Lecture 37 - Bipolar Junction Transistor *(cont.)*

December 4, 2002

Contents:

1. Common-emitter short-circuit current-gain cut-off frequency, f_T

Reading material:

del Alamo, Ch. 11, §11.4.2

Key questions

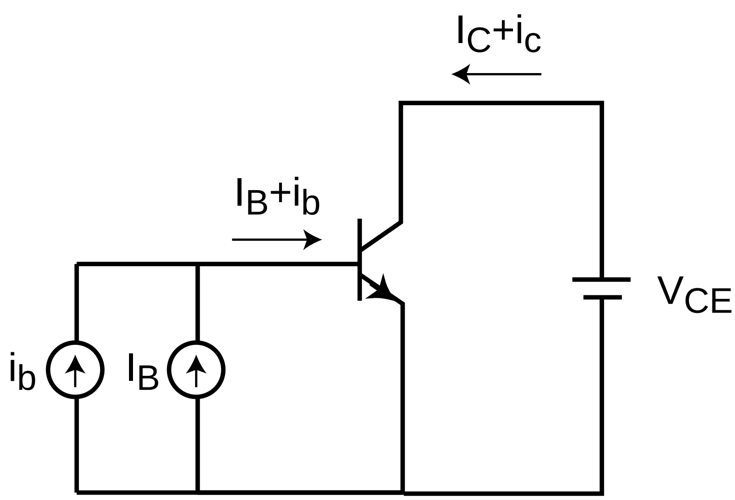
- How is the frequency response of a transistor assessed?
- What determines the frequency response of an ideal BJT?

1. Common-emitter short-circuit current-gain cut-off frequency, f_T

f_T: high-frequency figure of merit for transistors

Short-circuit means from the small-signal point of view.

BJT is biased in FAR.



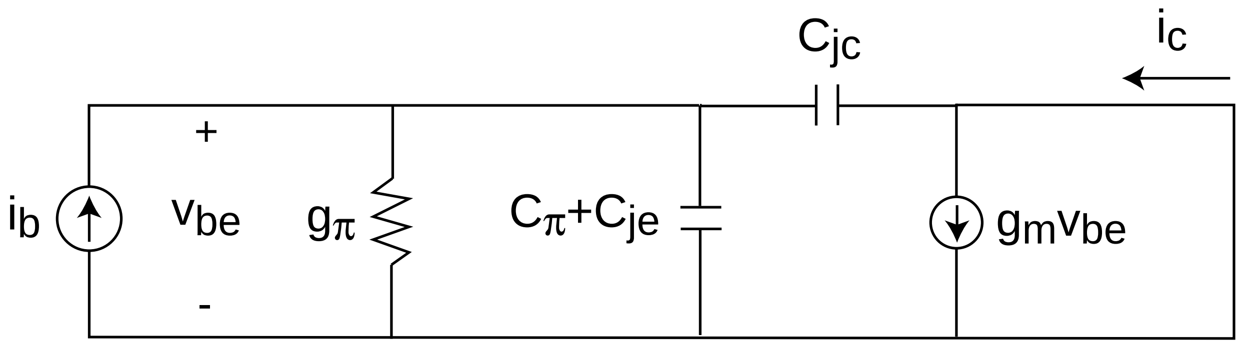
Focus on small-signal current gain:

$$h_{21} = \frac{i_c}{i_b}|_{v_{ce}=0}$$

For low frequency, $h_{21} \rightarrow \beta_F$, for high frequency h_{21} rolls off due to capacitors.

Definition of f_T: frequency at which $|h_{21}| = 1$.

Small-signal equivalent circuit model:



$$
\begin{aligned}
i_c &= g_m v_{be} - j\omega C_{jc} \\
i_b &= [g_\pi + j\omega(C_\pi + C_{je} + C_{jc})]v_{be}
\end{aligned}
$$

Then:

$$h_{21} = \frac{g_m - j\omega C_{jc}}{g_\pi + j\omega(C_\pi + C_{je} + C_{jc})}$$

Magnitude of h_{21}:

$$|h_{21}| = \frac{\sqrt{g_m^2 + \omega^2 C_{jc}^2}}{\sqrt{g_\pi^2 + \omega^2 (C_\pi + C_{je} + C_{jc})^2}}$$

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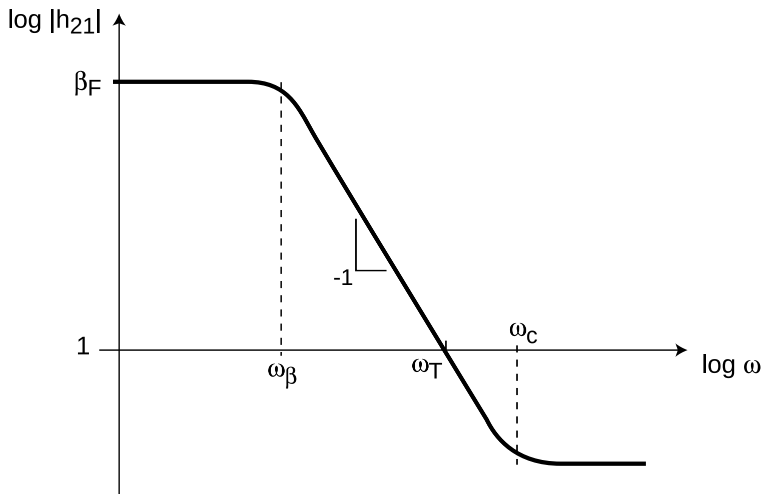
Bode plot of $|h_{21}|$:



Three regimes in $|h_{21}|$:

- low frequency, $\omega \ll \omega_\beta$:

$$|h_{21}| \simeq \frac{g_m}{g_\pi} = \beta_F$$

- intermediate frequency, $\omega_\beta \ll \omega \ll \omega_c$:

$$|h_{21}| \simeq \frac{g_m}{\omega (C_\pi + C_{je} + C_{jc})}$$

- high frequency, $\omega \gg \omega_c$:

$$|h_{21}| \simeq \frac{C_{jc}}{C_\pi + C_{je} + C_{jc}}$$

Angular frequencies that separate three regimes:

$$\omega_\beta = \frac{g_\pi}{C_\pi + C_{je} + C_{jc}}$$

$$\omega_c = \frac{g_m}{C_{jc}}$$

Angular frequency at which $|h_{21}| = 1$:

$$\omega_T = \frac{g_m}{C_\pi + C_{je} + C_{jc}}$$

In terms of frequency:

$$f_T = \frac{g_m}{2\pi(C_\pi + C_{je} + C_{jc})}$$

Note:

$$\omega_\beta = \frac{\omega_T}{\beta_F}$$

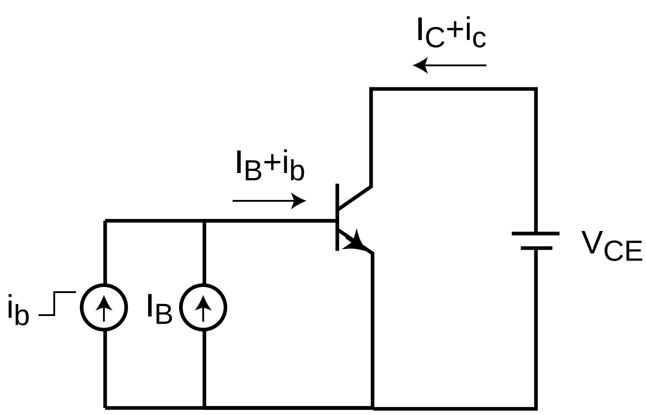
□ Physical meaning of f_T

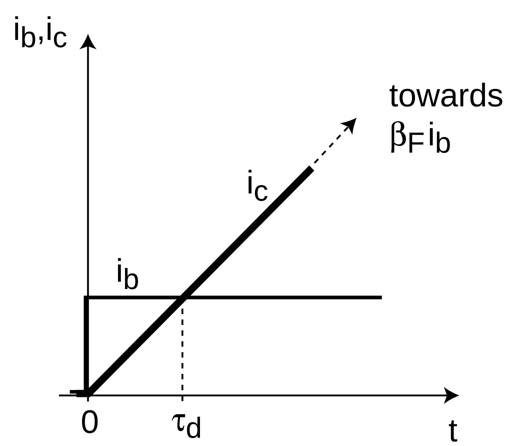
$1/2\pi f_T$ has units of time. Define *delay time*:

$$\tau_d = \frac{1}{2\pi f_T} = \frac{C_\pi}{g_m} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m} = \tau_{tB} + \frac{\tau_{tE}}{\beta_F} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m}$$

Four delay components in τ_d.

Consider response of BJT to a step-input base current:





At $t = 0$

$$I_B \rightarrow I_B + i_b$$

As $t \rightarrow \infty$

$$V_{BE} \rightarrow V_{BE} + v_{be}$$

$$I_C \rightarrow I_C + i_c = I_C + \beta_F i_b.$$

How much time does it take for i_C to reach its final value?

Charge must be delivered to four regions in BJT:

- *Quasi-neutral emitter*

$$q_e = \tau_{tE} i_b$$

- *Quasi-neutral base*

$$q_b = \tau_{tB} i_c$$

- *Emitter-base depletion region*

$$q_{je} = C_{je} v_{be} = \frac{C_{je}}{g_m} i_c$$

- *Base-collector depletion region*

$$q_{jc} = C_{jc} v_{bc} = C_{jc} v_{be} = \frac{C_{jc}}{g_m} i_c$$

Charge delivered at constant rate to base. Time that it takes for all charge to be delivered:

$$\tau_\beta = \frac{q_e + q_b + q_{je} + q_{jc}}{i_b} = \tau_{tE} + \beta_F(\tau_{tB} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m}) = \frac{1}{2\pi f_\beta}$$

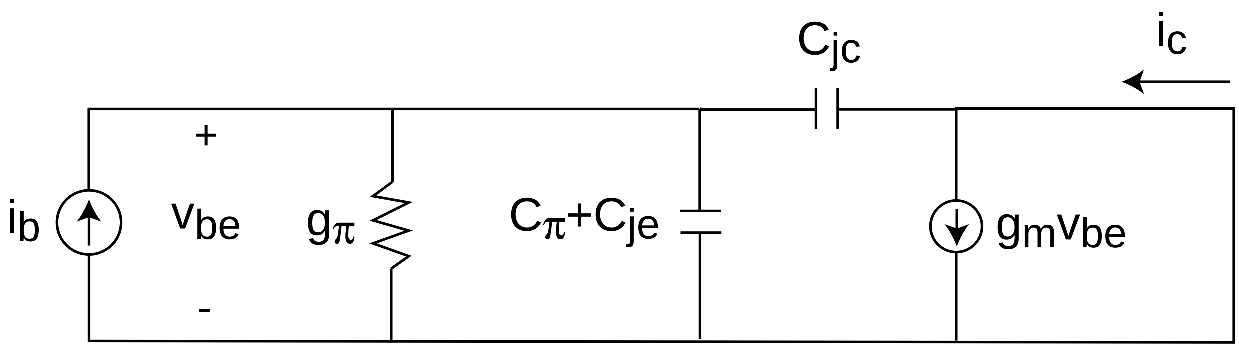
How much time does it take for i_C to build up to $I_C + i_b$?

Since $i_c = \beta_F i_b$,

$$\tau_d = \frac{\tau_\beta}{\beta_F} = \frac{\tau_{tE}}{\beta_F} + \tau_{tB} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m} = \frac{1}{2\pi f_T}$$

- $\tau_d = \frac{1}{2\pi f_T}$: delay time before i_C increases to $I_C + i_b$
- $\tau_\beta = \frac{1}{2\pi f_\beta}$: delay time before i_C increases to $I_C + \beta_F i_b$

With sinusoidal input:



$f \uparrow \Rightarrow$ fraction of i_b that goes into capacitors$\uparrow \Rightarrow v_{be} \downarrow \Rightarrow i_c \downarrow$.

At f_T : $|i_c| = |i_b|$

□ Key dependencies of f_T in ideal BJT

⋆ f_T dependence on I_C:

Rewrite f_T:

$$f_T = \frac{g_m}{2\pi(C_\pi + C_{je} + C_{jc})} = \frac{1}{2\pi\tau_F}\frac{1}{1 + \frac{kT}{q\tau_F}\frac{C_{je}+C_{jc}}{I_C}}$$

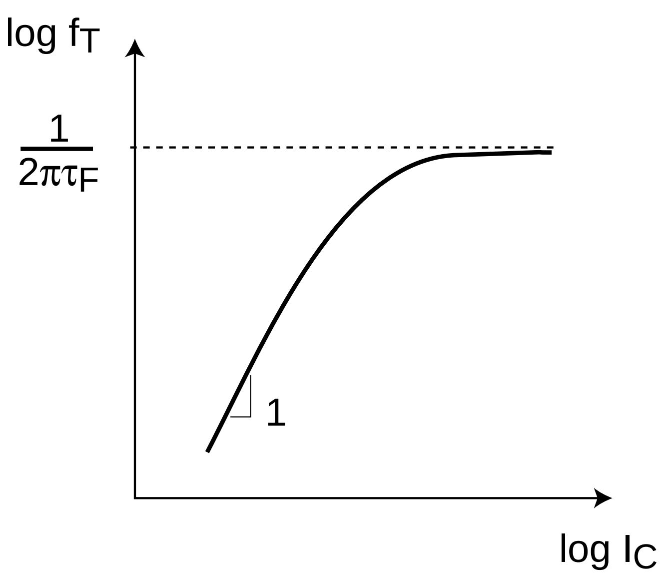
Two limits:

- Small I_C: limited by depletion capacitances

$$f_T \simeq \frac{q}{2\pi kT}\frac{I_C}{C_{je} + C_{jc}}$$

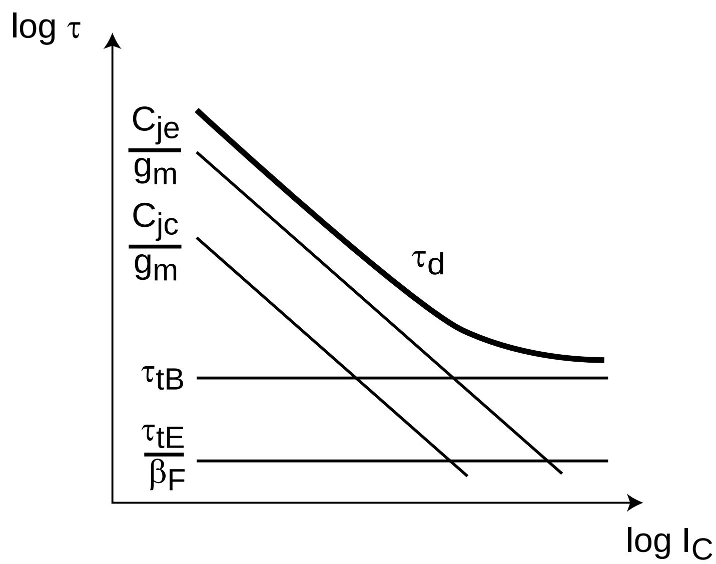
- Large I_C: limited by intrinsic delay (dominated by τ_{tB})

$$f_T \simeq \frac{1}{2\pi\tau_F}$$

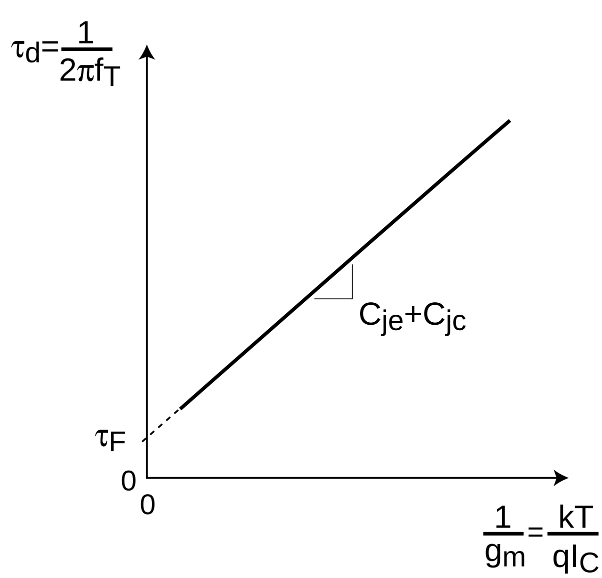


Alternative view of I_C dependence:

$$\tau_d = \frac{\tau_{tE}}{\beta_F} + \tau_{tB} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m} = \frac{1}{2\pi f_T}$$



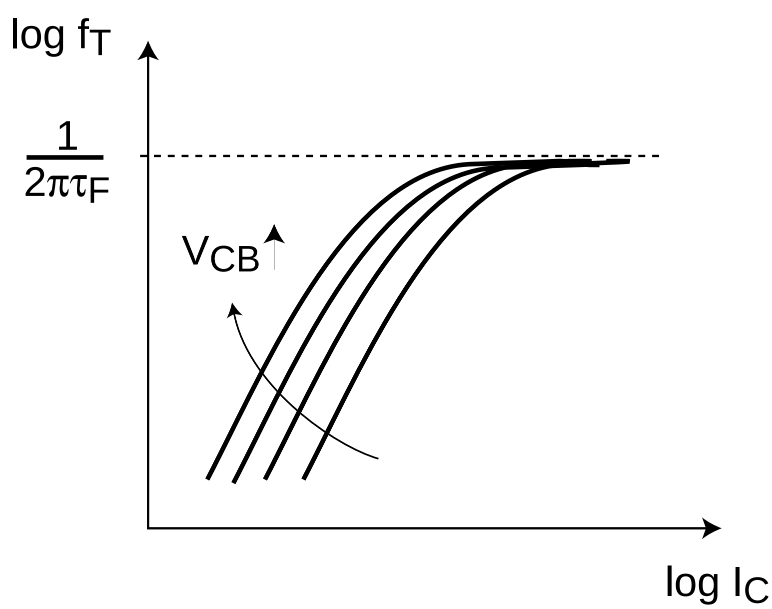
Standard experimental technique to extract τ_F and $C_{je} + C_{jc}$:



⋆ f_T dependence on V_{BC}:

$V_{CB} \uparrow$ (B-C junction is more reverse biased) $\Rightarrow C_{jc} \downarrow \Rightarrow f_T \uparrow$

[but only in low I_C regime of f_T]



Key conclusions

- f_T: high-frequency figure of merit for transistors: frequency at which $|h_{21}| = 1$.
- f_T of ideal BJT:

$$f_T = \frac{g_m}{2\pi(C_\pi + C_{je} + C_{jc})}$$

- *Delay time*, $\tau_d = \frac{1}{2\pi f_T}$: time it takes for step increase in i_B to yield an identical step increase in i_C.