Lecture 36 - Bipolar Junction Transistor *(cont.)*

December 2, 2002

Contents:

1. Current-voltage characteristics of ideal BJT *(cont.)*
2. Charge-voltage characteristics of ideal BJT
3. Small-signal behavior of ideal BJT

Reading material:

del Alamo, Ch. 11, §§11.2 (11.2.5), 11.3, 11.4 (11.4.1)

Announcements:

Note special schedule for the end of the semester:

- Dec. 5: lecture
- Dec. 6: lecture
- Dec. 9: guest lecture by Prof. H. Tuller
- Dec. 11: recitation

All in regular room and regular meeting times.

Key questions

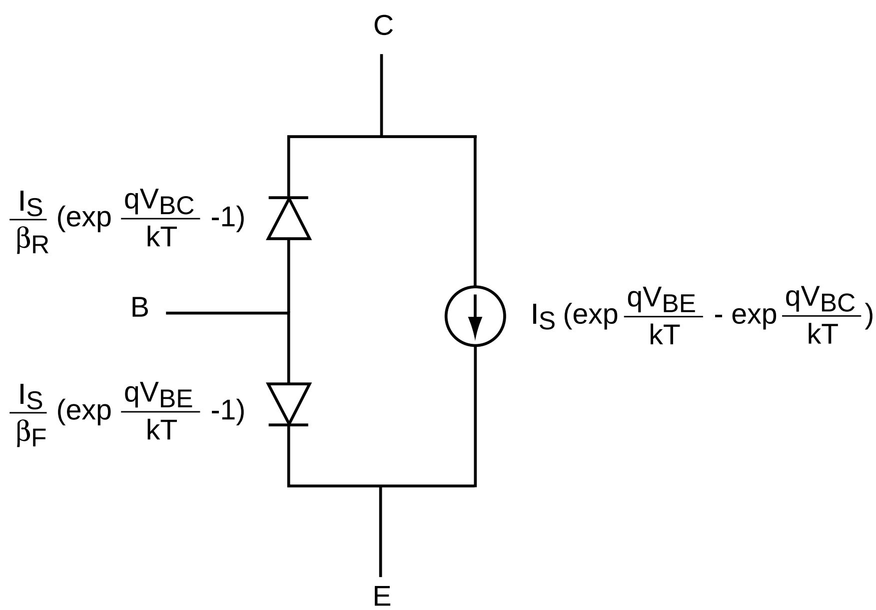
- How do the output characteristics of the ideal BJT look like?
- How do the charge-voltage characteristics of the ideal BJT look like?
- What is the topology of the small-signal equivalent circuit model of the ideal BJT in FAR?
- What are the key dependencies of its elements?

1. Current-voltage characteristics of ideal BJT *(cont.)*

Ideal BJT current equations (superposition of forward active + reverse):

$$
\begin{aligned}
I_C &= I_S(\exp\frac{qV_{BE}}{kT} - \exp\frac{qV_{BC}}{kT}) - \frac{I_S}{\beta_R}(\exp\frac{qV_{BC}}{kT} - 1) \\
I_B &= \frac{I_S}{\beta_F}(\exp\frac{qV_{BE}}{kT} - 1) + \frac{I_S}{\beta_R}(\exp\frac{qV_{BC}}{kT} - 1) \\
I_E &= -\frac{I_S}{\beta_F}(\exp\frac{qV_{BE}}{kT} - 1) - I_S(\exp\frac{qV_{BE}}{kT} - \exp\frac{qV_{BC}}{kT})
\end{aligned}
$$

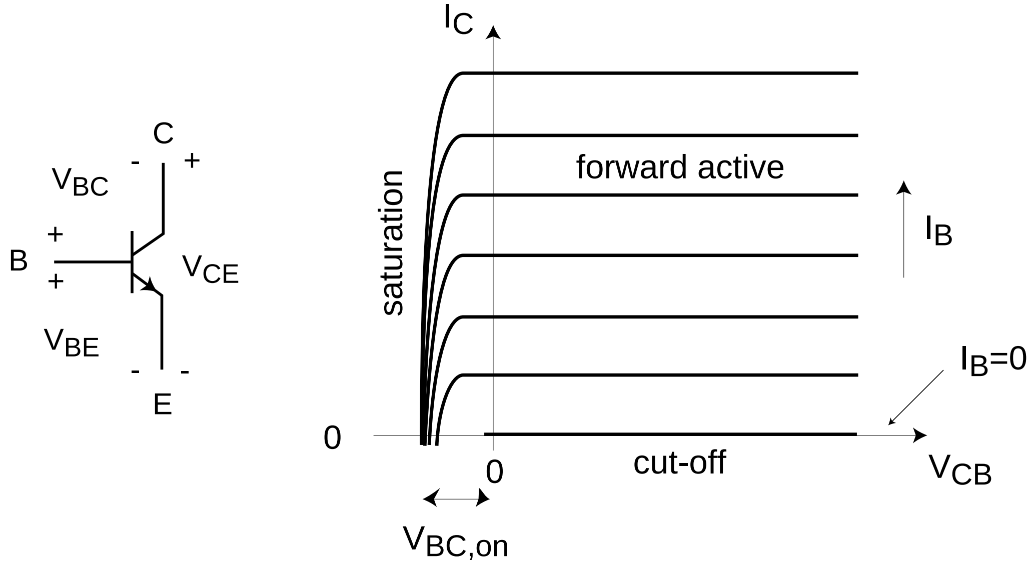
Equivalent circuit model representation:



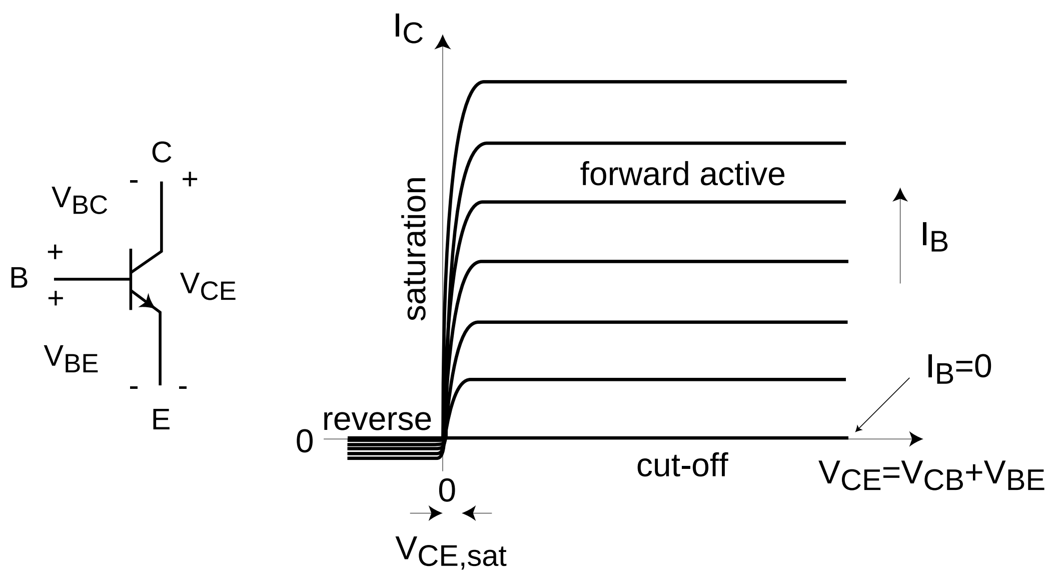
Complete model has only three parameters: I_S, β_F, and β_R.

□ Common-emitter output I-V characteristics

vs. V_{CB}:

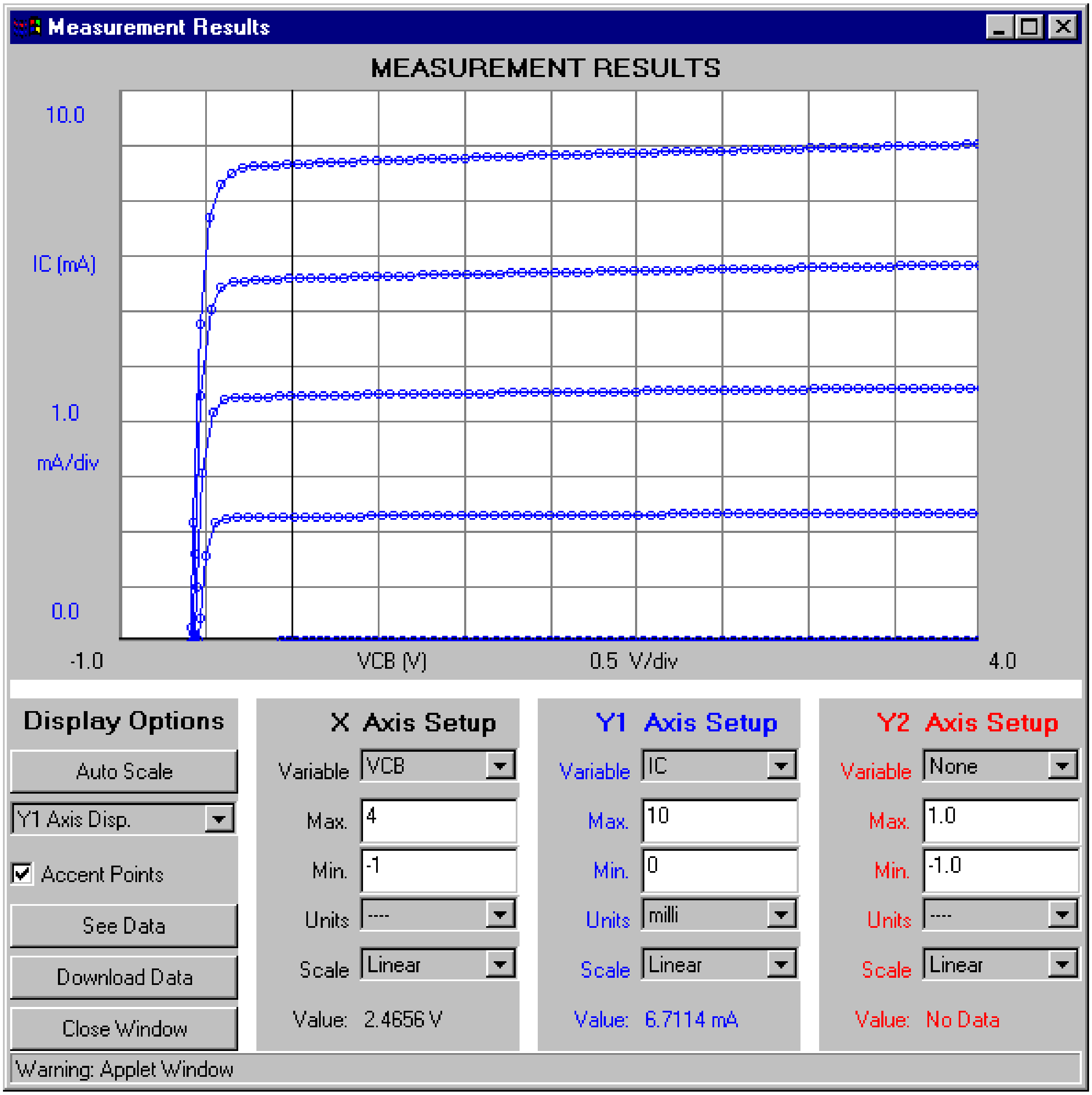


vs. V_{CE}:

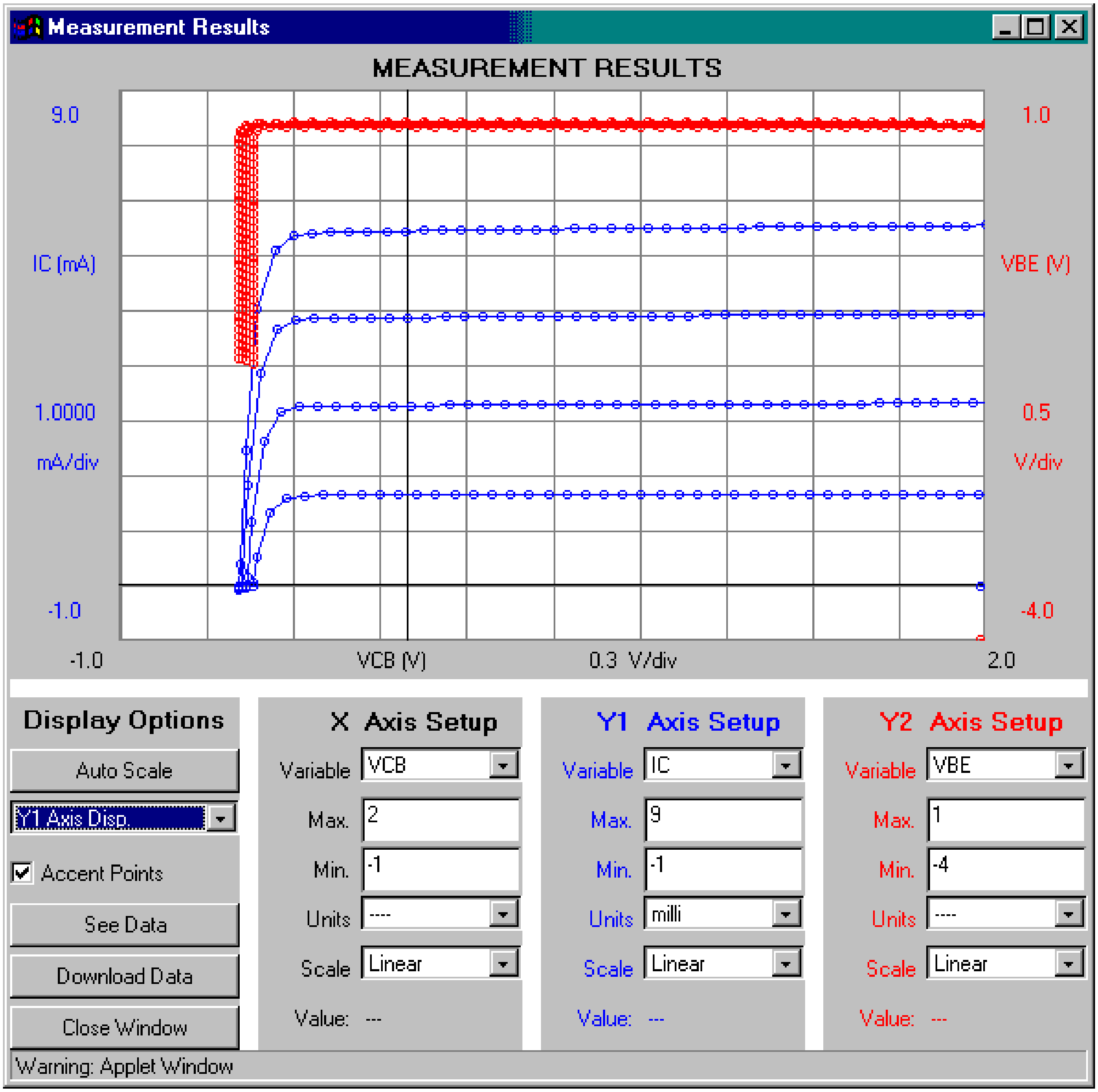


$$V_{CEsat} = -V_{BCon} + V_{BEon}$$

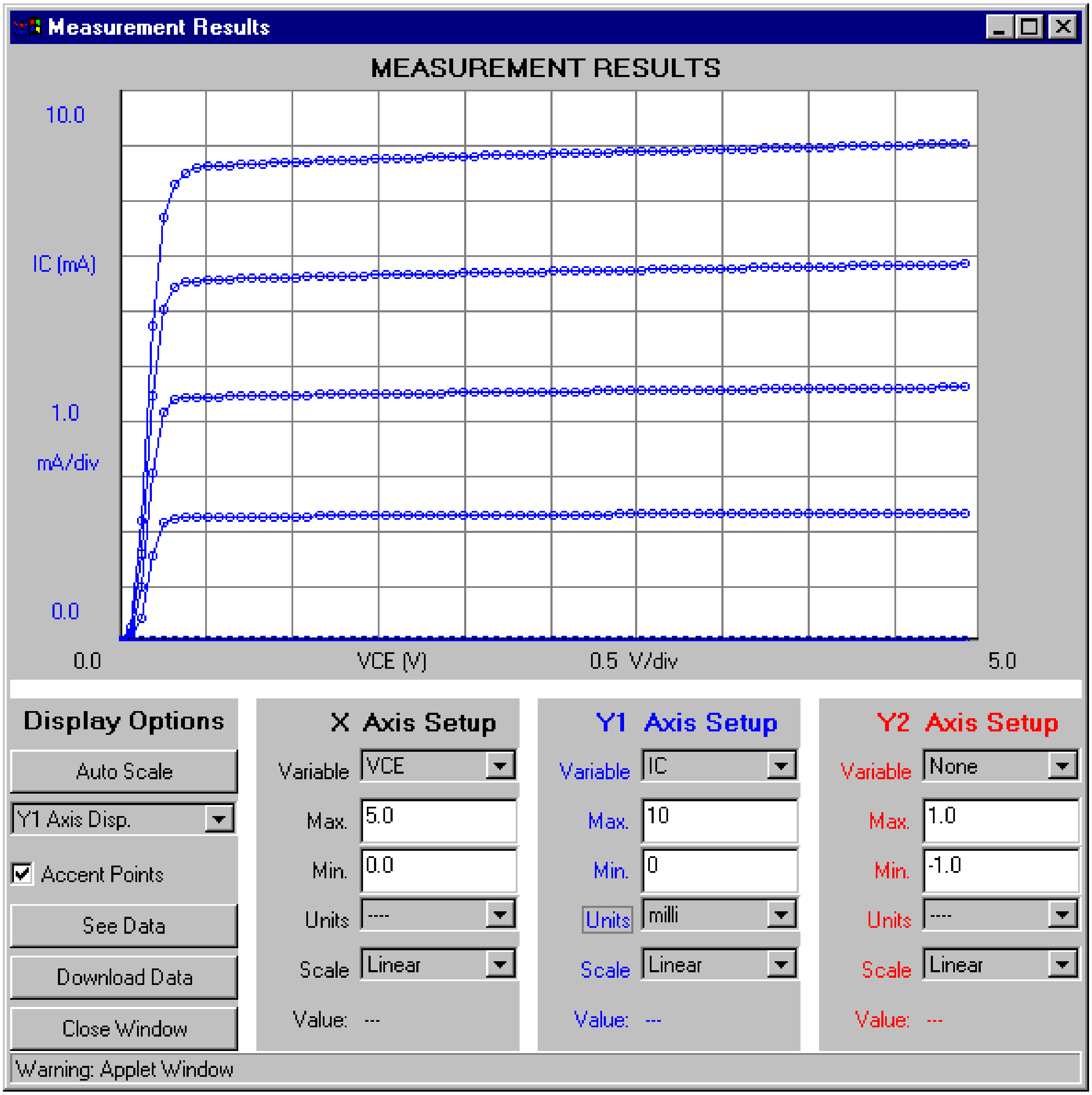
I_C vs. V_{CB} with I_B as parameter:



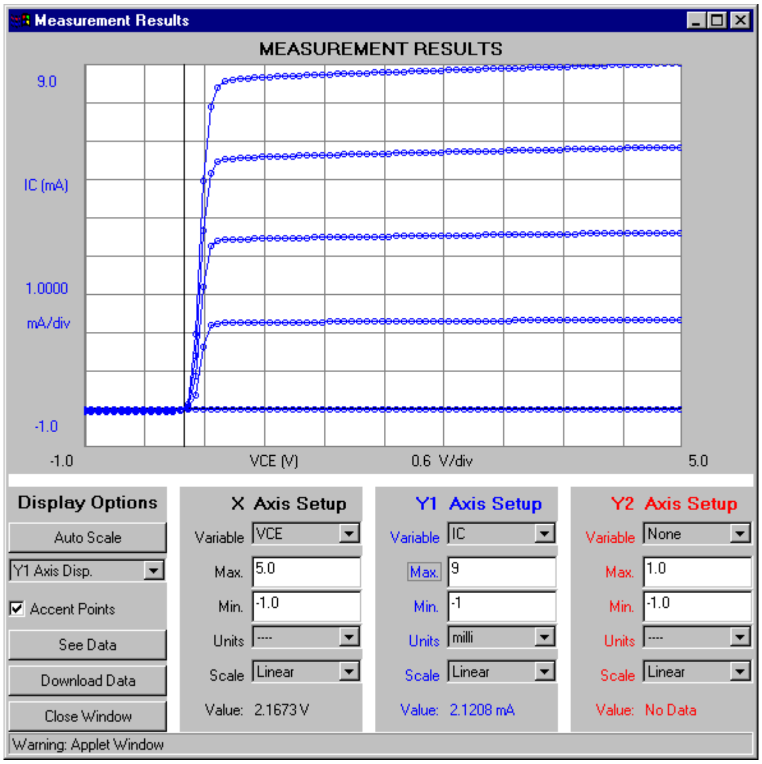
Where is the reverse regime?

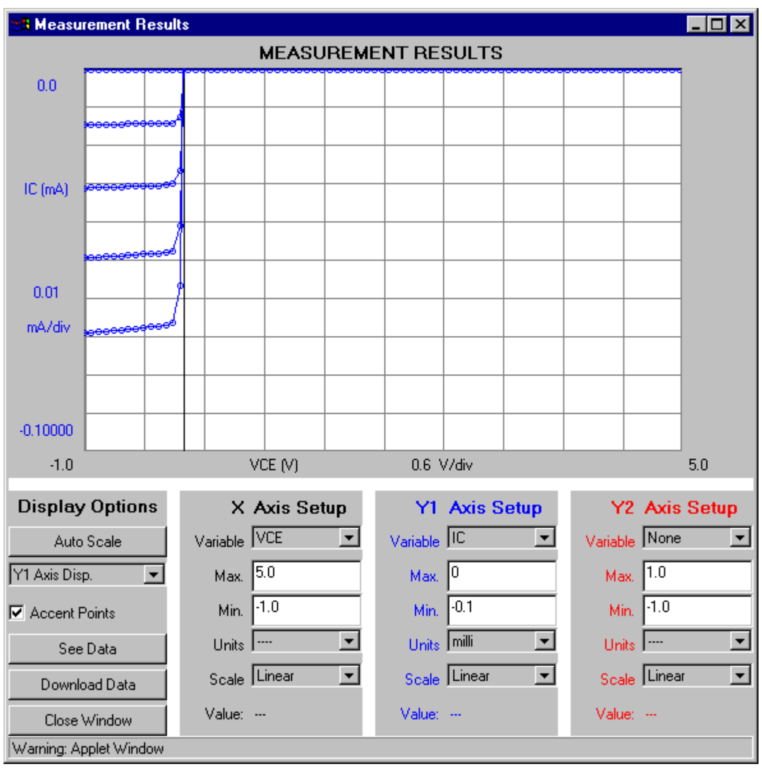


Common-emitter output characteristics:



Zoom into inverse regime:



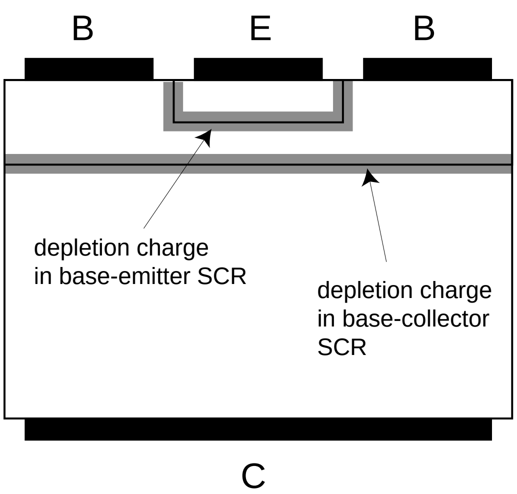


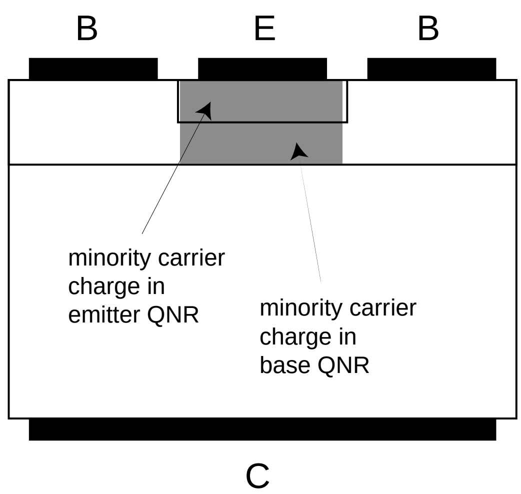
2. Charge-voltage characteristics of ideal BJT

In BJT, two types of stored charge:

- depletion layer charge
- minority carrier charge

In forward-active regime:





□ Depletion layer charge

In B-E and B-C SCR's, respectively:

$$Q_{jE} = A_E\sqrt{\frac{2\epsilon q N_E N_B(\phi_{biE} - V_{BE})}{N_E + N_B}}$$

$$Q_{jC} = A_C\sqrt{\frac{2\epsilon q N_B N_C(\phi_{biC} - V_{BC})}{N_B + N_C}}$$

ϕ_{biE} and ϕ_{biC} are respective built-in potentials.

Since $N_E \gg N_B \gg N_C$,

$$Q_{jE} \simeq A_E\sqrt{2\epsilon q N_B(\phi_{biE} - V_{BE})}$$

$$Q_{jC} \simeq A_C\sqrt{2\epsilon q N_C(\phi_{biC} - V_{BC})}$$

Depletion capacitance:

$$C_{je} = \frac{\partial Q_{jE}}{\partial V_{BE}} \simeq A_E\sqrt{\frac{\epsilon q N_B}{2(\phi_{biE} - V_{BE})}} = \frac{C_{jeo}}{\sqrt{1 - \frac{V_{BE}}{\phi_{biE}}}}$$

$$C_{jc} = \frac{\partial Q_{jC}}{\partial V_{BC}} \simeq A_C\sqrt{\frac{\epsilon q N_C}{2(\phi_{biC} - V_{BC})}} = \frac{C_{jco}}{\sqrt{1 - \frac{V_{BC}}{\phi_{biC}}}}$$

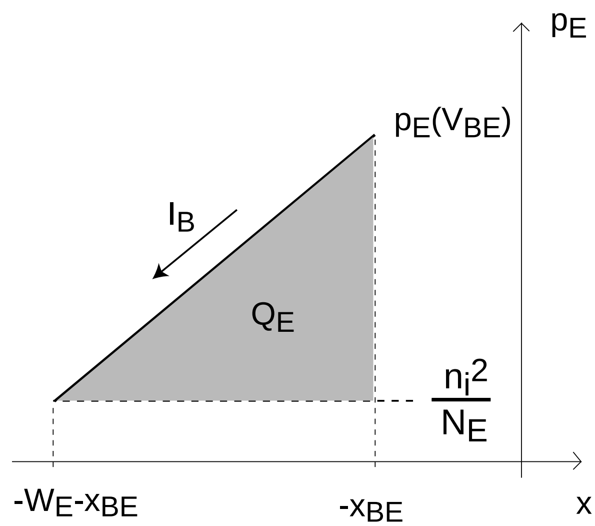
□ Minority carrier charge

Excess minority carriers in QNR's ⇒ excess majority carriers to keep quasi-neutrality ⇒ diffusion capacitance.

Key result from pn diode: in "short" or "transparent" QNR:

$$\textit{stored charge} = \textit{minority carrier transit time} \times \textit{injected minority carrier current}$$

• For **emitter** in FAR:

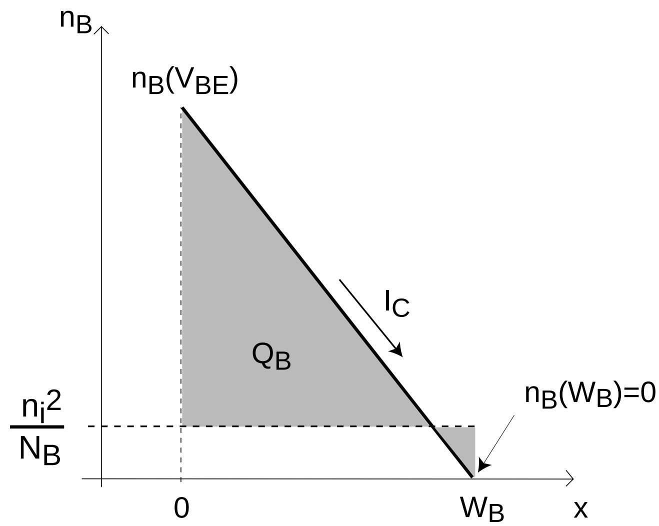


$$Q_E = \tau_{tE} I_B$$

with hole transit time:

$$\tau_{tE} = \frac{W_E^2}{2D_E}$$

• For **base** in FAR:



$$Q_B = \tau_{tB} I_C$$

with electron transit time:

$$\tau_{tB} = \frac{W_B^2}{2D_B}$$

Comments:

- Units of Q_E and Q_B are C.
- Q_E and Q_B scale with A_E.

Total minority carrier charge in FAR:

$$Q_F = Q_E + Q_B = \tau_{tE} I_B + \tau_{tB} I_C = (\frac{\tau_{tE}}{\beta_F} + \tau_{tB}) I_C = \tau_F I_C$$

$$\tau_F \equiv \textit{intrinsic delay}\ [\mathrm{s}]$$

τ_F is overall time constant for minority carrier storage in BJT in FAR:

$$\tau_F = \frac{\tau_{tE}}{\beta_F} + \tau_{tB}$$

Note: emitter contribution to τ_F is τ_{tE}/β_F because I_B is β_F times smaller than I_C.

If V_{BE} changes, Q_E and Q_B change $\Rightarrow$ capacitive effect:

$$C_F = \frac{dQ_F}{dV_{BE}} = \tau_F \frac{qI_C}{kT}$$

Location of this capacitance? Think of which terminals supply stored charge (minority and majority carriers):

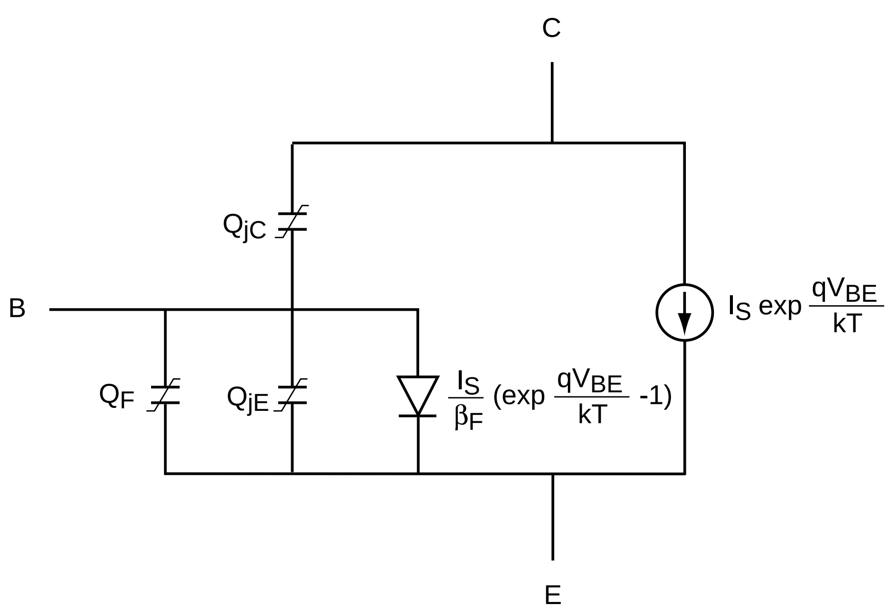
For Q_E:

- minority carriers (holes) injected from *base*
- majority carriers (electrons) come from *emitter* contact

For Q_B:

- minority carriers (electrons) injected from *emitter*
- majority carriers (holes) come from *base* contact

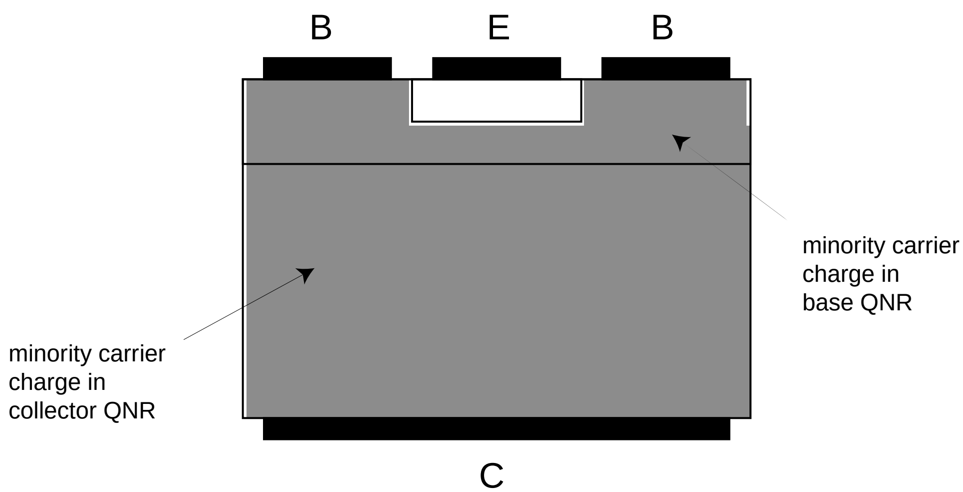
Equivalent-circuit model:



Similar picture in reverse regime: charge storage in base and collector

$$Q_R = \tau_R I_E$$

τ_R a bit complicated because it accounts for charge storage in *intrinsic* and *extrinsic* base and collector regions.

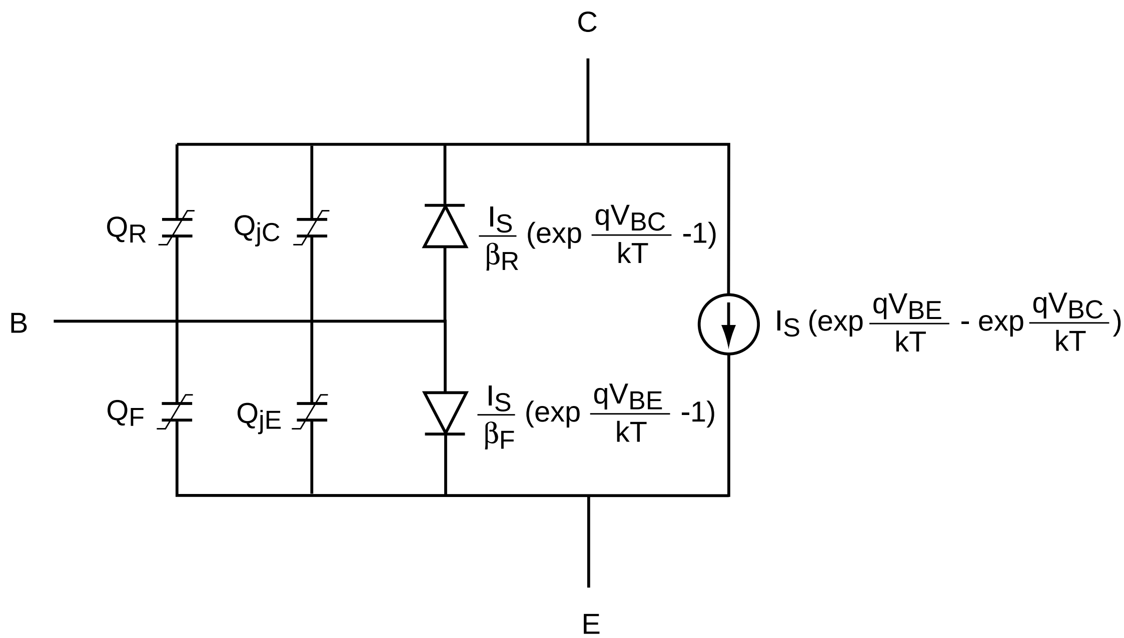


Diffusion capacitance:

$$C_R = \frac{dQ_R}{dV_{BC}} = \tau_R \frac{qI_E}{kT}$$

Located between base and collector terminals.

By superposition, complete equivalent circuit model valid in all four regimes:



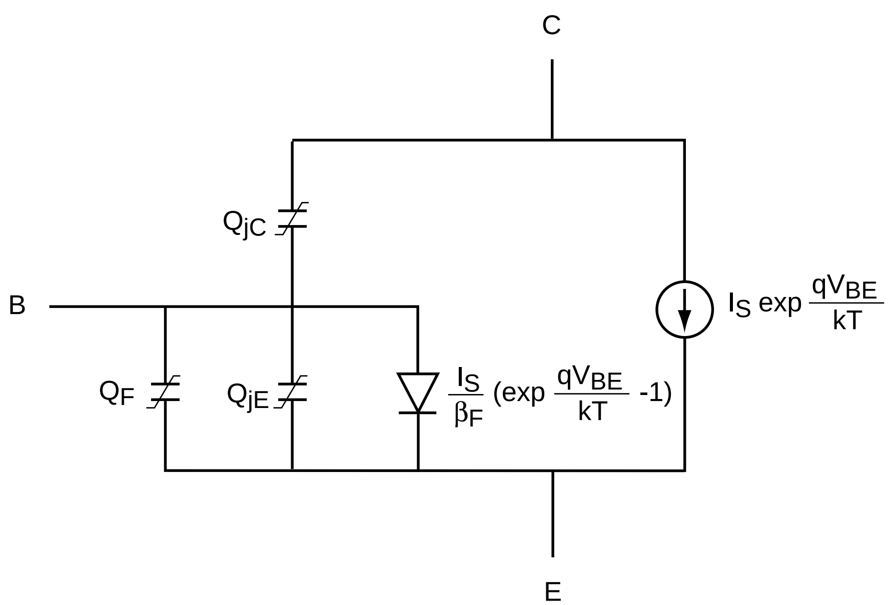
3. Small-signal behavior of ideal BJT

In analog (and digital) applications, interest in behavior of BJT to *small-signal* applied on top of bias

$\Rightarrow$ small-signal equivalent circuit model.

□ Small-signal equivalent circuit model in FAR

Must *linearize* hybrid-π model in FAR:



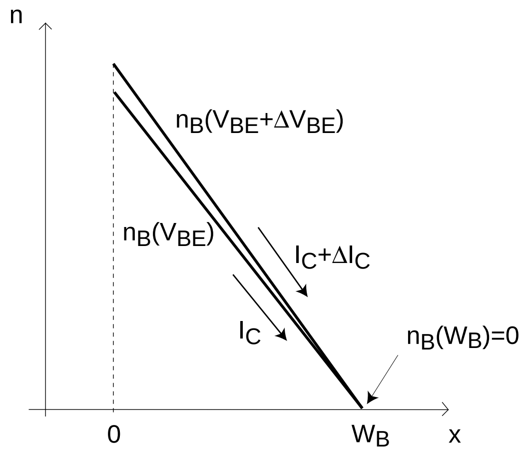
-Non-linear voltage-controlled current source linearized to *linear voltage-controlled current source.*

-Diode linearized to *resistor*.

-Charge storage elements linearized to *capacitors*.

- *Linearized voltage-controlled current source*

Apply small signal v_{be} on top of bias V_{BE}.



Collector current:

$$I_C+i_c = I_S \exp\frac{q(V_{BE}+v_{be})}{kT} \simeq I_S \exp\frac{qV_{BE}}{kT}(1+\frac{qv_{be}}{kT}) = I_C(1+\frac{qv_{be}}{kT})$$

Small-signal collector current:

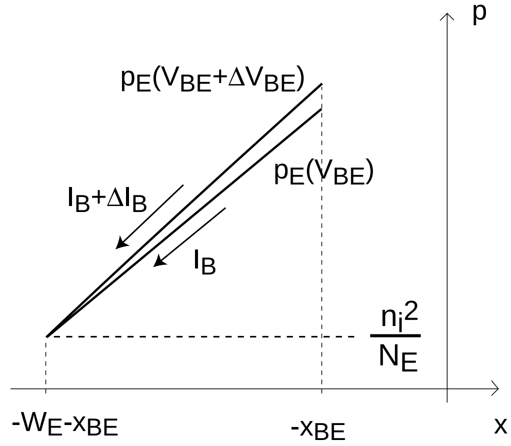
$$i_c = \frac{qI_C}{kT}v_{be}$$

Define *transconductance*:

$$g_m = \frac{qI_C}{kT}$$

g_m depends only on absolute value of I_C and T (unlike MOSFET, where g_m depends on device geometry)

• *Linearized diode*



Base current:

$$I_B + i_b = I_S \exp \frac{q(V_{BE} + v_{be})}{kT} \simeq \frac{I_S}{\beta_F} \exp \frac{qV_{BE}}{kT}(1 + \frac{qv_{be}}{kT})$$

Small-signal base current:

$$i_b = \frac{qI_B}{kT} v_{be}$$

Define *conductance*:

$$g_\pi = \frac{qI_B}{kT} = \frac{q}{kT}\frac{I_C}{\beta_F} = \frac{g_m}{\beta_F}$$

Then, in general,

$$g_\pi \ll g_m$$

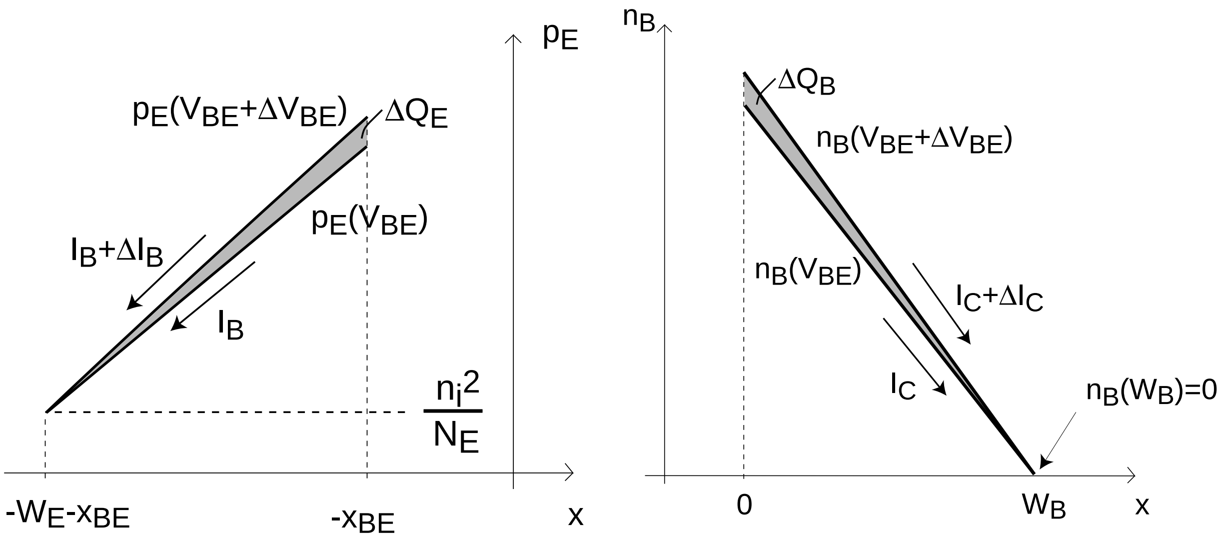
- *Capacitors*

$$Q_{jE} \rightarrow C_{je}$$

$$Q_{jC} \rightarrow C_{jc}$$

$$Q_F \rightarrow C_\pi$$

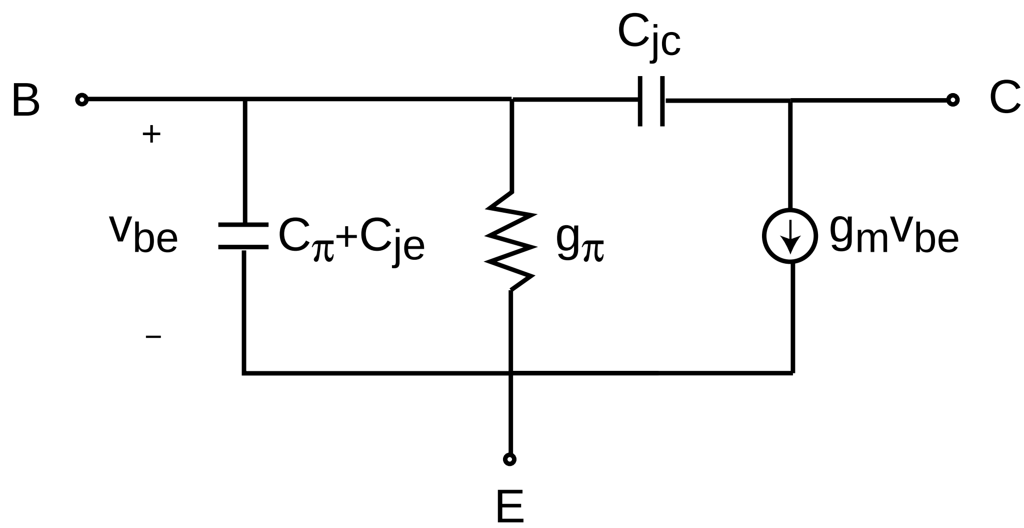
Two components in C_π:



Note:

$$C_\pi = \tau_F g_m$$

- Small-signal equivalent circuit model for ideal BJR in FAR:



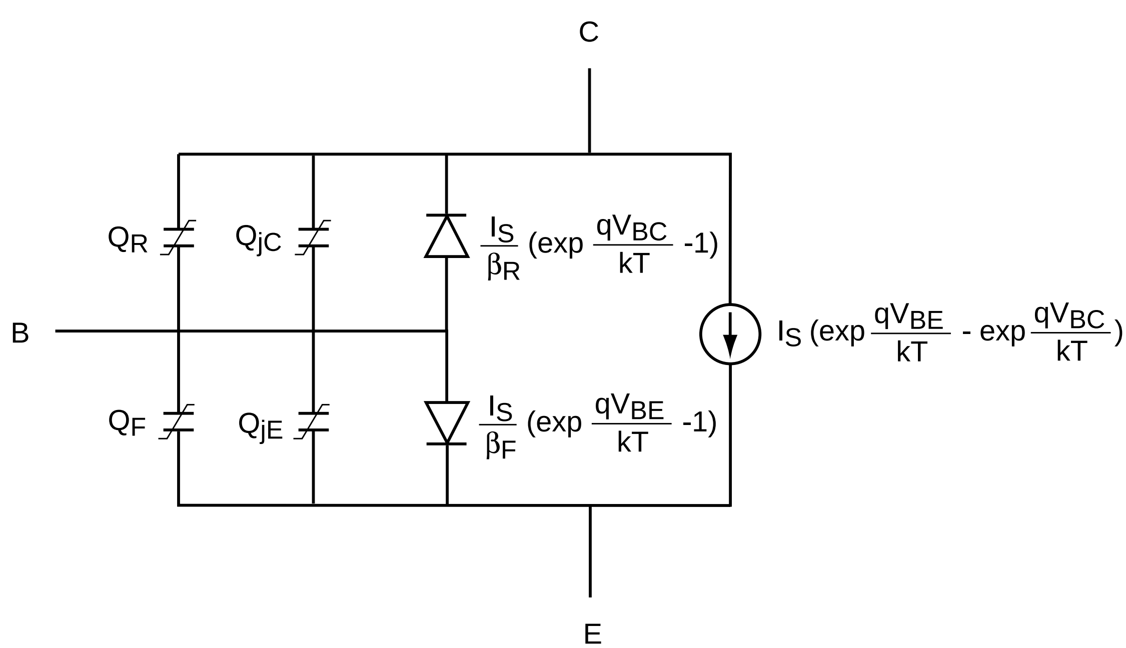
Key conclusions

- In BJT, two types of stored charge: depletion layer charge and minority carrier charge.
- Depletion layer charge accounted through depletion capacitances.
- Minority carrier charge accounted through time constant τ_F (*intrinsic delay*):

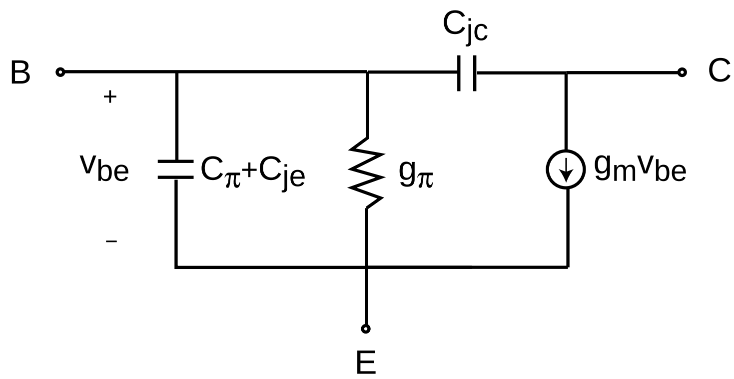
$$\tau_F = \frac{\tau_{tE}}{\beta_F} + \tau_{tB}$$

  Emitter contribution to τ_F is β_F times smaller than τ_{tE} because I_B is β_F times smaller than I_C.

- Non-linear hybrid-π model for ideal BJT including charge storage elements:



- Small-signal equivalent circuit model of ideal BJT in FAR:



with:

$$g_m = \frac{qI_C}{kT} \qquad g_\pi = \frac{qI_B}{kT} = \frac{g_m}{\beta_F} \qquad C_\pi = \tau_F g_m$$