MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.231, Physics of Solids I

Problems 1 and 2 are

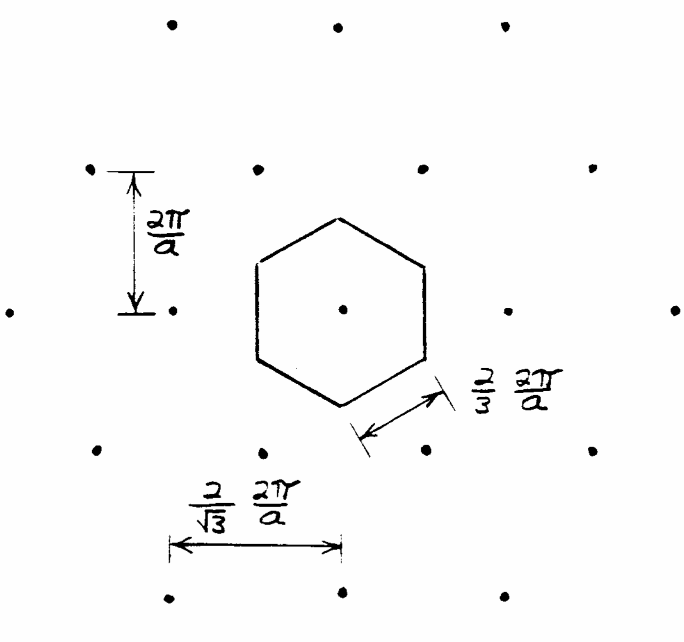
Due in Ses #35

Problem Set #8

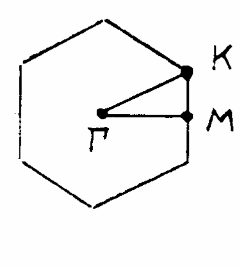
You should do problem 3 to study for the exam, but you don't need to hand it in.

Problem 1: Free Electron Bands in Two Dimensions

For a hexagonal lattice in two dimensions, sketch the first seven free electron energy bands along the line M-Γ-M. Give the degeneracy of each curve on your figure. Calculate the two lowest distinct energies at the Γ point and at the M point in terms of the characteristic energy $\lambda \equiv \frac{\hbar^2}{2m}\left[\frac{2\pi}{\sqrt{3}a}\right]^2$. [Imagine a 3 dimensional space with coordinate axes k_x, k_y, and ε. Consider a free electron energy dispersion relation (a paraboloid) centered at one of the points of the reciprocal lattice, $\varepsilon = \frac{\hbar^2}{2m}(\vec{k}-\vec{G})^2$. Visualize its intersection with a plane normal to k_x and k_y erected on the M-Γ-M line.]



RECIPROCAL LATTICE



SYMMETRY POINTS OF THE BRILLOUIN ZONE

Problem 2: Idealized Electron Dynamics

A single electron is placed at $\vec{k} = 0$ in an otherwise empty band of a bcc solid. The energy of the band is given by

$$\varepsilon(\vec{k}) = -\alpha - 8\gamma \cos(\tfrac{1}{2}k_x a) \cos(\tfrac{1}{2}k_y a) \cos(\tfrac{1}{2}k_z a).$$

At t=0 a uniform electric field $\vec{E} = E_0 \hat{x}$ is turned on.

a) Find an expression for the motion of the electron in k-space. Use a reduced zone picture. Find the motion of the electron in real space assuming it to be localized at the origin at t=0.

b) Why can motions such as described in a) never occur in real solids?

Problem 3: An Alloy

A certain alloy has an average of 1.23 conduction electrons per atom. Assume that the alloy may be regarded as a nearly free electron metal which crystallizes into a simple hexagonal structure with c/a = 1.000.

a) Sketch the Fermi surface in all occupied zones.

b) For which direction of the magnetic field will the de Haas-van Alphen period be a maximum?

c) As the weak periodic potential is increased from an infinitesimal value, what happens to the Fermi surface?

d) Assume that a large uniaxial stress (compression) is applied along the c-axis so that large changes in the c/a ratio can be achieved. At what values of c/a will there be dramatic changes in the Fermi surface? Describe these Fermi surface changes.