac{q}{kT}\frac{1}{I}\frac{dI}{dV}$$

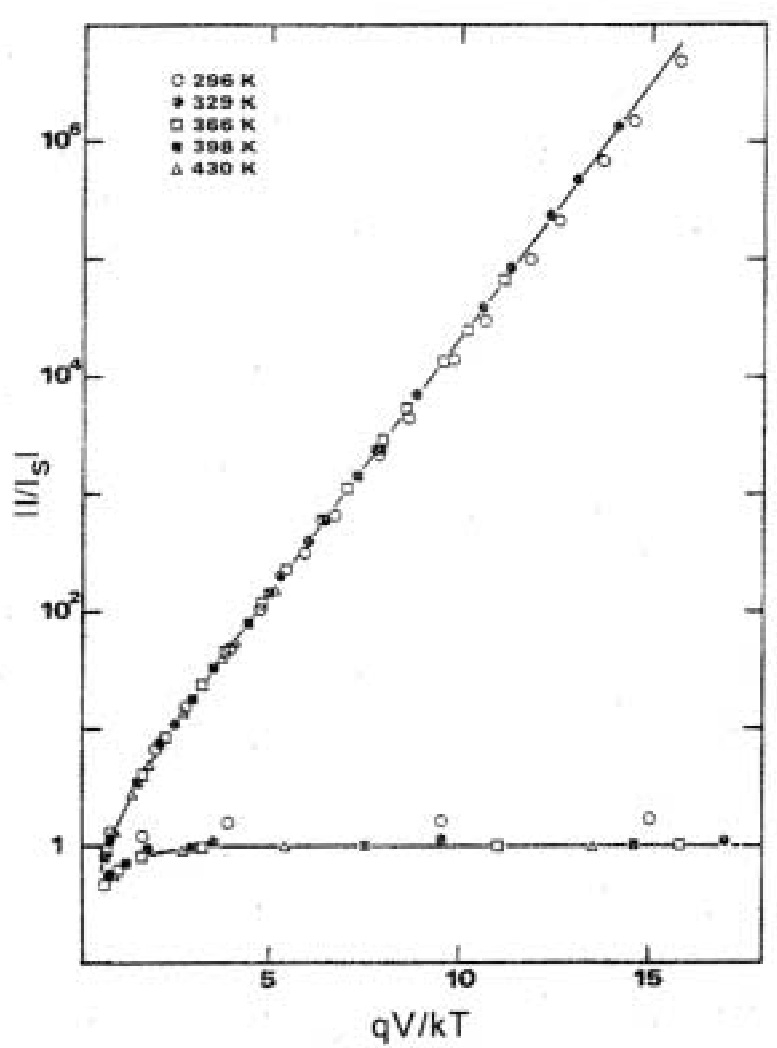


Universality of exponential relationship:

$$I = I_S(\exp\frac{qV}{kT} - 1)$$

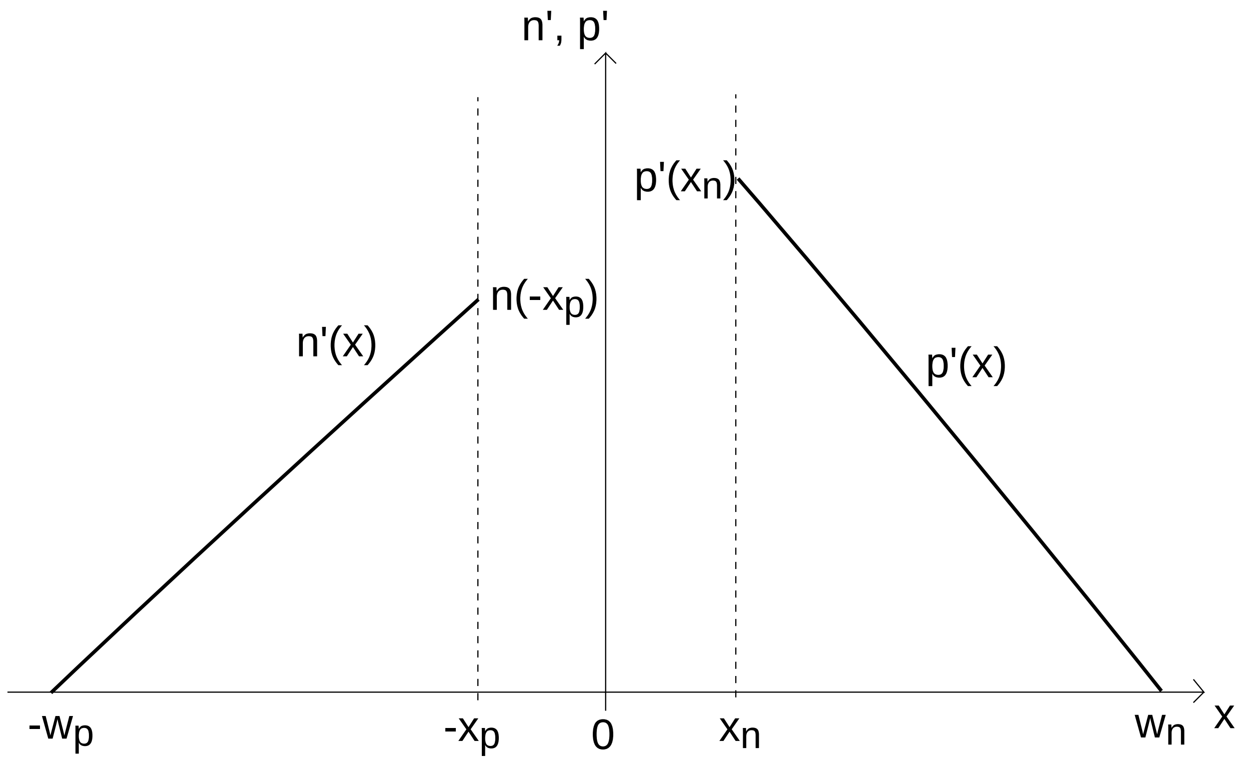
Then:

$$\frac{I}{I_S} = \exp\frac{qV}{kT} - 1$$



[Cappelletti 1985]

In short diodes with $S = \infty$, G&R takes place at surfaces:



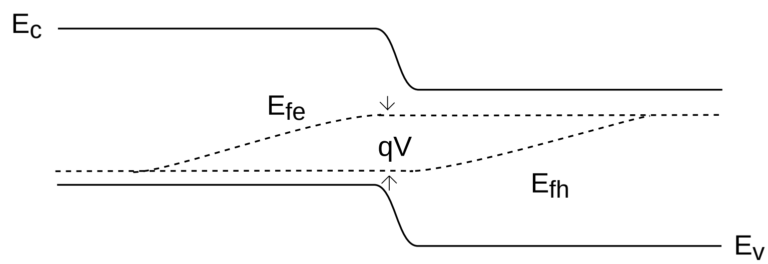
$$v_e^{diff}(-x_p) = -\frac{D_e}{w_p - x_p}$$

$$v_h^{diff}(x_n) = \frac{D_h}{w_n - x_n}$$

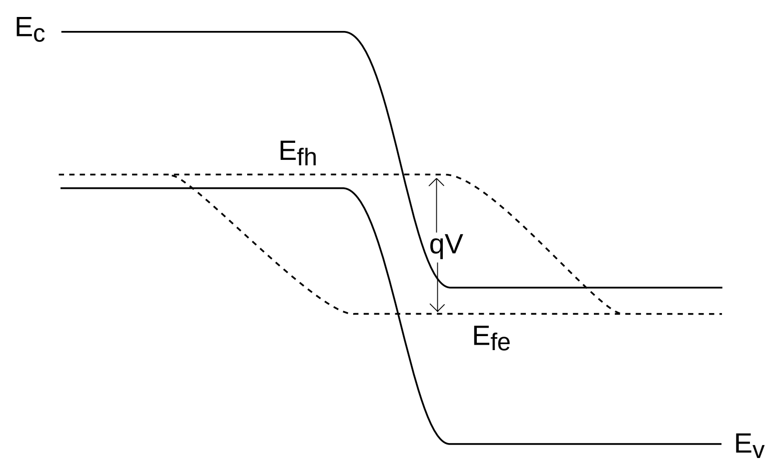
J_s is:

$$J_s \simeq q n_i^2 \left(\frac{1}{N_A}\frac{D_e}{w_p - x_p} + \frac{1}{N_D}\frac{D_h}{w_n - x_n}\right)$$

Quasi-Fermi levels across long diode:



forward bias



reverse bias

Inside SCR:

$$E_{fe} - E_{fh} = qV$$

Then:

$$np = n_i^2 exp\frac{qV}{kT}$$

From here can get also BC's.

Key conclusions

- Rectifying characteristics of pn diode arise from boundary conditions at edges of SCR.
- Excess minority carrier concentration at edges of depletion region:

$$n'(-x_p) = \frac{n_i^2}{N_A}(\exp\frac{qV}{kT} - 1),\ p'(x_n) = \frac{n_i^2}{N_D}(\exp\frac{qV}{kT} - 1)$$

- I-V characteristics of ideal pn diode:

$$I = I_s(\exp\frac{qV}{kT} - 1)$$

- Quasi-Fermi levels flat across SCR:

$$np = n_i^2 \exp\frac{qV}{kT} \qquad \text{inside SCR}$$