Lecture 12 - Carrier Flow *(cont.)*

September 30, 2002

Contents:

1. Minority-carrier-type situations *(cont.)*
2. Dynamic situations

Reading assignment:

del Alamo, Ch. 5, §§5.6-5.7

Seminar:

October 1: *Submicron Scaling of InP Bipolar Transistors: Device Design, Scaling Laws, Technology Roadmaps, and Advanced Fabrication Processes* by M. Rodwell, University of California at Santa Barbara; Rm. 34-101, 4 PM.

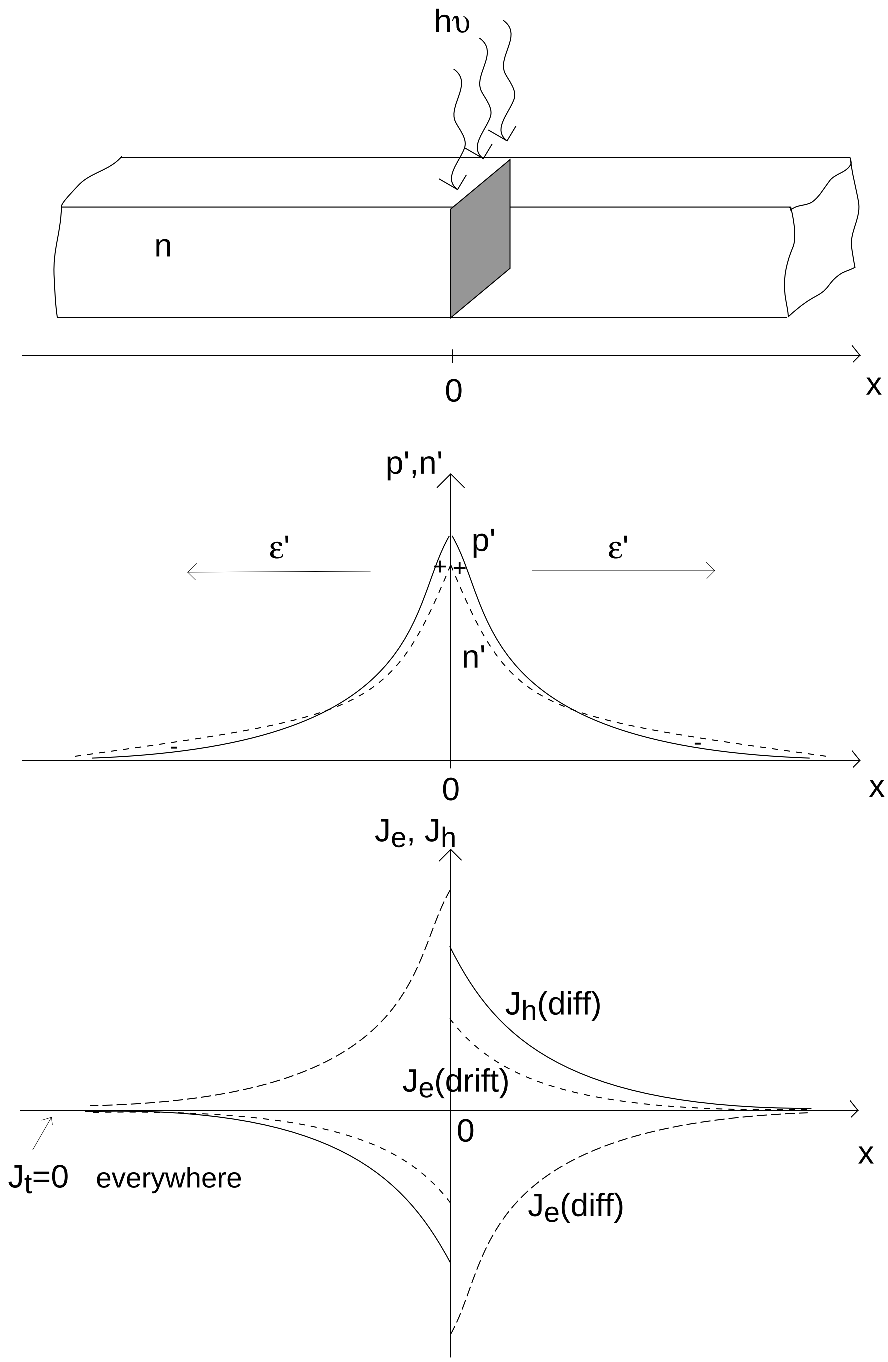
Key questions

- What is the length scale for minority-carrier type situations?
- What is the characteristic time constant of minority-carrier-type situations? Always?
- What do majority carriers do in minority-carrier type situations?

1. Minority-carrier-type situations *(cont.)*

□ Example 1:

Diffusion and bulk recombination in a "long" bar



Solution

STEP 1. Minority carrier flow problem (for $x \geq 0$):

$$\frac{d^2p'}{dx^2} - \frac{p'}{L_h^2} = 0$$

with

$$L_h = \sqrt{D_h \tau}$$

solution of the form:

$$p' = A \exp \frac{x}{L_h} + B \exp \frac{-x}{L_h}$$

B.C. at $x = 0$:

$$\frac{g_l}{2} = \frac{1}{q} J_h(0) = -D_h \frac{dp'}{dx}|_{x=0}$$

Then:

$$p' = \frac{g_l L_h}{2D_h} \exp \frac{-x}{L_h}$$

STEP 2. Hole current:

Assuming $J_h(drift) \ll J_h(diff)$

$$J_h \simeq -qD_h \frac{dp'}{dx} = \frac{qg_l}{2} \exp \frac{-x}{L_h}$$

STEP 3. Total current:

$$J_t = 0 \quad everywhere$$

STEP 4. Electron current:

$$J_e = -J_h = -\frac{qg_l}{2} \exp \frac{-x}{L_h}$$

STEP 5. Electron profile:

$$n' \simeq p' = \frac{g_l L_h}{2D_h} \exp \frac{-x}{L_h}$$

STEP 6. Electron diffusion current:

$$J_e(diff) = qD_e \frac{dn'}{dx} = -\frac{qg_l}{2}\frac{D_e}{D_h}\exp\frac{-x}{L_h}$$

STEP 7. Electron drift current:

$$J_e(drift) = J_e - J_e(diff)$$
$$= \frac{qg_l}{2}\frac{D_e - D_h}{D_h}\exp\frac{-x}{L_h}$$

Note: if $D_e = D_h \Rightarrow J_e(drift) = 0$

STEP 8. Average velocity of hole diffusion:

$$v_h^{diff} = \frac{J_h^{diff}(x)}{qp(x)} \simeq \frac{J_h(x)}{qp'(x)} = \frac{D_h}{L_h}$$

independent of x.

[will use when deriving I-V characteristics of pn junction diode]

Now verify assumptions

STEP 9. Verify *quasi-neutrality*: $|\frac{p'-n'}{p'}| \ll 1$

Compute $\mathcal{E}'$ from $J_e(drift)$:

$$\mathcal{E}' = \frac{J_e(drift)}{q\mu_e n_o} = \frac{kT}{q}\frac{g_l}{2n_o}\frac{D_e - D_h}{D_e D_h}\exp\frac{-x}{L_h}$$

From Gauss' law, get difference between n' and p':

$$p' - n' = -\frac{\epsilon kT}{q^2 n_o}\frac{g_l}{2L_h}\frac{D_e - D_h}{D_e D_h}\exp\frac{-x}{L_h}$$

Then

$$|\frac{p' - n'}{p'}| = (\frac{L_D}{L_h})^2\frac{D_e - D_h}{D_e}$$

If characteristic length of problem is much longer than L_D (Debye length), quasi-neutrality applies in minority-carrier-type situations.

Put numbers: for $N_D = 10^{16}\ cm^{-3}$, $L_D \sim 0.04\ \mu m$, $L_h \sim 400\ \mu m$, and $(L_D/L_h)^2 \sim 10^{-8}$.

STEP 10. Verify $J_h(drift) \ll J_h(diff)$

$$|\frac{J_h(drift)}{J_h(diff)}| = |\frac{q\mu_h p' \mathcal{E}'}{-qD_h \frac{dp'}{dx}}| = \frac{1}{2}\frac{p'}{n_o}\frac{D_e - D_h}{D_e}$$

as good as low-level injection

STEP 11. *Limit to injection* to maintain LLI: $p'(0) \ll n_o$

$$g_l \ll \frac{2D_h n_o}{L_h}$$

STEP 12. Verify linearity between v^{drift} and $\mathcal{E}'$

At $x = 0$ (worst point):

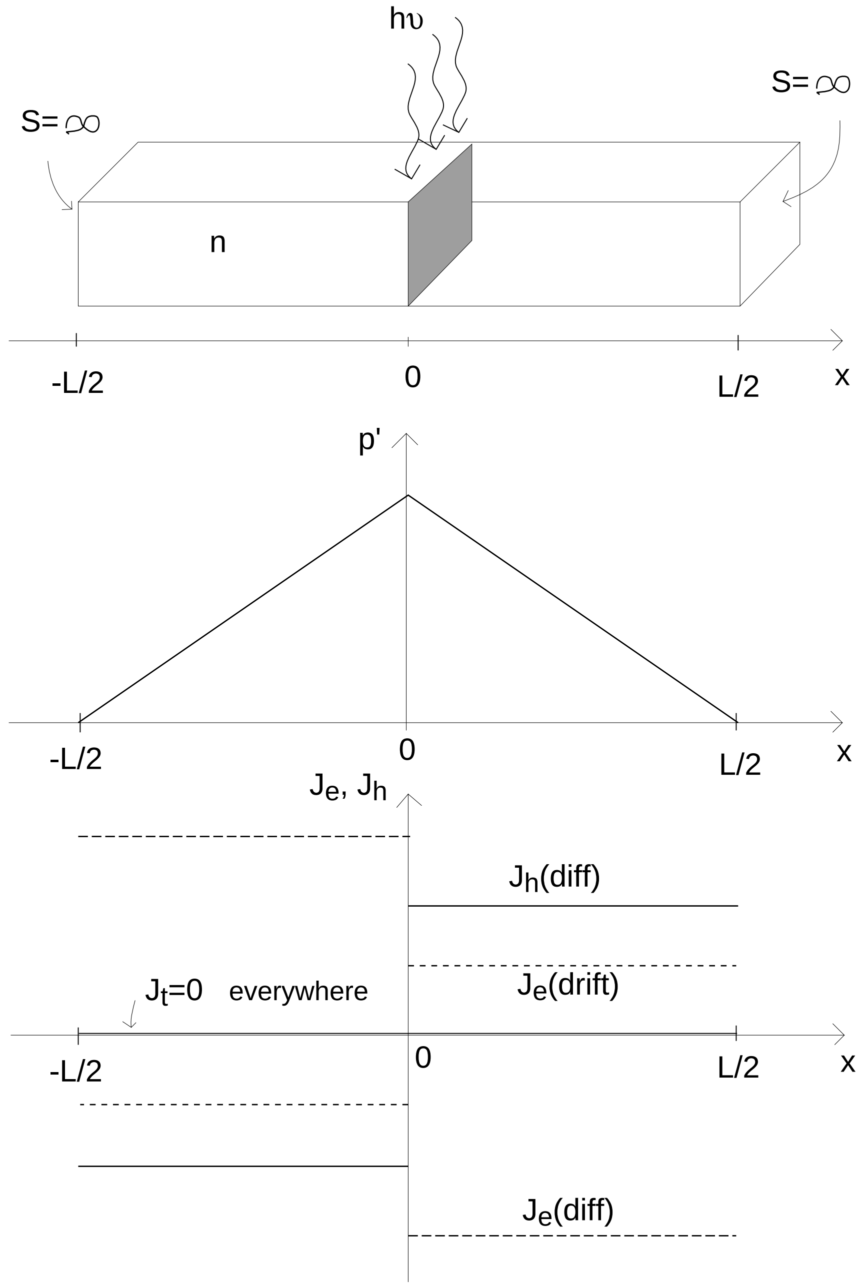
$$\mu_e \mathcal{E}' = \frac{g_l}{2n_o}\frac{D_e - D_h}{D_h} \ll \frac{D_e - D_h}{L_h}$$

$\sim 1000\ cm/s\ \ \ll\ \ v_{sat}.$

□ Example 2: Diffusion and surface recombination in a "short" or "transparent" bar

Uniform doping: $\mathcal{E}_o = 0$; static conditions: $\frac{\partial}{\partial t} = 0$

Bar length: $L \ll L_h$; $S = \infty$ at bar ends.

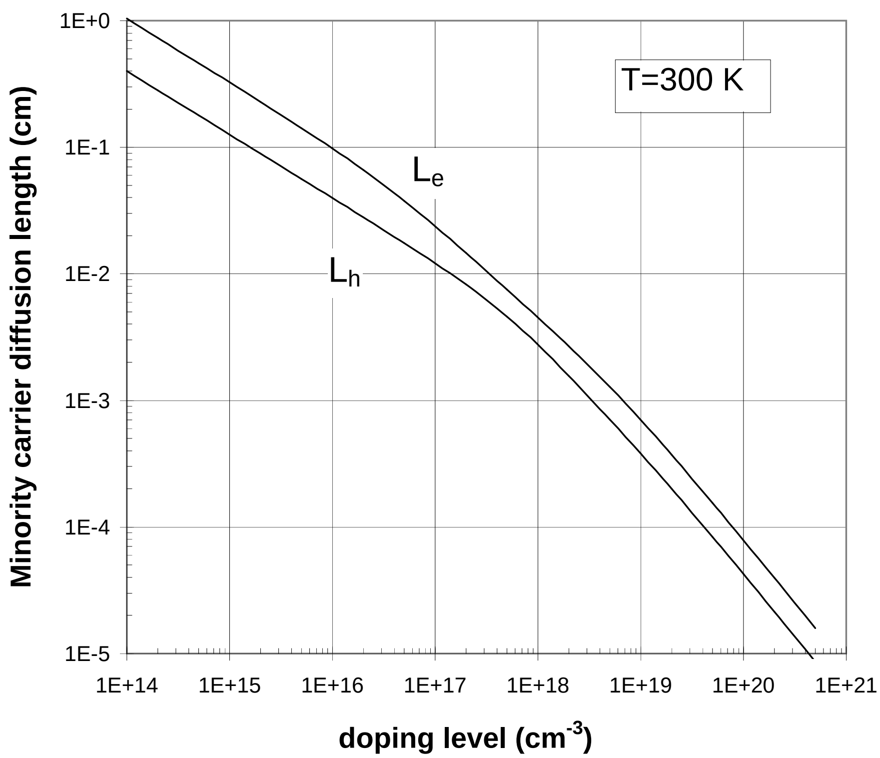


Length scales of minority-carrier situations

□ *Diffusion Length*: mean distance that a carrier diffuses in a bulk semiconductor before recombining

$$L_{diff} = \sqrt{D\tau}$$

L_{diff} strong function of doping:



□ *Sample size*, L

- If $L \gg L_{diff}$, L_{diff} is characteristic length of problem
- If $L \ll L_{diff}$, L is characteristic length of problem

2. Dynamic situations

□ MAJORITY CARRIER SITUATIONS: characteristic time constant is *dielectric relaxation time* $\sim\ sub-ps$

⇒ nearly always quasi-static

□ MINORITY CARRIER SITUATIONS: characteristic time constant dominated by minority carrier physics

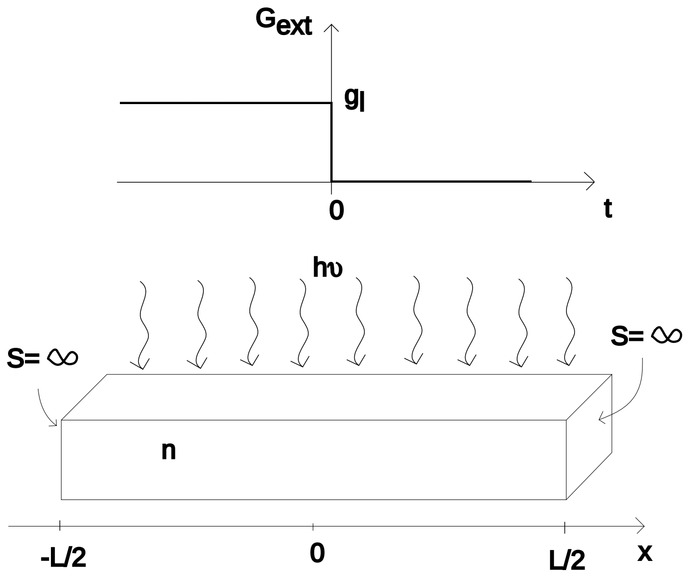
⇒ Substantial memory effects

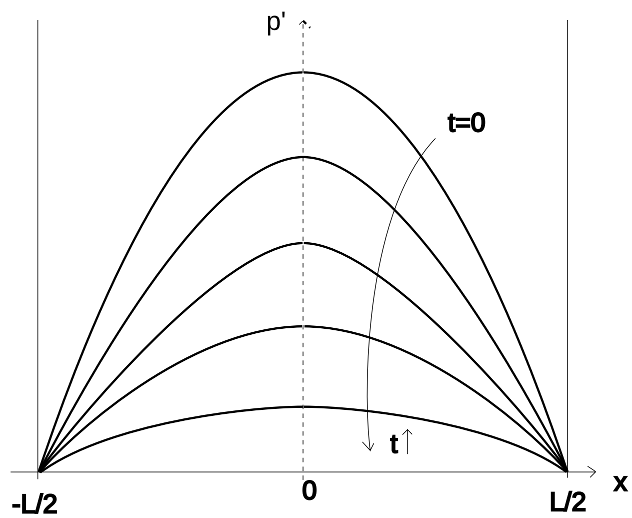
- in uniform situations characteristic time constant is *carrier lifetime*
- in non-uniform situations?

□ Example: Transient in semiconductor bar with $S = \infty$

Uniformly-doped n-type bar.

Switch-off transient after uniform illumination





Two recombination paths:

- Bulk recombination: time constant τ (carrier lifetime)
- Surface recombination: limited by carrier diffusion to surfaces; time constant: $\propto L, \ \propto 1/D$

Combined time constant: $< \tau$

□ For $t \leq 0$ (steady-state solution under illumination):

$$D_h \frac{d^2 p'}{dx^2} - \frac{p'}{\tau} + G_{ext} = 0$$

Boundary conditions:

$$\frac{dp'}{dx}|_{x=0} = 0$$

$$p'(\pm\frac{L}{2}) = 0$$

Solution:

$$p'(x, t = 0) = g_l \tau (1 - \frac{\cosh \frac{x}{L_h}}{\cosh \frac{L}{2L_h}})$$

□ For $t \geq 0$:

$$D_h \frac{\partial^2 p'}{\partial x^2} - \frac{p'}{\tau} = \frac{\partial p'}{\partial t}$$

Solve by method of separation of constants:

$$p'_n(x,t) = \exp\frac{-t}{\tau}\sum_n K_n \exp\frac{-D_h t}{\lambda_n^2}\cos\frac{x}{\lambda_n} \qquad \text{for } n = 1,2,3,...$$

K_n are proper weighting coefficients and

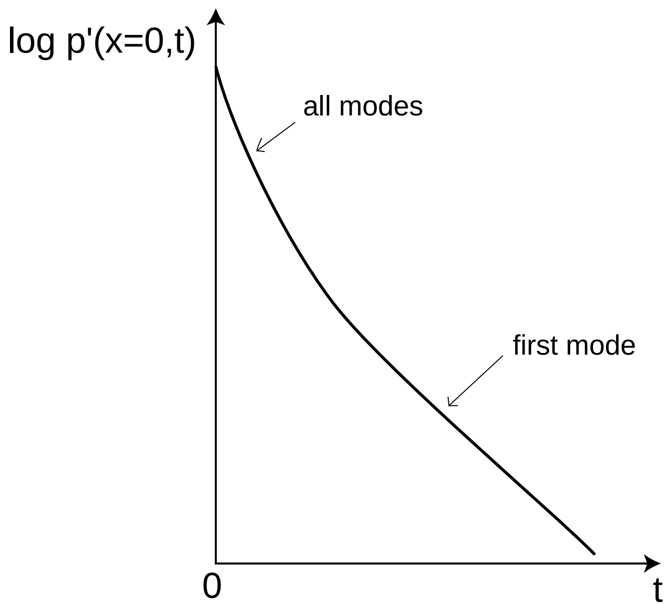
$$\lambda_n = \frac{L}{(2n-1)\pi} \qquad \text{for } n = 1,2,3,...$$

Time decay is not simple exponential but sum of individual exponentials. Time constant of nth component:

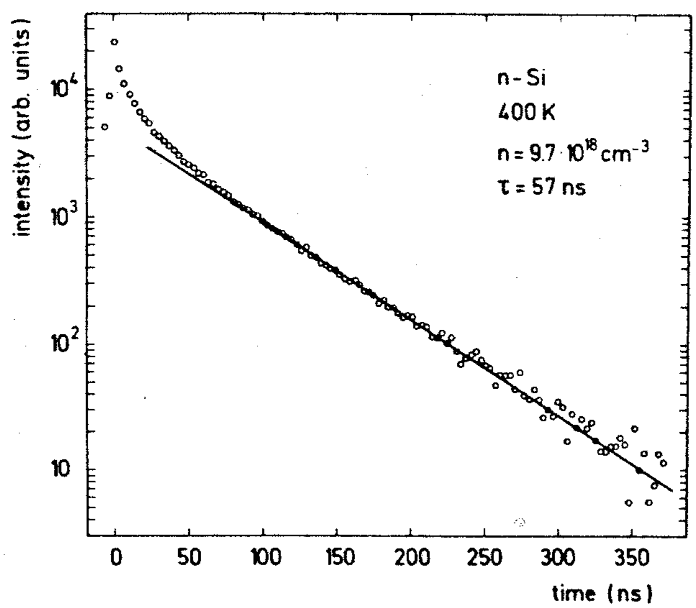
$$\frac{1}{\tau_n} = \frac{1}{\tau} + D_h[\frac{(2n-1)\pi}{L}]^2 > \frac{1}{\tau} \qquad \text{for } n = 1,2,3,...$$

$$n \uparrow \rightarrow \ \tau_n \downarrow$$

High-order components decay quickly $\Rightarrow$ initial fast decay followed by slow decay dominated by 1st order time constant



This is seen in experiments:



After short time, decay dominated by first mode with time constant:

$$\frac{1}{\tau_1} = \frac{1}{\tau} + D_h(\frac{\pi}{L})^2$$

This is the dominant time constant of the problem.

In a general way:

$$\frac{1}{\tau_1} = \frac{1}{\tau} + \frac{1}{\tau_t}$$

with $\tau_t \equiv$ *transit time* or average time for excess carrier to reach surface

$$\tau_t = \frac{L^2}{\pi^2 D_h}$$

Surface recombination speeds up excess minority carrier decay by providing additional recombination paths:

$$\tau_1 < \tau$$

In the limit of very slow bulk recombination,

$$\tau_1 \simeq \tau_t$$

Getting the excess carriers to the surface becomes the bottleneck to the recombination rate.

Key conclusions

- Two characteristic lengths in minority-carrier type situations:
  - *diffusion length*, $L = \sqrt{D\tau}$, average distance that a carrier diffuses in a bulk semiconductor before recombining;
  - *sample size*, L
  - whichever one is smallest, L or L_{diff}, dominates behavior of minority carriers.
- Majority-carrier type situations can be considered quasi-static.
- Minority-carrier type situations show substantial memory.
- Time constants in minority-carrier type situations:
  - carrier lifetime
  - transit time $\propto L^2/D$
  - whichever one is smallest dominates
- Minority-carrier type situations called that way because:
  - length and time scales of problem dominated by minority carrier behavior (diffusion, recombination, and drift)
  - role of majority carriers is to preserve quasi-neutrality and total current continuity
- Order of magnitude of key parameters in Si at $300K$:
  - Diffusion length: $L_{diff} \sim 0.1 - 1000\ \mu m$ (depends on doping level).

Self study

- Work out example 2: diffusion and surface recombination in a "short bar" (§5.6.2)