Lecture 29 - The "Long" Metal-Oxide-Semiconductor Field-Effect Transistor *(cont.)*

November 13, 2002

Contents:

1. Small-signal equivalent circuit model

Reading assignment:

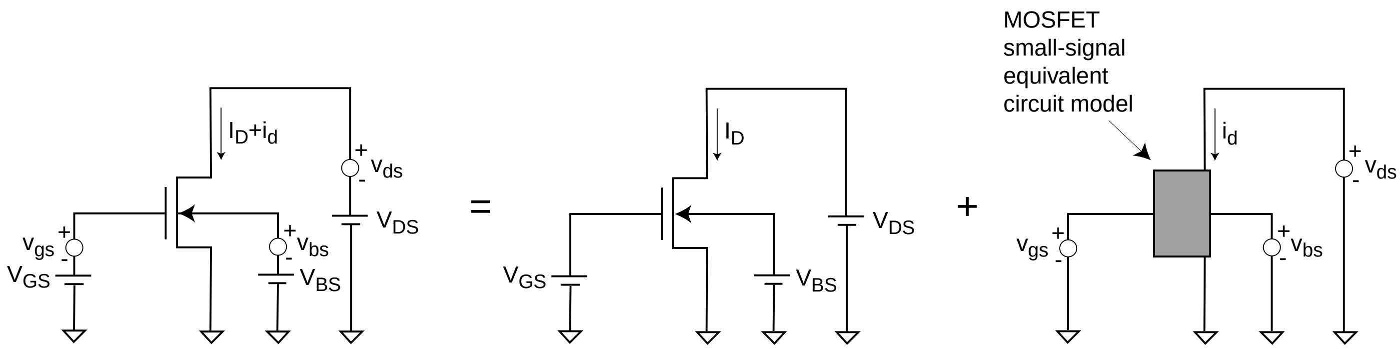
Tsividis, Ch. 8, §§8.1-8.4

Key questions

- What is the topology of a small-signal equivalent circuit model for the MOSFET?
- What are the key bias and geometry dependencies of all small-signal elements in the model?

1. Small-signal equivalent circuit model

In many applications, interested in response of device to *small signal* applied on top of bias:



Key points:

- Small signal is *small* ⇒ non-linear device behavior becomes linear.
- Can separate response of MOSFET to bias and small signal.
- Since response is linear, *superposition* applies ⇒ effects of different small-signals independent from each other.

Mathematically:

$$i_D(V_{GS}, V_{DS}, V_{BS}; v_{gs}, v_{ds}, v_{bs}) \simeq$$

$$I_D(V_{GS}, V_{DS}, V_{BS}) + i_d(v_{gs}, v_{ds}, v_{bs})$$

i_d linear on small-signal drives:

$$i_d \simeq g_m v_{gs} + g_d v_{ds} + g_{mb} v_{bs}$$

Define:

$$g_m \equiv \textit{transconductance}\ [S]$$
$$g_d \equiv \textit{output}\ \text{or}\ \textit{drain conductance}\ [S]$$
$$g_{mb} \equiv \textit{back transconductance}\ [S]$$

Approach to computing g_m, g_d, and g_{mb}:

$$g_m \simeq \frac{\partial I_D}{\partial V_{GS}}\Big|_{V_{DS},V_{BS}}$$

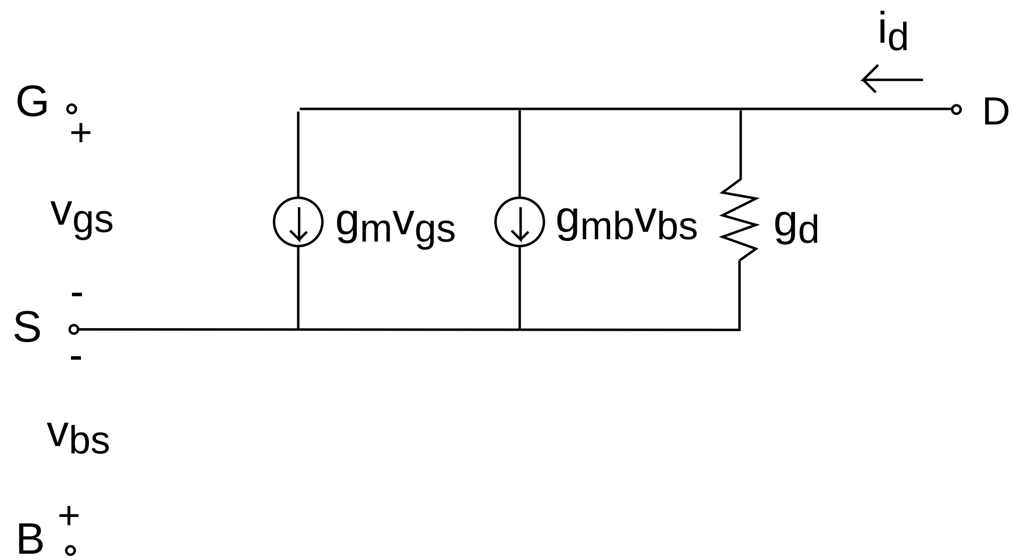
$$g_d \simeq \frac{\partial I_D}{\partial V_{DS}}\Big|_{V_{GS},V_{BS}}$$

$$g_{mb} \simeq \frac{\partial I_D}{\partial V_{BS}}\Big|_{V_{GS},V_{DS}}$$

For mathematical simplicity, do in absence of body effect.

Small-signal low-frequency equivalent circuit model:

$$i_d \simeq g_m v_{gs} + g_d v_{ds} + g_{mb} v_{bs}$$



□ **Transconductance**, g_m:

$$g_m \simeq \frac{\partial I_D}{\partial V_{GS}}\Big|_{V_{DS},V_{BS}}$$

- Linear regime:

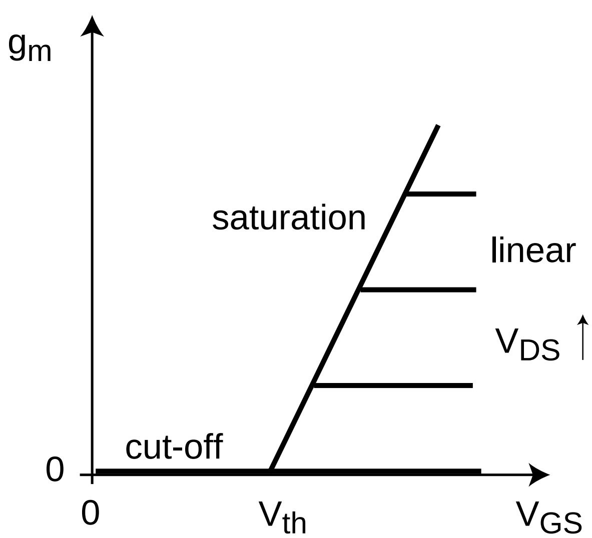
$$I_D = \frac{W}{L}\mu_e C_{ox}(V_{GS} - V_{th} - \frac{1}{2}V_{DS})V_{DS}$$

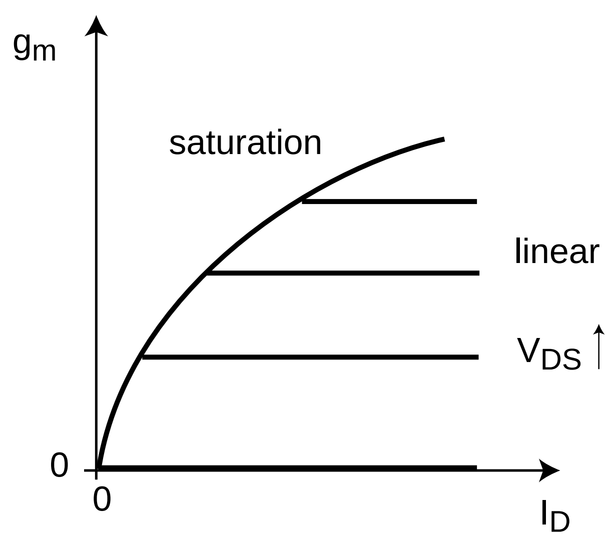
$$g_m = \frac{W}{L}\mu_e C_{ox} V_{DS}$$

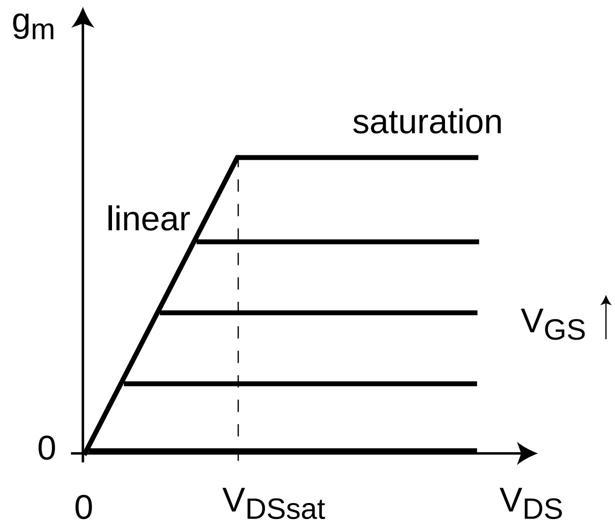
- Saturation regime:

$$I_{Dsat} = \frac{W}{2L}\mu_e C_{ox}(V_{GS} - V_{th})^2$$

$$g_m = \frac{W}{L}\mu_e C_{ox}(V_{GS} - V_{th}) = \sqrt{2\frac{W}{L}\mu_n C_{ox} I_D}$$






gm
saturation
linear
VGS
0
0
VDSsat
VDS

□ **Output conductance**, g_d:

$$g_d \simeq \frac{\partial I_D}{\partial V_{DS}}\Big|_{V_{GS},V_{BS}}$$

- Linear regime:

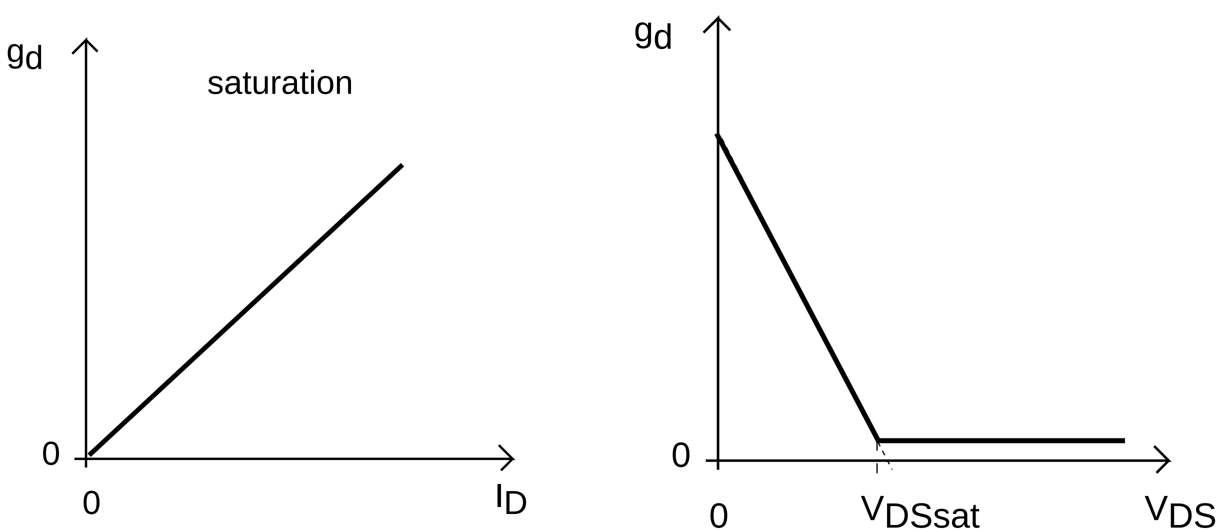
$$I_D = \frac{W}{L}\mu_e C_{ox}(V_{GS} - V_{th} - \frac{1}{2}V_{DS})V_{DS}$$

$$g_d = \frac{W}{L}\mu_e C_{ox}(V_{GS} - V_{th} - V_{DS})$$

- Saturation regime:

$$I_{Dsat} = \frac{W}{2L}\mu_e C_{ox}(V_{GS} - V_{th})^2(1 + \frac{V_{DS} - V_{DSsat}}{\mathcal{E}_p L})$$

$$g_d = \frac{W}{2L}\mu_e C_{ox}(V_{GS} - V_{th})^2\frac{1}{\mathcal{E}_p L} \simeq \frac{I_{Dsat}}{\mathcal{E}_p L}$$



□ **Backgate transconductance**:

$$g_{mb} \simeq \frac{\partial I_D}{\partial V_{BS}}|_{V_{GS},V_{DS}}$$

Back-gate transconductance arises from V_{th} dependence on V_{BS}:

$$g_{mb} \simeq \frac{\partial I_D}{\partial V_{th}}|_{V_{GS},V_{DS}} \frac{\partial V_{th}}{\partial V_{BS}}|_{V_{GS},V_{DS}} = -\frac{\partial I_D}{\partial V_{GS}}|_{V_{th},V_{DS}} \frac{\partial V_{th}}{\partial V_{BS}}|_{V_{GS},V_{DS}}$$

Then:

$$g_{mb} \simeq -g_m \frac{\partial V_{th}}{\partial V_{BS}}|_{V_{GS},V_{DS}}$$

Since:

$$V_{th}(V_{SB}) = V_{tho} + \gamma(\sqrt{\phi_{sth} + V_{SB}} - \sqrt{\phi_{sth}})$$

Then:

$$\frac{\partial V_{th}}{\partial V_{BS}}|_{V_{GS},V_{DS}} = -\frac{\gamma}{2\sqrt{\phi_{sth} + V_{SB}}}$$

Then:

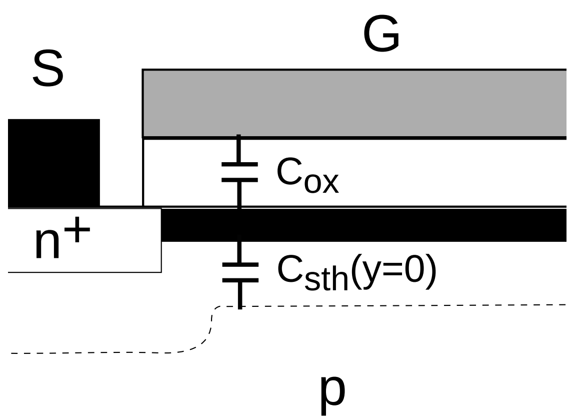
$$g_{mb} \simeq \frac{\gamma g_m}{2\sqrt{\phi_{sth} + V_{SB}}}$$

g_{mb} inherits all dependences of g_m.

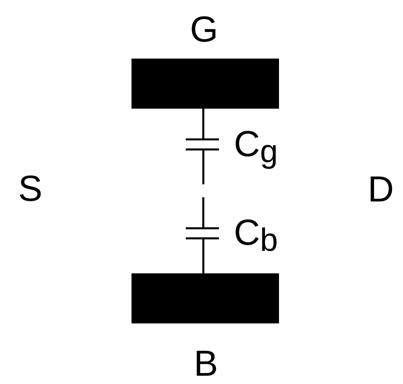
Can be rewritten as:

$$\frac{g_{mb}}{g_m} \simeq \frac{C_{sth}(V_{SB}, y = 0)}{C_{ox}}$$

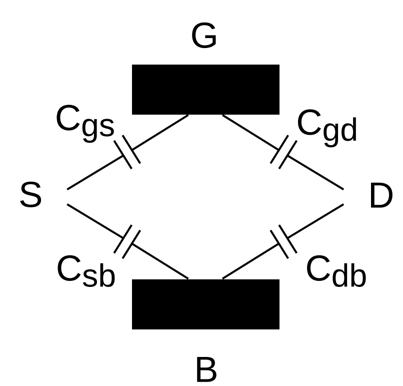
g_{mb}: another manifestation of electrostatic control of inversion layer by body.



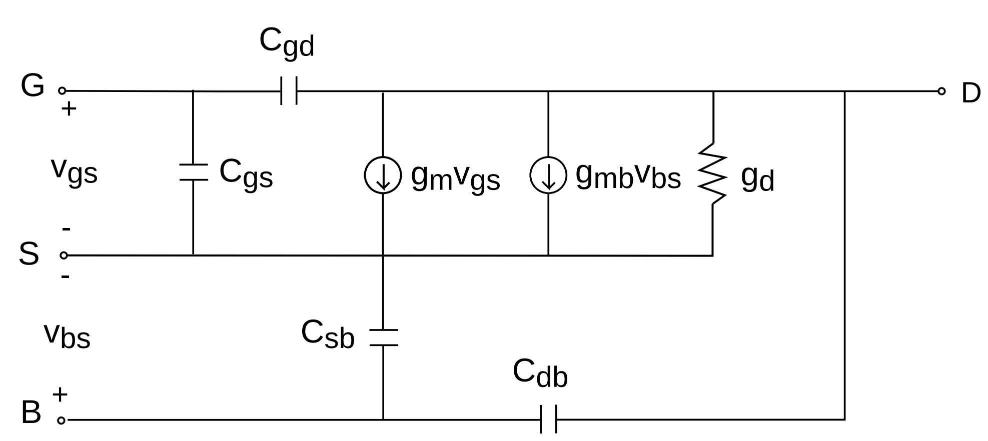
At **high frequency**, $\Delta V \Rightarrow \Delta Q \Rightarrow$ capacitive effect.



But inversion layer has distributed charge:

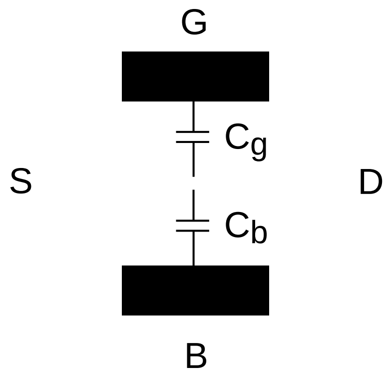


High-frequency small-signal equivalent circuit model:



□ *Intrinsic capacitances*

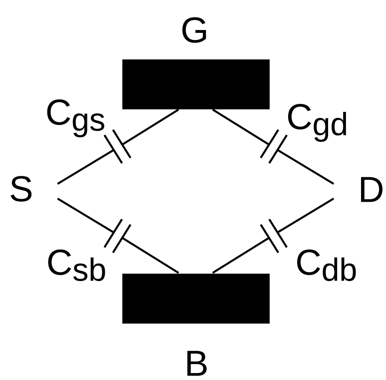
• When lumped:



To first order, can write C_b in terms of C_g:

$$\frac{C_b}{C_g} \simeq \frac{C_{sth}}{C_{ox}} \;\Rightarrow\; C_b \simeq \frac{C_{sth}}{C_{ox}} C_g = \frac{\gamma}{2\sqrt{\phi_{sth} + V_{SB}}} C_g$$

• Distributed:



$$C_{gs} = -\frac{\partial Q_I}{\partial V_{GS}} \qquad C_{gd} = -\frac{\partial Q_I}{\partial V_{GD}}$$

$$C_{sb} \simeq \frac{\gamma}{2\sqrt{\phi_{sth} + V_{SB}}} C_{gs} \qquad C_{db} \simeq \frac{\gamma}{2\sqrt{\phi_{sth} + V_{SB}}} C_{gd}$$

To compute capacitances, must compute total integrated charge in inversion layer, Q_I.

For mathematical simplicity, do in absence of body effect:

$$Q_I = W \int_0^L Q_i(y) dy = W \int_0^{V_{DS}} Q_i(V) \frac{dy}{dV} dV$$

From MOSFET current lecture,

$$J_e = -\frac{I_D}{W} = \mu_e Q_i \frac{dV}{dy}$$

Then:

$$\frac{dy}{dV} = -\frac{W \mu_e}{I_D} Q_i$$

Therefore:

$$Q_I = -\frac{W^2 \mu_e}{I_D} \int_0^{V_{DS}} Q_i^2(V) dV$$

Also,

$$Q_i(V) = -C_{ox}(V_{GS} - V - V_{th})$$

Assume V_{th} is independent of V (weak body effect).

All together:

$$Q_I = -\frac{W^2\mu_e C_{ox}^2}{I_D}\int_0^{V_{DS}}(V_{GS}-V-V_{th})^2 dV = \frac{W^2\mu_e C_{ox}^2}{I_D}\frac{1}{3}(V_{GS}-V-V_{th})^3\Big|_0^{V_{DS}}$$

$$= \frac{1}{3}W^2\mu_e C_{ox}^2\frac{(V_{GS}-V_{DS}-V_{th})^3-(V_{GS}-V_{th})^3}{\frac{W}{L}\mu_e C_{ox}(V_{GS}-V_{th}-\frac{1}{2}V_{DS})V_{DS}}$$

Note also,

$$(V_{GS}-V_{th}-\frac{1}{2}V_{DS})V_{DS} = \frac{1}{2}[(V_{GS}-V_{th})^2-(V_{GD}-V_{th})^2]$$

Then:

$$Q_I = -\frac{2}{3}WLC_{ox}\frac{(V_{GS}-V_{th})^3-(V_{GD}-V_{th})^3}{(V_{GS}-V_{th})^2-(V_{GD}-V_{th})^2}$$

Further mathematical simplification:

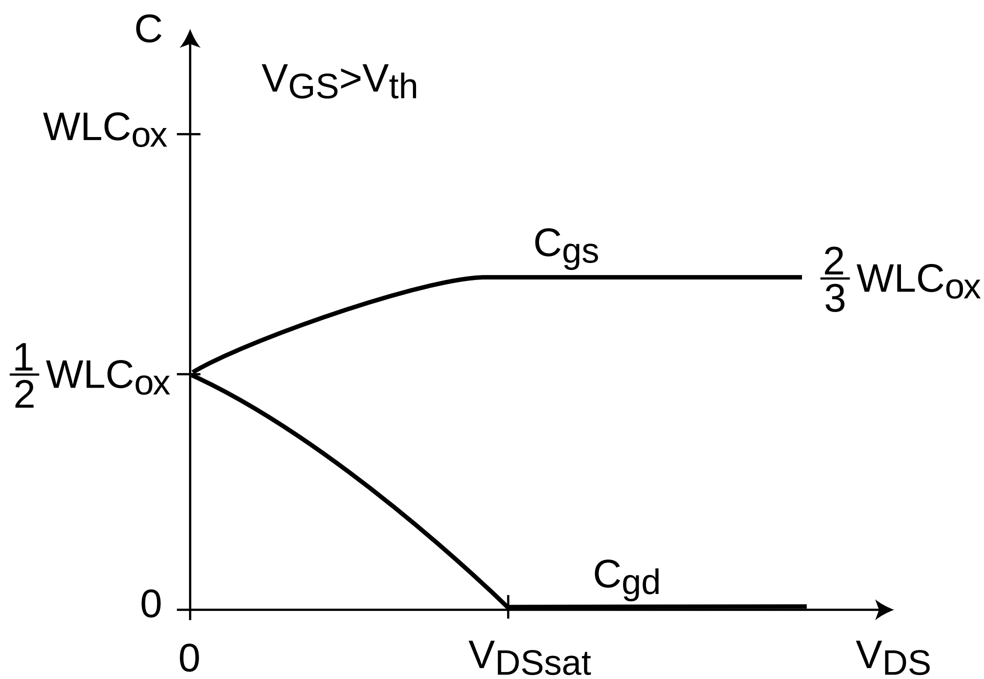
$$Q_I = -\frac{2}{3}WLC_{ox}\frac{(V_{GS}-V_{th})^2+(V_{GS}-V_{th})(V_{GD}-V_{th})+(V_{GD}-V_{th})^2}{(V_{GS}-V_{th})+(V_{GD}-V_{th})}$$

Compute capacitances:

$$C_{gs} = -\frac{\partial Q_I}{\partial V_{GS}}|_{V_{GD}} = \frac{1}{2}WLC_{ox}(V_{GS}-V_{th})\frac{V_{GS}-V_{th}-\frac{2}{3}V_{DS}}{(V_{GS}-V_{th}-\frac{1}{2}V_{DS})^2}$$

$$C_{gd} = -\frac{\partial Q_I}{\partial V_{GD}}|_{V_{GS}} = \frac{1}{2}WLC_{ox}(V_{GS}-V_{DS}-V_{th})\frac{V_{GS}-V_{th}-\frac{1}{3}V_{DS}}{(V_{GS}-V_{th}-\frac{1}{2}V_{DS})^2}$$

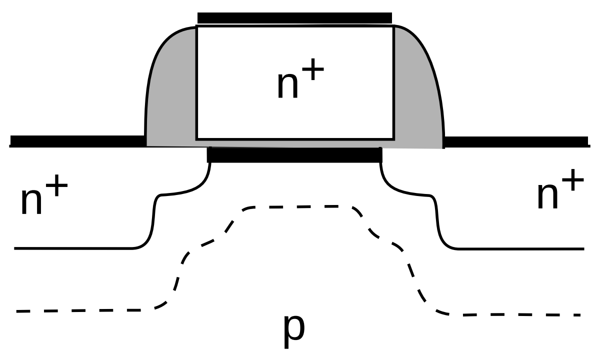
Evolution with V_{DS}:



Limits:

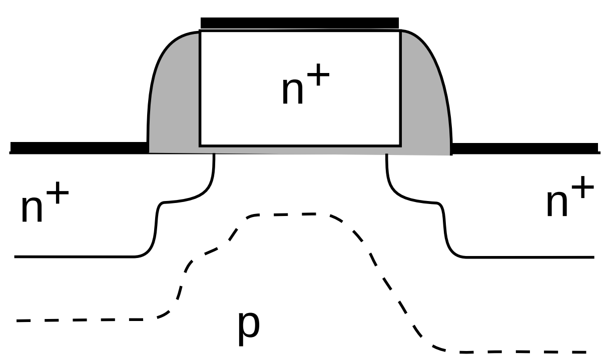
- Linear regime (small V_{DS}): inversion layer charge uniformly distributed across channel.

$$C_{gs} \simeq C_{gd} \simeq \frac{1}{2} W L C_{ox}$$



- Saturation regime ($V_{DS} > V_{DSsat}$): channel pinched-off at drain end.

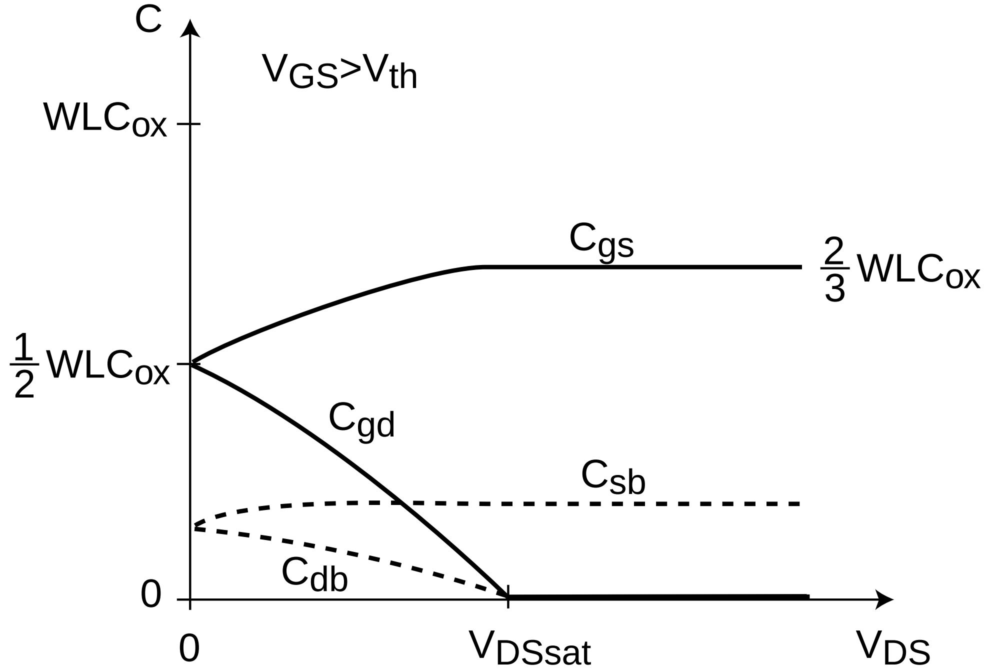
$$C_{gs} \simeq \frac{2}{3} W L C_{ox} \qquad C_{gd} \simeq 0$$



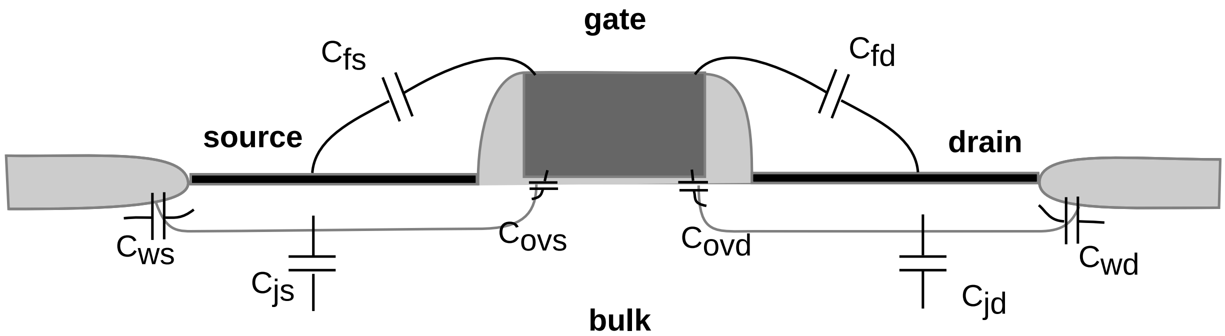
Body capacitances:

$$C_{sb} \simeq \frac{\gamma}{2\sqrt{\phi_{sth} + V_{SB}}} C_{gs}$$

$$C_{db} \simeq \frac{\gamma}{2\sqrt{\phi_{sth} + V_{SB}}} C_{gd}$$



□ *Extrinsic capacitances*



- Source and drain junction capacitances (including sidewall capacitances.
- Source and drain overlap capacitances.
- Gate fringing capacitances.

All together:

$$C_{gs} = C_{gs}^{int} + C_{ovs} + C_{fs}$$
$$C_{gd} = C_{gd}^{int} + C_{ovd} + C_{fd}$$
$$C_{sb} = C_{sb}^{int} + C_{js} + C_{ws}$$
$$C_{db} = C_{db}^{int} + C_{jd} + C_{wd}$$

In saturation:

$$C_{gs} \simeq \frac{2}{3} W L C_{ox} + C_{ovs} + C_{fs}$$
$$C_{gd} \simeq C_{ovd} + C_{fd} \ll C_{gs}$$
$$C_{sb} \simeq \frac{\gamma W L C_{ox}}{3\sqrt{\phi_{sth} + V_{SB}}} + C_{js} + C_{ws}$$
$$C_{db} \simeq C_{jd} + C_{wd}$$

Key conclusions

- *Transconductance.* In saturation regime:

$$g_m \propto \sqrt{I_D}$$

- *Drain conductance.* In saturation regime:

$$g_d \propto \frac{I_D}{L}$$

- *Back transconductance*:

$$g_{mb} \simeq g_m \frac{C_{sth}(V_{SB}, y = 0)}{C_{ox}}$$

- In saturation, C_{gs} dominated by intrinsic gate-inversion layer capacitance:

$$C_{gs} \simeq \frac{2}{3} W L C_{ox}$$

- In saturation, C_{gd} dominated by overlap and fringe capacitances:

$$C_{gd} \simeq C_{ovd} + C_{fd} \ll C_{gs}$$

- C_{sb} dominated by source-body junction capacitance:

$$C_{sb} \simeq C_{js}$$

- C_{db} dominated by drain-body junction capacitance:

$$C_{db} \simeq C_{jd}$$