Lecture 22 - The Si surface and the Metal-Oxide-Semiconductor Structure *(cont.)*

October 25, 2002

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1. Ideal MOS structure in thermal equilibrium *(cont.)*
2. Ideal MOS structure outside thermal equilibrium

Reading assignment:

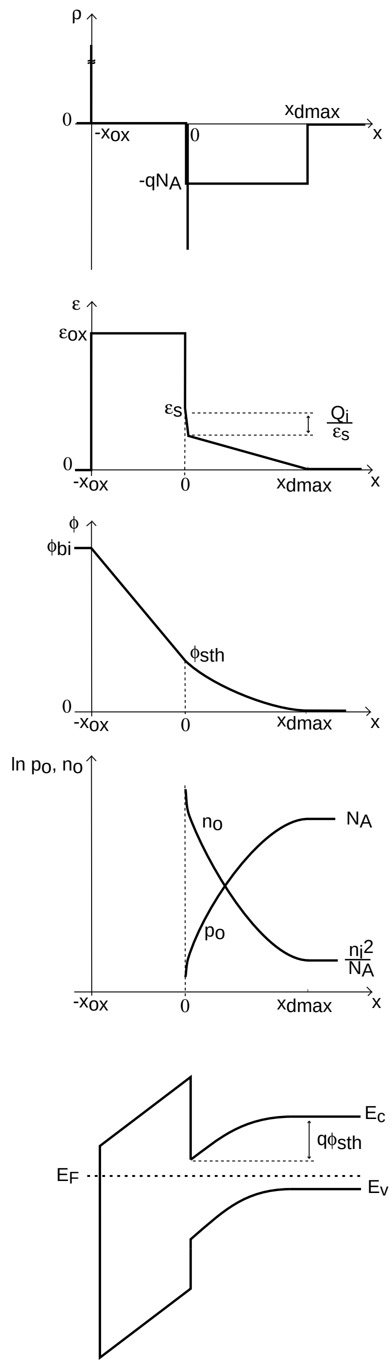
del Alamo, Ch. 8, §§8.2 (8.2.3, 8.2.4), 8.3 (8.3.1)

Key questions

- What happens if a voltage is applied to the metal with respect to the semiconductor in a MOS structure?
- How much voltage needs to be applied to bring the MOS structure to the onset of inversion?
- How does the inversion charge evolve with voltage?

1. Ideal MOS structure in thermal equilibrium

□ Inversion



$$Q_s = Q_d + Q_i$$

Do *sheet-charge approximation*: inversion layer much thinner than any other vertical dimensions of the problem.

Two consequences:

1. ϕ_s depends on Q_d but is independent of Q_i (ϕ does not change while crossing a sheet of charge):

$$Q_d = -qN_A x_d \simeq -\sqrt{2\epsilon_s q N_A \phi_s}$$

   [same relationship as in depletion]

2. ϕ_s in inversion rather insensitive to actual value of W_M:

$$\phi_s \simeq \phi_{sth}$$

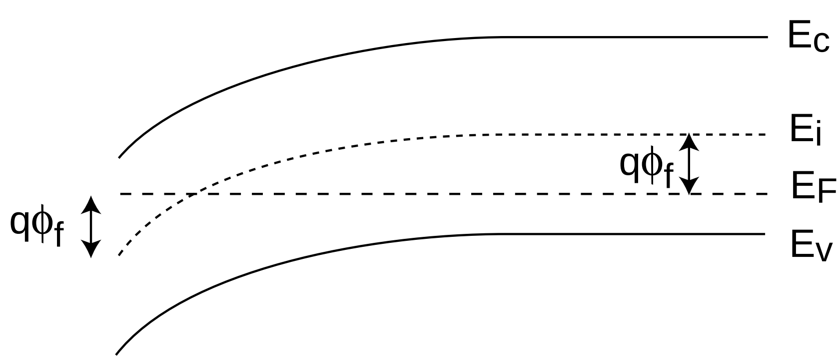
   ϕ_{sth} is *surface potential at threshold*

Rough estimate of ϕ_{sth}: value that brings surface right at edge of inversion, that is, $n_o(0) = N_A$:

$$n_o(x=0)|_{threshold} = \frac{n_i^2}{N_A} \exp \frac{q\phi_{sth}}{kT} = N_A$$

Then:

$$\phi_{sth} = 2\frac{kT}{q} \ln \frac{N_A}{n_i} \equiv 2\phi_f$$



Within the *sheet-charge approximation*, electrostatic problem is easy to solve:

- In inversion, ϕ_s independent of $W_M \Rightarrow x_d$ independent of W_M:

$$x_d \simeq x_{dmax} = \sqrt{\frac{2\epsilon_s \phi_{sth}}{qN_A}}$$

Total depletion region charge:

$$Q_d \simeq Q_{dmax} = -qN_A x_{dmax} = -\sqrt{2\epsilon_s q N_A \phi_{sth}}$$

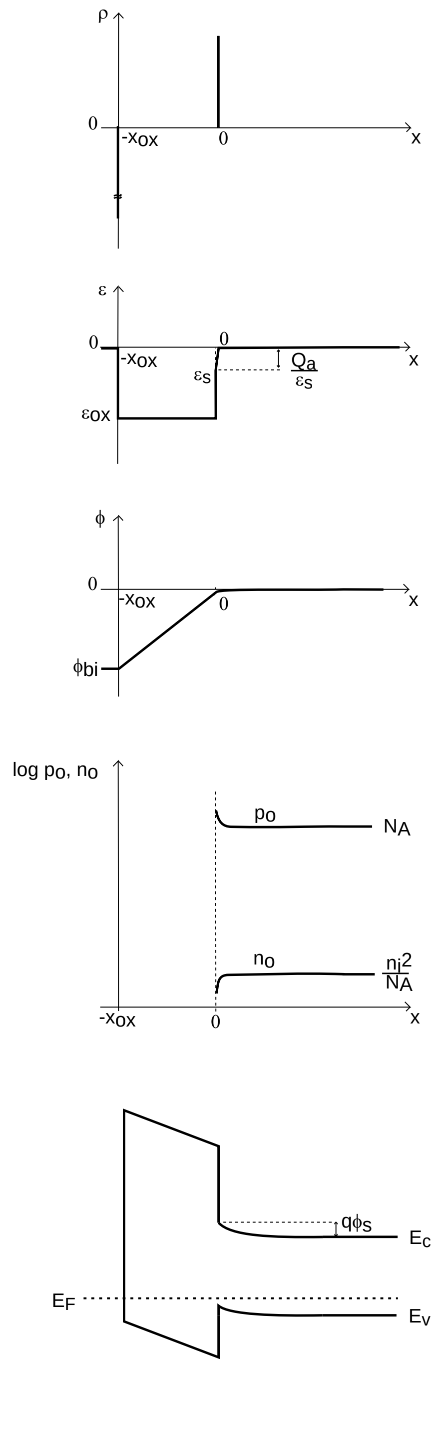
- From fundamental electrostatics relation in inversion:

$$\phi_{bi} = \phi_s - \frac{Q_s}{C_{ox}} = \phi_{sth} - \frac{Q_i + Q_{dmax}}{C_{ox}}$$

Derive expression for Q_i in inversion:

$$Q_i = -C_{ox}(\phi_{bi} - \phi_{sth}) - Q_{dmax}$$

□ Accumulation



$$Q_s = Q_a$$

Sheet-charge approximation again:

$$\phi_s \simeq 0$$

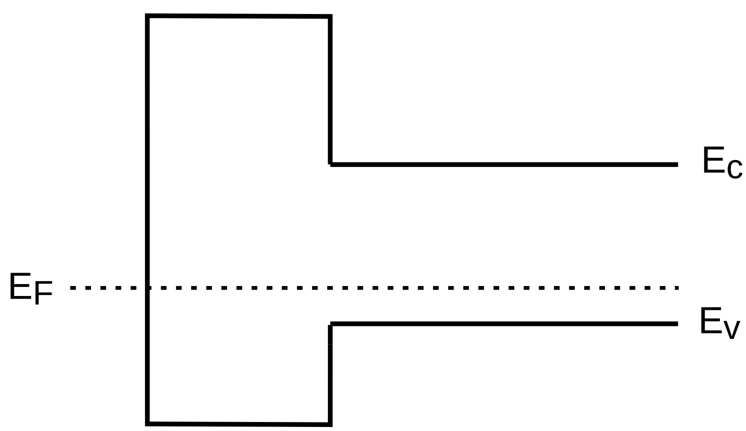
Use fundamental electrostatics equation again:

$$Q_a \simeq -C_{ox}\phi_{bi}$$

□ **Flatband**

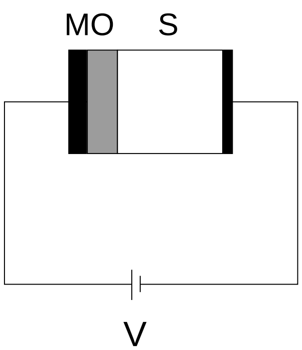
If $\phi_{bi} = 0 \Rightarrow \; Q_s = 0, \; \mathcal{E}_s = 0, \; \mathcal{E}_{ox} = 0, \phi_s = 0$

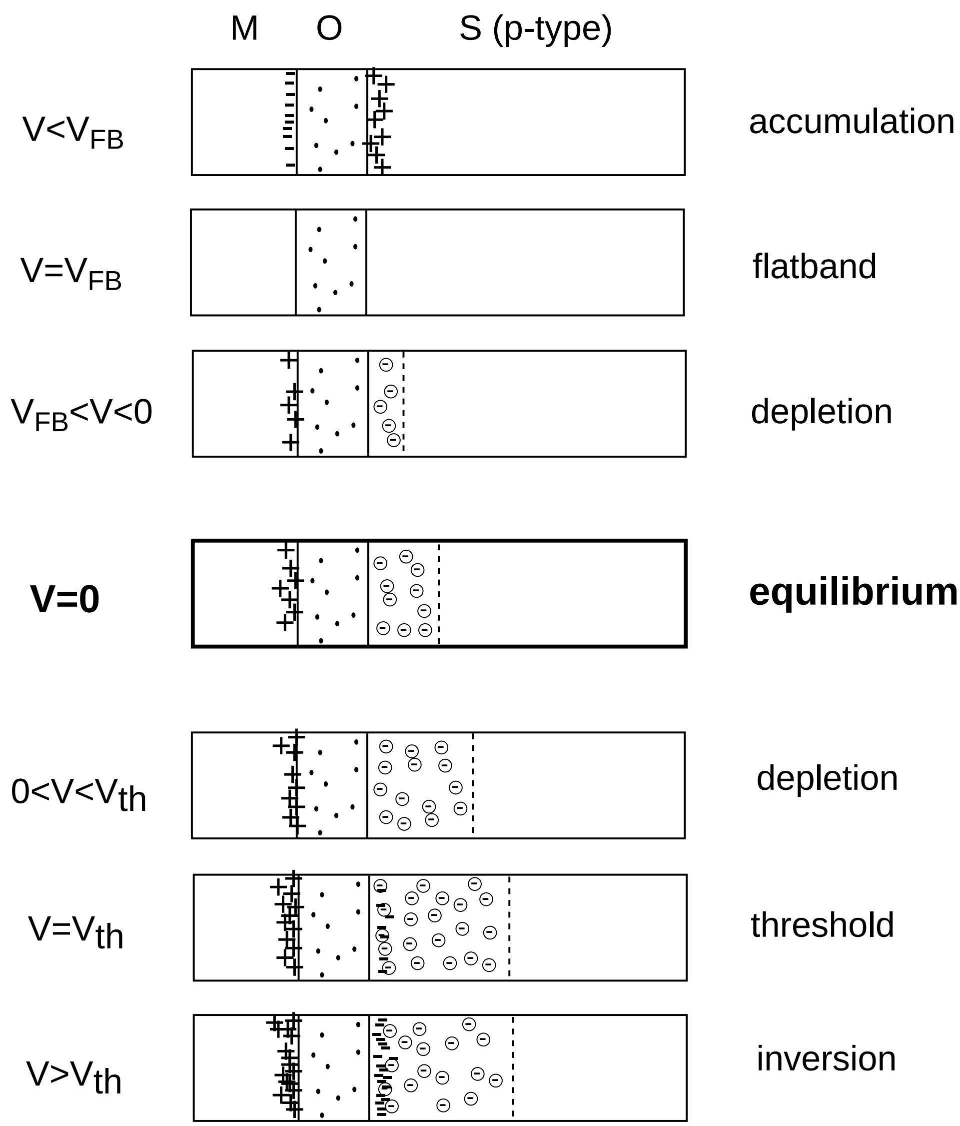
Perfect charge neutrality everywhere.

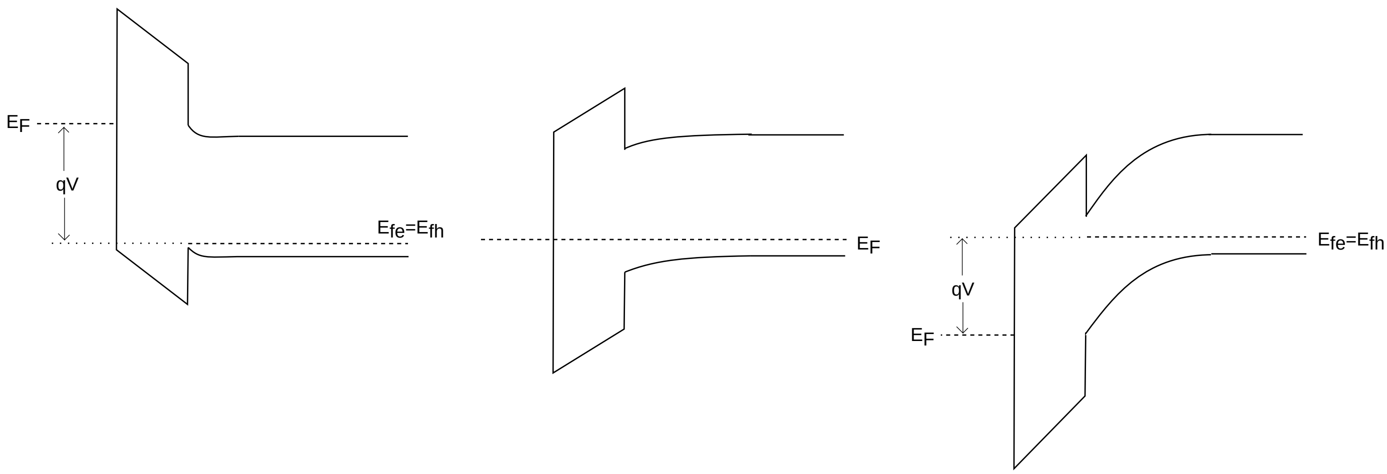


2. Ideal MOS structure outside equilibrium

□ Apply voltage V to metal with respect to semiconductor:







a) accumulation (V<0) b) thermal equilibrium (depletion) c) inversion (V>0)

- MOS structure can swing all the way from accumulation to inversion.
- semiconductor remains in quasi-equilibrium (no carrier flow, no carrier injection)

$$E_{fe} = E_{fh} = E_F$$

- electrostatics identical to TE but potential difference across structure changed:

$$\phi_{bi} \rightarrow \phi_{bi} + V$$

□ Total potential build-up across structure:

$$\phi_{bi} + V = \phi_s - \frac{Q_s}{C_{ox}}$$

Several important results:

- *Flatband voltage*: $\phi_s = 0$, $Q_s = 0$:

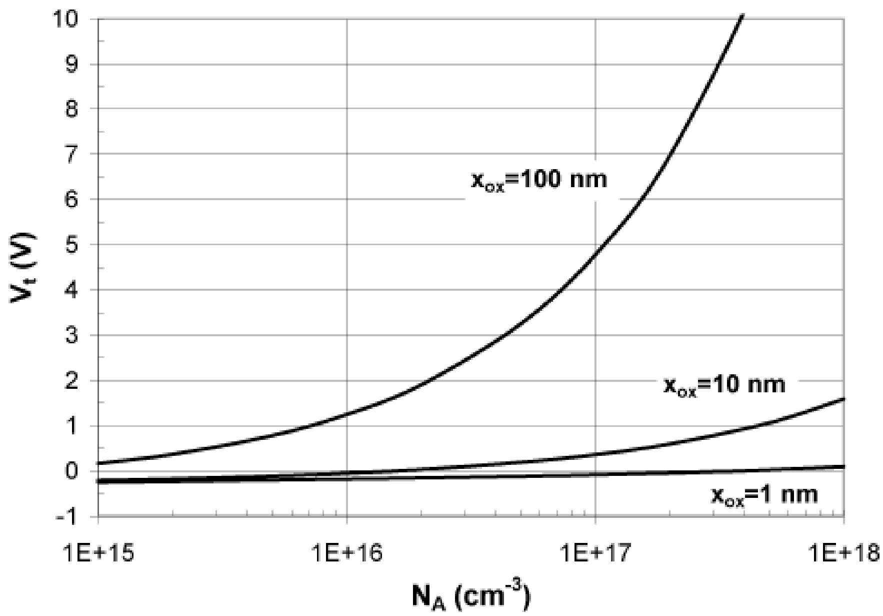
$$V_{FB} = -\phi_{bi}$$

- *Threshold voltage* for inversion: $Q_i = 0$, $Q_d = Q_{dmax}$, $\phi_s = \phi_{sth}$:

$$V_{th} = -\phi_{bi} + \phi_{sth} - \frac{Q_{dmax}}{C_{ox}} = V_{FB} + \phi_{sth} + \gamma\sqrt{\phi_{sth}}$$

$$V_{th} = -\phi_{bi} + \phi_{sth} - \frac{Q_{dmax}}{C_{ox}} = V_{FB} + \phi_{sth} + \gamma\sqrt{\phi_{sth}}$$

V_t plays key role in MOSFET operation.



Key dependencies: $N_A \uparrow \rightarrow V_t \uparrow$

$$x_{ox} \uparrow \rightarrow V_t \uparrow$$

• In inversion ($V > V_{th}$):

$$Q_i = -C_{ox}(V - V_{th}) \qquad \text{for } V \geq V_{th}$$

Once reached inversion, the inversion charge increases *linearly* with the applied voltage in excess of V_{th}.

□ Poisson-Boltzmann formulation

In uncompensated uniformly-doped p-type semiconductor:

$$\rho = q(p - n - N_A)$$

Poisson equation:

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon_s}(p - n - N_A)$$

Quasi-equilibrium prevails for electrons and holes:

$$n = n_{oB} \exp \frac{q\phi}{kT} \simeq \frac{n_i^2}{N_A} \exp \frac{q\phi}{kT}$$
$$p = p_{oB} \exp \frac{-q\phi}{kT} \simeq N_A \exp \frac{-q\phi}{kT}$$

Charge neutrality in bulk:

$$p_{oB} - n_{oB} - N_A = 0$$

All together - *Poisson-Boltzmann equation*:

$$\frac{d^2\phi}{dx^2} = -\frac{qN_A}{\epsilon_s}[(\exp \frac{-q\phi}{kT} - 1) - \frac{n_i^2}{N_A^2}(\exp \frac{q\phi}{kT} - 1)]$$

Double integration of this equation leads to complete solution.

First integration tricky [see notes]:

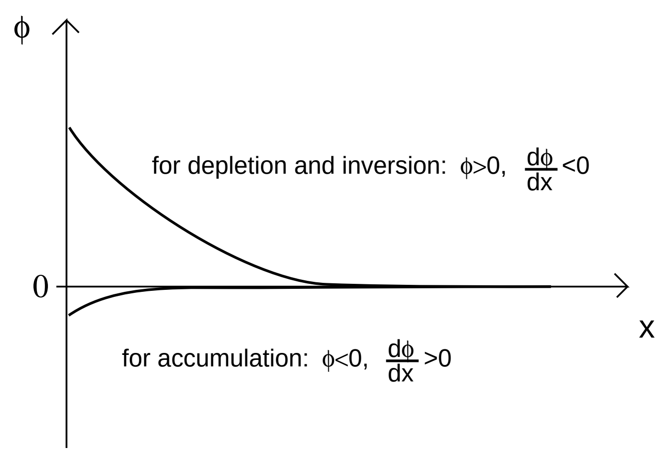
$$\frac{d\phi}{dx} = -\sqrt{\frac{2kTN_A}{\epsilon_s}}F(\phi)$$

with:

$$F(\phi) = \frac{\phi}{|\phi|}[(\exp\frac{-q\phi}{kT} + \frac{q\phi}{kT} - 1) + \frac{n_i^2}{N_A^2}(\exp\frac{q\phi}{kT} - \frac{q\phi}{kT} - 1)]^{1/2}$$

Check signs:

- if $\phi > 0$, $\frac{\phi}{|\phi|} = +1$, and $\frac{d\phi}{dx} < 0$ (depletion and inversion)
- if $\phi < 0$, $\frac{\phi}{|\phi|} = -1$, $\frac{d\phi}{dx} > 0$ (accumulation)



Second integration from surface ($x = 0$, $\phi = \phi_s$) towards bulk:

$$\int_{\phi_s}^{\phi} \frac{d\phi}{F(\phi)} = -\sqrt{\frac{2kTN_A}{\epsilon_s}}x$$

Complete solution, but..., in general, not analytical.

Even without second integration, we have interesting results:

- *Electric field*:

$$\mathcal{E} = -\frac{d\phi}{dx} = \sqrt{\frac{2kTN_A}{\epsilon_s}}F(\phi)$$

- *Electric field at surface*:

$$\mathcal{E}_s = \sqrt{\frac{2kTN_A}{\epsilon_s}}F(\phi_s)$$

- *Charge in semiconductor*:

$$Q_s = -\epsilon_s \mathcal{E}_s = -\sqrt{2\epsilon_s kTN_A}F(\phi_s)$$

- *Relation between* V *and* ϕ_s:

$$V = -\phi_{bi} + \phi_s + \frac{\sqrt{2\epsilon_s kTN_A}}{C_{ox}}F(\phi_s) = V_{FB} + \phi_s + \sqrt{\frac{kT}{q}}\gamma F(\phi_s)$$

Key conclusions

- In inversion, $\phi_s \simeq \phi_{sth}$, roughly independent of metal.
- In accumulation, $\phi_s \simeq 0$, roughly independent of metal.
- At Si/SiO_2 interface, semiconductor can swing all the way from accumulation to inversion by the application of a voltage.
- To first order, surface potential in inversion and accumulation does not change with V.
- *Flatband voltage*: voltage that produces flatband:

$$V_{FB} = -\phi_{bi}$$

- *Threshold voltage*: voltage beyond which an inversion layer of electrons is formed:

$$V_{th} = V_{FB} + 2\phi_f + \gamma\sqrt{2\phi_f}$$

- Beyond threshold, absolute inversion charge increases linearly with voltage:

$$Q_i = -C_{ox}(V - V_{th})$$

Self study

- Mathematics of Poisson-Boltzmann formulation