Due Date: Wednesday, December 3, 2003

1. Consider an economy where investors demand a required return of 15 percent per year, capital depreciates at 10 percent per year, and output (y) is related to capital (k) according to the production function $y = k^{0.5}$. Show that the optimal capital stock when capital investment can be expensed under the corporate income tax is $k^* = 4$.

2. Consider an economy in which firms produce output using two capital inputs, equipment (E) and structures (S), according to the production technology

$$Y = E^{.25}S^{.25}.$$

Assume that the price of output is fixed at unity and that both equipment and structures are in infinitely elastic supply with a price of unity. Investors demand a required return of 10 percent on all capital investments, and both equipment and structures depreciate at a rate of 15 percent per year. Firms are 100 percent equity financed. In the initial setting, the tax rate on corporate profits is 50 percent and equipment investment can be expensed, while investments in structures are eligible for depreciation allowances equal to true economic depreciation.

(a) Find the numerical values that the pretax marginal products of equipment and structures must satisfy.

(b) Calculate the firm's output if both equipment and structures could be expensed, and compare this with the output in the case when equipment can be expensed while structures are depreciated using true economic depreciation.

(c) Assuming that firms are in a steady state in which their gross investment is precisely equal to the depreciation on their existing capital stock, compare the revenue that would be collected by the corporate income tax when both types of investment can be expensed, and when only equipment can be expensed while structures receive true economic depreciation. Recall that in computing tax revenue, you should calculate τ*[F(E,S) - Depreciation Deductions].

3. In September 1998 the U.S. Treasury Department began issuing "Series I" inflation-indexed saving bonds. These bonds are purchased using after-tax dollars. At the moment, they accrue interest at a nominal interest rate of $2.0 + \pi$ percent per year, where π is the rate of inflation. The bonds mature in 30 years; after 30 years they no longer pay interest. The accruing interest on these bonds is not taxed until the bonds are cashed in, at which time the owner is liable for income tax on the full amount of accrued interest.

(a) Obtain an expression for the "effective accrual tax rate" on a Series I savings bond, assuming that the bond is held to maturity. This tax rate equals that interest income tax rate that, if it were applied to the annual or quarterly income generated by the bond, would lead to the same terminal wealth as the actual tax system. How does the effective accrual tax rate depend on the rate of inflation over the holding period, and the marginal tax rate that the saver faces when the bonds are redeemed?

(b) Now assume that the rate of inflation for the next 30 years is exactly 3.0 % per year, and that an individual investor faces a marginal income tax rate of 33%. Assume, further, that the yield to maturity on a 30 year "ordinary" Treasury bond is 6.0 percent. (This problem has a short self life -- the Treasury is about to stop issuing 30-year bonds.) Note that the interest on an ordinary Treasury bond is taxed on accrual, which means that it is taxed each year as it accrues. Obtain expressions for the after-tax wealth that an individual will have, after 30 years, if he or she invests $1000 in an ordinary Treasury bond and in a Series I bond. What is the effective after-tax yield on the "ordinary" Treasury bond, and how does it compare with the effective after-tax yield on the Series I bond?

(c) A high-ranking Treasury official asserts that "Individuals can purchase up to $30,000 per year of Series I bonds. For most individuals, this is substantially more than their annual saving. Therefore, the availability of Series I bonds virtually eliminates the income tax burden on personal saving." Would you agree with this claim? How would you evaluate it?

4. (OPTIONAL) Consider a world economy consisting of N identical countries, each endowed with one unit of land. The world contains one unit of capital, which is freely mobile between countries. Land, in contrast, is immobile. All countries have identical production technologies given by

$$Y = K^{.25}L^{.75}$$

where K and L denote capital and land, respectively. The output price is normalized to unity.

(a) Find the equilibrium interest rate and total income received by capitalists and landowners when none of the jurisdictions tax either capital or land. Evaluate these quantities for N = 2 and N = 100. (Note that N = 100 means there is more land in the world than N=2!)

(b) Now consider the impact of a tax at rate θ on capital income in country 1. Assume that revenues are used to purchase good Y, and that the government's purchases do not affect the production technology in country 1. The after-tax rate of return to capital invested in country 1 is now $(1-\theta)F_K$. Find new expressions for the after-tax interest rate, total landowner and capitalist income, and government revenue in country 1 as functions of N and θ.

(c) For $\theta = .25$, find the change in total capital and total land income, and the revenue raised in country 1, if N = 2 and N = 100. What happens to the pretax marginal product of capital in the countries without taxes? How do landowners in country 1 fare as a result of the tax? What do these examples suggest about the usefulness of the "small open economy" assumption that world interest rates are fixed, so capital taxes are shifted to land?