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Actor Semantics of PLANNER-73

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Abstract

Work on PLANNER-73 and actors has led to the development of a basis for semantics of programming languages. Its value in describing programs with side-effects, parallelism, and synchronization is discussed. Formal definitions are written and explained for sequences, cells, and a simple synchronization primitive. In addition there is discussion of the implications of actor semantics for the controversy over elimination of side-effects.

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Introduction

A formalism for semantics is usually developed and presented in some context which provides motivation. It might be designed for the purpose of being the formal definition of a specific language as a guide to programmers and implementors. The formalism might be used as the means for describing properties of a particular program, or as a basis for a set of proof rules for verifying properties of programs. The motivation for this paper is development of a semantic base not for some particular programming language, but rather for a kind of computation.* The systems of interest are those which involve parallelism, side-effects, and synchronization. The concepts which must be available to express properties of these systems will be presented and examples given. However, no particular programming language will be used.

Our approach to the semantics will be centered on the concept of an actor. An actor is simply an object which can send or receive messages. All elements of a system can be viewed as actors. The only activity possible is transmission of one actor to another actor. The energy for computation comes from an activator which can follow the script of an actor. All programs and systems will be referred to as actors. The existence of more than one activator indicates parallelism.

Continuations

Once an actor, m, is transmitted to another actor, t, the activator proceeds by following the script of t using information from m. For this to be of any use as a model of computation, it must be possible for m to have some structure. In many cases m will simply contain a message. It will often also contain a continuation part. Then m may be represented as

(apply: message (then-to: continuation)).

If a message is sent without a continuation then all further instructions for computation must be determined by the target. Sending a continuation makes it possible to indicate another actor to which control can eventually be passed (i.e., an actor whose script the activator may follow).

The reason that it is useful to be able to talk about continuations in a semantic formalism is that otherwise much of the control structure has to remain implicit in something like an evaluator or a compiler. Control structure is a pattern of passing messages in a computation. Ideally it should be possible to distinguish between, say, recursive and iterative computations as being two different patterns of message passing behavior which can be precisely characterized.

* Actor semantics was not developed to explain or define PLANNER-73. PLANNER-73 is a language dedicated to the direct realization of this semantic base.

An ordinary functional call to a function **f** can be implemented by passing to **f** a transmission with an explicit continuation. The continuation represents the actor to which the value of **f** should be sent. In addition, other kinds of control structures can be expressed, such as jumps (in which no continuation is present) and co-routines [see Appendix].

Distribution of Information

Information can be represented in various ways in an actor system. Often it is desirable to represent knowledge redundantly with different uses of the same knowledge appearing in different guises in several places. How one might choose to distribute it depends on one's purposes and the various uses to which the knowledge can be put. Concentrating on use as the key to representation of data structures contrasts with the approach of concentrating on the notion of type [van Wijngaarden, 1969].

For example, in defining lists and the usual operations on lists, there is flexibility in how the information about the structure of lists may be distributed among these operations and the data objects themselves. One possibility is that the function call denoted by **(length L)** can directly produce the length of the list, L. That is, the operation, **length**, could be defined in such a way that it knows enough about how lists are constructed (and about how they can be decomposed) to be able to operate on the list directly and to compute its length. For example, the operator **length** could repeatedly ask the list L for its next element and keep a count of the number of times it succeeds.

Alternatively, the operator **length** might know nothing about lists and when sent one would have to depend on the list itself to understand its own structure (and to understand it in terms of the concept "length" among others). What happens in this case is that **(length L)** turns around and sends a message to L asking for its "length." This makes sense for fixed length lists since for such lists the information never changes and need only be computed once. This message-passing style of definition is particularly useful for "polymorphic" operators in an *interactive* system where new kinds of objects which share some of the properties of lists can be defined at any time. It is desirable for **(length L)** to continue working and to extend correctly to new data types that are continually being defined. When information can be put wherever it seems natural, polymorphic operators can be defined without details of type checking. Instead the operator can be applied to any type of data and it is up to the data to determine whether it is a reasonable operator and if so how to perform its operation. Depending on circumstances, either style can lead to a more efficient implementation, but both should be representable.

Behavior of an Actor

Actors are defined by their behaviors. A behavior is an irreflexive partial order of the set of events which represent transmissions. The behavior involving only a single activator (process)[3] is a totally ordered sequence of events. A description of the behavior of an actor has at least two uses. First, it defines the actor. Secondly, it specifies the properties of the actor which can be relied on in a proof about properties of another system in which it is contained.

The event oriented approach to semantics is well suited to the expression of meanings of programs with side-effects and parallelism. Rather than attempting to record the current state of the world in something like a state vector [McCarthy, 1964], changes in state are reflected in changes over time in the behaviors of the actors. The event oriented approach is particularly useful when there is parallelism as well as change in state. Recording of all possible global states (including control information) for all the parallel processes can involve a combinatorial explosion of states. In order to describe the effect of a change in a single cell using global states, one would have to make the corresponding changes in the state information for each of the processes which could be active.

Level of Detail in Describing Computations

Another degree of flexibility available in actor semantics involves the amount of detail to be included in specifications. That is, an actor can be specified (or its set of possible behaviors can be specified) in as much or as little detail as is desired. In describing an existing actor system this level of detail is fixed by the choice of a set of distinguished actors contained in the actor system. Only events involving these distinguished actors will be recorded as behavior of the system. To specify desired behavior of a system one writes axioms which any behavior of the system must satisfy. At a very high level of detail these axioms will essentially be input-output constraints, i.e. *what* is to be done, not *how*. If more detailed specifications are made, the possible behaviors which will realize those input-output constraints become more restricted. Generally, in order to prove that a set of specifications is satisfied by a particular system, one examines the behaviors of the system at a level of detail greater than that of the specifications and proves that these behaviors realize the behavior that is required.

These points should give some indication of how the simple actor concept can be used as a building block to define the higher level concepts which are useful descriptive tools. After the formal definitions of behaviors of actors in the following section, we will give semantic definitions of several familiar systems, illustrating the application of these concepts.

3 Note that while the activator as a source of energy for computation may be analogous to some notion of process, the activator is definitely not meant to connote any of the following concepts: program counters, pushdown stacks, or address spaces.

Behaviors

Since actors are defined by their behaviors, the concept of behavior will have to be defined. Behaviors consist of events.

Definition: An event is a four-tuple, $\langle t\ m\ \alpha\ ec\rangle$ where t is the target, m is the transmission, α is the activator, and *ec* is the event count of the activator α.

As stated earlier, an event is a description of the transmission of m to t by α. Since the events of a single activator are totally ordered in time, an event count can be used to completely characterize a particular transmission. This means that if identical transmissions occur involving some particular t, m, and α then the two can in fact be distinguished by the proper time for α at which they occurred.

The event count *ec'* denotes the time immediately after *ec*. Therefore $\langle t_1\ m_1\ \alpha\ ec\rangle$ immediately precedes $\langle t_2\ m_2\ \alpha\ ec'\rangle$ and we can write $\langle t_1\ m_1\ \alpha\ ec\rangle$ --> $\langle t_2\ m_2\ \alpha\ ec'\rangle$. For events with different activators, $\langle t_1\ m_1\ \alpha_1\ ec_1\rangle$ --> $\langle t_2\ m_2\ \alpha_2\ ec_2\rangle$ if α_2 is created in the event $\langle t_1\ m_1\ \alpha_1\ ec_1\rangle$. (Some actors have the ability to create new activators.) The transitive closure of the relation --> is referred to as causally precedes i.e. for events E_1 and E_2, E_1 --> E_2 is read as "E_1 causally precedes E_2." This is an ordering which can reasonably be considered to be an intrinsic property of the actor system in question. Other orderings may be imposed by axioms for primitive actors. (For example a synchronization primitive introduces constraints in the causally precedes relation.)

Definition: A history is a set of events partially ordered by the transitive closure of the relation -->.

An external environment for an actor system is a set of actors constituting the "outside" of the system. This includes an assignment of an initial set of activators and initial events for each activator which will determine the "beginning of history" for this system and environment. These initial events will always be about transmissions which cross the boundary and enter the actor system, i.e. events which have a target inside the system and a message outside the system. Histories are written for systems with arbitrary but fixed external environments.

The behavior of an arbitrary system of actors can be extracted from the history by choosing only those events which involve certain distinguished actors. Note: we will consider any new actor created by an event involving an actor in the system Σ to be in Σ also.

Definition: For any history of the actor system Σ the <u>behavior</u> of system Σ with respect to the set of distinguished actors $\Delta \subset \Sigma$ is the subset of the events of the history containing events <t m α ec> where either

1. t is in Σ and m is not.
2. m is in Σ and t is not.
3. Either t or m is in Δ.

Thus an event will be in the behavior if it describes either a transmission across the boundary of Σ (i.e. with target inside and transmission outside or vice versa) or a transmission with a distinguished actor as either target or message. This is how the level of detail of a behavior is fixed by the choice of Δ. Note that if the set Δ is empty then the minimally detailed behavior results, i.e. the behavior of Σ as seen from the outside.

Two actor systems are equivalent with respect to some set of distinguished actors if and only if they have identical behaviors with respect to that same set of actors. The choice of distinguished actors can make a difference in equivalence of actor systems. For example, two programs for factorial can be seen to be equivalent at the level of input-output transformation (i.e. with no actors distinguished) even if a larger set of distinguished actors would reveal one to be a "counting-up" factorial and the other to be "counting-down."

If multiplication, represented by *, is considered to be a distinguished actor then in any behavior of one factorial there might be a series of events <* [1 1] α ec> --> <* [2 1] α ec'> --> <* [3 2] α ec"> and so on, while in the other there might be a series <* [n 1] α ec> --> <* [n-1 n] α ec'> --> <* [n-2 n*n-1] α ec">.

<u>*Behavioral Specifications of Systems*</u>

Since behaviors of most actors are infinite objects it will not often be possible to write out completely the set of behaviors of any given (or desired) actor. In fact, many behaviors of a system will contain incidental information about events in the environment. Instead, characteristics common to *all behaviors* of *any actor system containing the actor in question* can be specified in the form of axioms about events and the causally precedes relation. These are not Hoare [1969] style axioms about transformations of predicates over the occurrence of an operation or of a line of code, but rather are about the kinds of events that can possibly occur in a computation and how they must be ordered.

When an actor is being specified as a primitive, that is, when it is not defined in terms of other actors, the axioms describe conditions which must be satisfied by any history. Any actor can be described from the outside but if an actor is to be implemented in terms of other actors it should be understood that between any two events there may be an arbitrary number of other events if we examine a history (or even a more detailed behavior). However, all events related by causally precedes will remain so related. When we do not wish to imply that E_2 immediately follows E_3 we will write, "?" in the event count. In general "?" can be used in any place in an event to indicate that we do not care about what actor, activator, or event count appears there.

Recall that if two events E_1 and E_2 are known to be in a behavior, we can write E_1 --> E_2 to indicate that E_1 causally precedes E_2. When E_1 and E_2 have not previously been mentioned, a statement of the form E_1 --> E_2 should be read as "whenever E_1 is in the history (behavior) then so is E_2 and E_1 causally precedes E_2," or alternatively as "if E_1 is in the history (behavior) it will cause E_2." This is a way in which we guarantee response in *finite time from* an actor.

We will continue to use the labelling convention on the transmission actors. The transmissions are of the following general form:

(apply: <u>message</u> (then-to: <u>continuation</u>) (else-to: <u>complaint-dept</u>))

We will often omit the else-to part when it is not of interest.

<u>Sequences</u>

The following axioms about the behaviors of sequences summarize some of the familiar properties of sequences while illustrating the use of axioms to constrain the kinds of behavior an actor can have. Note that since sequences are indeed functional objects they could have been described in more conventional mathematical notation [Scott and Strachey, 1971].

An actor which is a sequence of *n* elements has the property (among others) that it can be asked for its *i*th element. If a sequence is represented by $[x_1 \ldots x_n]$ this property can be specified by the following axiom:

<$[x_1 \ldots x_n]$ **(apply: [i] (then-to: c) (else-to: cd))** α **ec**>
    --> <**c (apply:** x_i**)** α **ec'**>   for $1 \leq i \leq n$

This means that at the level of detail at which sequences are "black boxes" the event in which the sequence $[x_1 \ldots x_n]$ is sent the message [i] and the continuation, c, always causes as the next event the transmission of the *i*th element of the sequence to that continuation. This transmission had three components: a message, a continuation, and a complaint department. This last actor is the one to which **'not-applicable** messages are sent if the target actor does not like its message. For instance, for $i > n$, it is the case that the sequence satisfies

<$[x_1 \ldots x_n]$ **(apply: [i] (then-to: c) (else-to: cd))** α **ec**>
    --> <**cd (apply: 'not-applicable)** α **ec'**>.

When using axioms as specifications of all possible behaviors we assume that for any messages not explicitly dealt with the actor being described causes 'not-applicable to be sent to the complaint department. Thus at this stage in the definition it can be assumed that sending any transmission (apply: x (then-to: c) (else-to: cd)) to a sequence where x is not a sequence containing an integer, causes 'not-applicable to be sent to cd.

Cells

Cells do not fit as neatly into a functional notation. A cell is created with some initial contents. It is possible to ask for the contents of a cell. Update messages to that cell can change its contents. Thus it is possible that several events in a single behavior can have identical targets (namely the cell) and identical transmissions and yet cause different events to occur. This occurs when the events have transmissions

(apply: 'contents (then-to: continuation))

but the contents have been changed between the two events. In each case the continuation is sent different actors.

This means that cells are side-effect primitives. Loosely speaking, an actor system has undergone a side-effect if its behavior has changed over time. In terms of functions, a side-effect has occurred when the same procedure p has been called on two different occasions with the same argument and has returned different values. This can be generalized to behaviors by looking at future events rather than just at values returned.

Cells correspond to side-effect primitives in various languages. Therefore, it is likely that most readers have relied on them in programming. We will now try to formulate exactly which properties are being relying on. First would be that creation of a cell (or allocation of space for a cell) with its initial value should be an operation guaranteed to take only finite amount of time. This is also a desirable property of updates and contents queries. (Any argument that a given program which contains assignment statements always terminates is based on these properties).

Axiom 0:

Cons-cell creates cells.

<cons-cell (apply: [x] (then-to: continuation) (else-to: cd_1)) α ec_1>
    --> <continuation (apply: $cell^*$) α ec_1'>

where cell is a newly created actor.

Axiom 1:

Cells can respond to contents queries and updates.

<cell (apply: 'contents (then-to: continuation) (else-to: cd)) α ec>

--> <continuation ? α ec'>.

<cell (apply: [← x] (then-to: continuation) (else-to: cd)) α ec>

--> <continuation (apply: cell) α ec'>.

[Notice that a cell returns itself,
not its contents after update]

Newly created actors are marked in events by an asterisk[*]. It is often useful to be able to tell explicitly that an actor, a, appears for the first time in event E since for any other event, E_1, in which a occurs it must be the case that E --> E_1.*

The next two properties that some procedures rely on are closely related to each other. Initially we expect contents queries to find the original contents in the cell. This should be true until an update transmission is sent. Similarly, once an update transmission is sent, say [← y] , we would expect to find y in the cell until it is updated. Thus we shall give two more axioms, one about retrieving the initial contents and the other about retrieving the last contents stored.

Axiom 2 says that if **cell** first appears in event E_1 and if no update occurs between E_1 and contents query E_2, then E_3 will be an event which sends the initial contents to the continuation.

Axiom 2

Initial Contents -- A cell created with x as its contents will respond to the message **'contents** with x if no new value has been explicitly placed there yet.

* This has implications for the relation *"knows about"* which can be formalized and which has implications for the definition of kinds of control structure and the explanation of how garbage collection fits into the model. [See Appendix]

Suppose that E_0 --> E_1 is in the behavior where

E_0 is <cons-cell (apply: [x] (then-to: c) (else-to: cd)) α_1 ec_1>

E_1 is <c (apply: cell*) α_1 ec_1'>

If there are events E_2, E_3 where

E_1 --> E_2 --> E_3

E_2 is <cell (apply: 'contents (then-to: c_1) (else-to: cd_1)) α ec>

E_3 is <c_1 (apply: y) α ec'>

and $\neg\exists E$ (E_1 --> E --> E_2)

where E is of the form

<cell (apply: [← ?] (then-to: ?) (else-to: ?)) ? ?>

then y = x.

Axiom 3 makes a similar statement about the last contents stored by an update, rather than about the initial contents. The only significant change from the statement of Axiom 2 is that E_0 is:

<cell (apply: [← x] (then-to: c) (else-to: cd)) α ec_1>.

Unfortunately these axioms are reasonable only in the absence of parallelism since we are ignoring the fact that --> only refers to causal relations between events. In programs without parallelism, the relation --> is necessarily a total ordering making these axioms sufficient. (Also, due to the predictable nature of the cell in the absence of parallelism its use does not really increase power [See Appendix].) But in general there may also be update events due to another activator which are occurring independently of the ordering of the first activator.

In a behavior such as:

$$\begin{tikzcd}
 & \langle\text{cons-cell (apply: [2] (then-to: c))}\ \alpha\ ec\rangle \arrow[d] & \\
 & \langle\text{c (apply: cell*)}\ \alpha\ ec'\rangle \arrow[dl] \arrow[dr] & \\
\langle\text{cell (apply: [← 3] (then-to: } c_1))\ \alpha_1\ ec_1\rangle \arrow[d] & & \langle\text{cell (apply: [← 4] (then-to: } c_2))\ \alpha_2\ ec_3\rangle \arrow[d] \\
\langle c_1\ \text{(apply: cell)}\ \alpha_1\ ec_1'\rangle \arrow[d] & & \langle c_2\ \text{(apply: cell)}\ \alpha_2\ ec_3'\rangle \\
\langle\text{cell (apply: 'contents (then-to: } c_3))\ \alpha_1\ ec_2\rangle \arrow[d] & & \\
\langle c_3\ \text{(apply: ?)}\ \alpha_1\ ec_2'\rangle & &
\end{tikzcd}$$

The message place marked by the question mark can represent either the message 3 or 4. Which actor it will be is not a property of the system. Axiom 3 would force it to be 3.

The obvious patch is to weaken the axiom from saying exactly what has to be returned to the continuation, to just restricting the set of actors it could be. This requires adjusting the last line of axiom 2 to say that y is in the set

$$\{ z \mid z = x \vee \exists E_4 \text{ such that } (E_4 \text{ is } \langle \text{cell (apply: } [\leftarrow z] \text{ (then-to: c) (else-to: cd))} \ ?\ ?\rangle \wedge \neg (E_4 \dashrightarrow E_2 \text{ or } E_2 \dashrightarrow E_4))\}.$$

That is, y can be the initial contents or any value stored in parallel with this activator.

Axiom 3 should be adjusted similarly, such that the contents could be either the contents stored in the last update event to causally precede the contents query, or the contents stored in any update which is parallel to the activator.

Now observe the following behavior:

$$\begin{tikzcd}
 & \langle \text{cons-cell (apply: [2] (then-to: c))}\ \alpha\ \text{ec}\rangle \arrow[d] & \\
 & \langle \text{c (apply: cell}^{*}\text{)}\ \alpha\ \text{ec}'\rangle \arrow[dl] \arrow[dr] & \\
\langle \text{cell (apply: } [\leftarrow 3] \text{ (then-to: } c_1\text{))}\ \alpha_1\ ec_1\rangle \arrow[d] & & \langle \text{cell (apply: } [\leftarrow 5] \text{ (then-to: ?))}\ \alpha_2\ ?\rangle \arrow[d] \\
\langle \text{cell (apply: 'contents (then-to: } c_2\text{))}\ \alpha\ ec_2\rangle \arrow[d] & & \langle \text{cell (apply } [\leftarrow 6] \text{ (then-to: ?))}\ \alpha_2\ ?\rangle \\
\langle c_2 \text{ (apply: } ?_1\text{)}\ \alpha_1\ ec_2'\rangle \arrow[d] & & \\
\langle \text{cell (apply: 'contents (then-to: } c_3\text{))}\ \alpha_1\ ec_3\rangle \arrow[d] & & \\
\langle c_3 \text{ (apply: } ?_2\text{)}\ \alpha_1\ ec_3'\rangle & &
\end{tikzcd}$$

In fact, our intuition about the contents of cells is that it is even more restricted than stated, because if $?_1$ has value 5, then $?_2$ can no longer have any value in {3, 5, 6}, but rather can only have values in {5, 6}. There are many examples in which we can make deductions that severely restrict the possible contents of a cell. These are actually deductions about constraints in addition to those imposed by --> as to the order in which events for a given cell could have occurred.

A first reaction to these examples might be that although the current definition jars with intuition, it may be reasonable to accept bizarre behavior once non-determinism is a possibility. Perhaps we simply should not rely on accidental timing characteristics of systems. However, there are occasions when such behavior of cells is relied upon in the presence of parallelism. For instance, several solutions to the mutual exclusion of critical sections which do not make use of semaphores [Dijkstra, 1965; Knuth, 1966], implicitly make use of the cell's retaining its last contents stored even in the presence of parallelism.

We found that the fact from which all our deductions about reasonable behaviors were made was that the contents retrieved had to be consistent with some possible total ordering of all references to that cell. In that ordering each event in which the contents is checked must agree with the last contents stored. Thus cells allow one to make deductions about the actual order in which events involving that cell occur even though the events may not be necessarily ordered by -->. This is the property of cells which makes it possible to use busy waiting to synchronize independent processes.

In order to axiomatize the behavior of cells in the presence of parallelism we define the last contents stored in a cell with respect to an arbitrary total ordering ->.

Definition: For the event E, the <u>last contents stored</u> in a cell, **cell**, with respect to an arbitrary total ordering, ->, is defined to be either

the initial contents, if **cell** is created in event E_0 and there is no event E_3 which is an update to **cell** and such that $E_0 \to E_3 \to E$

or

the actor stored in update event E_0, if $E_0 \to E$ and there is no other event E_3 updating **cell** such that $E_0 \to E_3 \to E$.

<u>Axiom 4:</u>

In any behavior containing an actor, **s**, created by **cons-cell**, there is at least one total ordering, $\Rightarrow_s$, on the events in that behavior which have **s** as target. This ordering has the following properties:

1. $E_1 \dashrightarrow E_2$ implies $E_1 \Rightarrow_s E_2$

2. If $E_1 \dashrightarrow E$

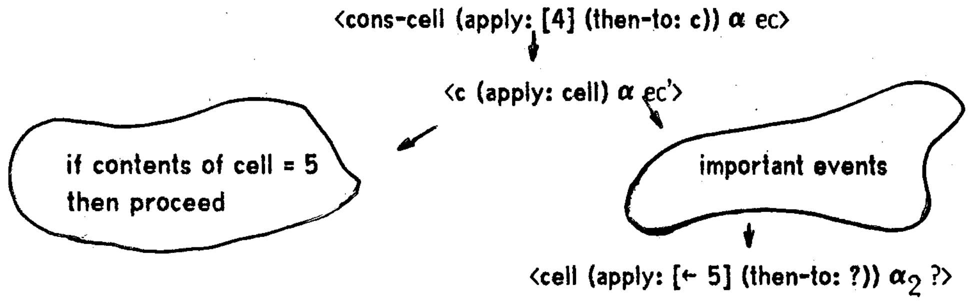
   where E_1 = **<s (apply: 'contents (then-to: c)) α ec>**

   and E = **<c (apply: x) α ec'>**

then x is the <u>last contents stored</u> in **s** with respect to $\Rightarrow_s$.

This is almost enough. From any single cell's point of view there is a total ordering of all events in which it is the target. Thus it is reasonable for the behavior of that cell to depend on that ordering. This is the meaning of Axiom 4.

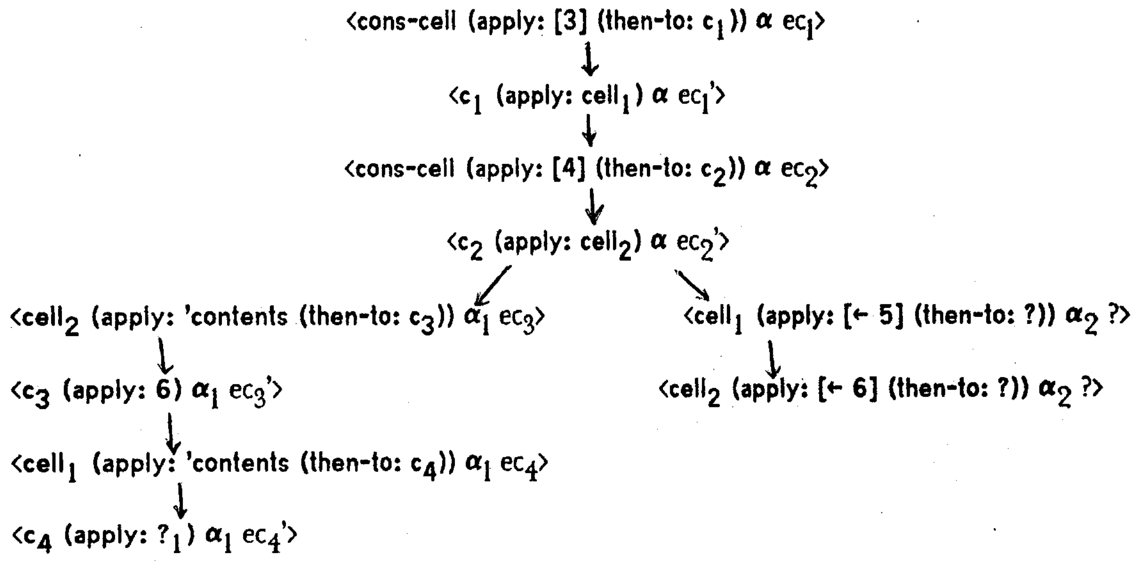
In fact, more information about actual ordering of events can be deduced from checking contents of cells. For instance if it is important for some set of events performed by α_2 to precede some event of α_1, a cell can be used as follows:



<cons-cell (apply: [4] (then-to: c)) α ec>
<c (apply: cell) α ec'>
if contents of cell = 5 then proceed
important events
<cell (apply: [← 5] (then-to: ?)) α2 ?>

The reason that this synchronization works is that in addition to knowing that if the contents of cell is found to be 5 then <cell (apply: [← 5] (then-to: ?)) α_2 ?> must have occurred, we also know from the causally precedes relation that the "important events" must have occurred. Thus --> and $\Rightarrow_{cell}$ can be used together to make deductions about ordering of events other than those involving **cell**.

The additional axiom is needed to constrain behaviors of cells in cases where there is more than one cell. The contents of any single cell must then be consistent not only with total ordering on events involving it but also with any partial ordering which can be derived from both --> and the total ordering implicit in other cells. As an example, the following case would not be sufficiently specified by the above axioms.



By axiom 4, $?_1$ could be 3 or 5. In fact, since $\textbf{cell}_2$ is known to have contents 6, $\textbf{cell}_1$ must have been updated to 5 and the only possible value for $?_1$ is 5.

Axiom 5:

In any behavior containing actors, s_i, created by **cons-cell**, it must be the case that there are orderings, $\Rightarrow_{s_i}$, satisfying axiom 4 such that the events of that behavior can be partially ordered by an irreflexive partial order, $\Rightarrow$, which has the following properties:

1. $E_1 \dashrightarrow E_2$ implies $E_1 \Rightarrow E_2$.

2. $E_1 \Rightarrow_{s_i} E_2$ implies $E_1 \Rightarrow E_2$.

There can still be non-determinism since although in any computation the events referring to the cell, s, are totally ordered by some $\Rightarrow_s$, in fact, axiom 4 does not completely determine that ordering. That is, there may be many total orderings on the events with s as target which have the properties enumerated in axiom 4, and which satisfy axiom 5.

Synchronization

A synchronization primitive imposes a total ordering on all events of activators which pass through it. This means that if some synchronization actor always encased a cell it would never be necessary to worry about unordered updates and contents queries. The synchronization would impose an ordering and the cell would always contain the last (in the causally precedes relation) contents stored.

This is part of a general form for specifying behaviors of any system using synchronization. There must be an axiom to the effect that an ordering is imposed, and the axioms about other properties of the system can be dependent on this ordering.

One way to protect a cell would be to define a primitive **cons-protected-cell** which can create protected cells. Externally **cons-protected-cell** should have behavior satisfying:

<**cons-protected-cell (apply: x (then-to: c))** α ec>
    --> <**c (apply: protected-cell***) α ec'>.

In order to describe the behavior of **protected-cell**, it is necessary to describe **cons-protected-cell** in some more detail. Although its means of synchronization can be left unspecified, the fact that it contains a locally known cell and that this cell is encased in a protective actor which is known to the world should be axiomatized.

In more detail, the behaviors of **cons-protected-cell** also satisfy:

<**cons-protected-cell (apply: x (then-to: c))** α ec>
    --> <**cons-cell (apply: x (then-to:** c_1***))** α ?>
where c_1 has behaviors which satisfy:
<c_1 **(apply: cell***) α ?>
    --> <**c protected-cell*** α ?>.

Now we can describe the behavior of **protected-cell**.

Axiom 1:

From the outside a protected cell should look like a cell.

<protected-cell (apply: 'contents (then-to: c)) α ec>
  --> <c (apply: ?) α ec'>
and
<protected-cell (apply: [← ?] (then-to: c)) α ec>
  --> <c (apply: protected-cell) α ec'>.

The rest of the properties of protected-cell can be inferred from further details about what it does with its messages:

Axiom 2:

Protected-cell sends messages to cell.

<protected-cell (apply: 'contents (then-to: c)) α ?>
  --> <cell (apply: 'contents (then-to: $c_{content}$*)) α ?>
where
<$c_{content}$ t α ec>
  --> <c t α ec'>.

and

<protected-cell (apply: [← x] (then-to: c)) α ?>
  --> <cell (apply: [← x] (then-to: c_x*)) α ?>
where
<c_x t α ?>
  --> <c (apply: protected-cell) α ?>.

Thus protected-cell has side-effects because its behavior reflects the behavior of cell.

Axiom 3:

Protected-cell adds ordering to a behavior.

If E_1 --> E_2 and E_3 --> E_4 where

E_1 = <protected-cell (apply: x_1 (then-to: c_1)) α_1 ?>
E_2 = <cell (apply: x_1 (then-to: c_{1x}*)) α_1 ?>
E_3 = <protected-cell (apply: x_2 (then-to: c_2)) α_2 ? >
E_4 = <cell (apply: x_2 (then-to: c_{2x}*)) α_2 ? >

then either $E_2 \dashrightarrow E_4$

or $E_4 \dashrightarrow E_2$.

This says that somewhere in the interface between **protected-cell** and **cell** there is locking and unlocking going on which causes all events with **cell** as target to be ordered by causally precedes. The exact mechanism cannot be seen at this level of detail but it exists.

<u>Axiom 4:</u>

Given E_1, E_2, E_3, and E_4 as above and $E_2 \dashrightarrow E_4$, then in fact

$E_2 \dashrightarrow \langle c_{1x}$ (apply: ?) α_1 ?$\rangle \dashrightarrow E_4$.

That is, the event in which a response is made from the cell triggers **protected-cell** and allows another transmission to the cell. Thus c_{1x} must know how to free the cell. Again, since the synchronization mechanism is primitive we cannot see the chain of events *causing* the unlocking. It is simply a property of this mechanism that it causes the protected cell to let someone else get to the cell.

Now, from the point of view of the cell, all events are totally ordered by causally precedes. If cells were always used inside protectors only the simpler single activator axioms for cells would be important. The more complex global ordering constraints can be deduced from a combined understanding of synchronization which imposes ordering and of cells which remember their contents.

Conclusions

We can express characteristics of a side-effect primitive in the presence of parallelism. Side-effects are described by delineation of possible changes in behavior rather than in a state vector. Synchronization can be used to impose additional ordering constraints.

Further research is in progress on applying this semantic formalism to structured synchronization actors, similar to monitors [Hoare, 1974] or serializers [Hewitt, 1974]. Once a sufficiently expressive set of descriptions is developed we will implement some systems, such as an airline reservation system or a solution to the readers-writers problem, and prove "correctness" relative to these descriptions.

The formalism can also be used in comparative studies of the behaviors characteristic of a variety of control structures. An example of this kind of work accompanies a result on elimination of cells presented in the Appendix.

Appendix

Concerning Side-Effects

A Controversy

There is some controversy at present over the desirability and necessity of use of side-effect primitives. There are at least two major aspects of this controversy. One relates to the clarity and reliability of code. There are programmers who would prefer to at least restrict, if not eliminate entirely, the use of side-effect primitives such as cells in programs. Styles of programming without side-effects are compared to styles dependent on side-effects later in this appendix.

Another aspect of the controversy is the effect on implementation requirements of use of particular kinds of side-effect primitives such as cells. Implementations of cells must handle possible cycles in the ***knows about*** relation for actors. Each actor ***knows about*** a finite number of other actors. We will say that there is a cycle in an actor system if there is some actor which ***knows about*** itself. These cycles make it necessary to use a copying garbage collection algorithm rather than a reference count algorithm.*

It is well known that it is impossible to create cycles in the pure λ-calculus. Furthermore, it is easy to see that generalizing the λ-calculus to allow transmissions to be explicitly sent and received [making co-routines efficient] does not generate any cycles. Since there does not seem to be any economical way to detect the formation of cycles during program execution one cannot rely solely on a reference count garbage collection scheme for programs when cells are in use.

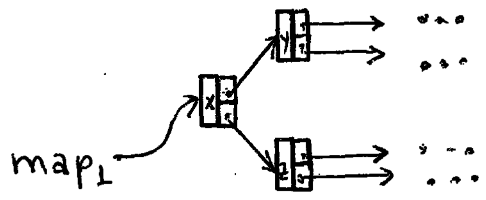
In this section a result is presented to the effect that cells can be eliminated from programs without parallelism and that this can be done within certain bounds on loss in efficiency. When considered in light of the actor semantics of this paper this fact can be seen to be true even in the presence of iteration, generators, and co-routines. We will present the result with informal argument rather than proof and then proceed with discussion of side-effects in various control and data structures.

Elimination of Cells

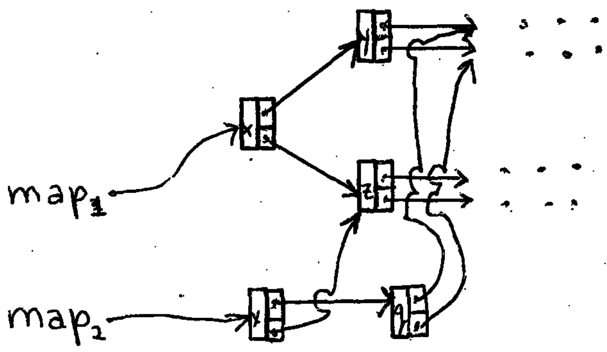
Theorem: There is a uniform procedure for eliminating cells from programs with side-effects, but no parallelism. It causes at most an $(n * (\ln n))$ vs n loss of efficiency in time, and at most a $(c * m)$ vs m increase in storage where c is a constant.

* The question of which type of garbage collection is preferable in a given implementation depends on a variety of considerations (for discussion see [Dennis, 1974]).

The general idea is that instead of having cells with contents which can be changed, the system has to keep track of these contents in some format which can be referenced by the cells when they want to know their contents. We call this object a *map*. Whenever the contents of a cell is needed, it can be found by asking the map. Whenever the value of a cell, n_1, is to be updated to v, this change must be simulated without an actual side-effect. That is, the existing map, map_1, must not be changed by the operation of simulating updating the contents of a cell. This can be accomplished by creating a *new* map, which we shall call map_2, whose behavior is identical to that of map_1 except that when asked for the contents of n_1 it must respond with v. The new map, map_2, must be the one examined in the future in the next update or content operations on cells. To see how this can be accomplished, assume that map_1 is stored as



Then if the third cell was to be changed to have contents q, map_2 could be stored as



All statements involving cells must have access to the current map. The way to give them this access is to always pass around the current map. This can be done in applicative programming languages by passing an additional argument, namely the map, to all functions. The map would then be available as a local variable in each procedure body.

In imperative languages this would be done by rewriting sequential code to pass the map from one statement to the next so that the simulated side-effects will propagate through the sequence. In both cases all creation of new cells, and all updating or accessing of the contents of cells will be simulated by operations on the current map.

In an actor based language in which the basic operation is the transmitting of a transmission actor to a target actor, *the current map must be included in all transmissions.*

We will continue to use the message passing metaphor to explain the bound on loss of efficiency. In accordance with this decision, time is measured by the number of transmissions which must occur. Transmissions can be thought of as taking one unit of time, although an arbitrary amount of real time may pass between transmissions, depending on how the program is being run. The addition of the map to each transmission only linearly increases the time taken by a factor of $(1 + \epsilon)$, where ϵ can be made arbitrarily small. The important inefficiency comes in the number of transmissions needed to look up information in the map. The bound is based on a tree structured implementation of the map. It is well known that a tree with n nodes can be searched in time proportional to $(\ln n)$. Thus at any time if there are m nodes in the map a simulation of a cell statement (creation, updating, accessing) causes extra transmissions in a number proportional to $(\ln m)$. At the nth transmission of the original computation there can be at most n transmissions in which a cell was created and the size of the map increased. Therefore a computation of n transmissions can be simulated in at worst $(n \ln n)$ transmissions.

The bound on space is based on one possible physical storage arrangement for the tree. If it is stored in the form used above, then if the original computation created m cells (realized physically in m locations) the simulation requires $3m$.

Implications of the Theorem

The result is directly applicable to implementation with reference count in the following way. If the only concern is that it be possible to run programs without having to do mark/sweep or copying garbage collection, then programs which do not involve parallelism in any language (SIMULA-67, LISP with **setq** and **rplaca**) could simply be translated to programs with maps and primitives which imitate cells using maps. The loss in efficiency in time is then known to be bounded by $(n \ln n)$ vs n. The space needed increases by a constant factor.*

Also, it is not necessary that this implementation create extra garbage when creating new maps and abandoning old ones. If the storage scheme chosen is the one illustrated above then garbage is recognizable at the time it becomes garbage. Part of the process of creating a new map can be the placing of new garbage into free storage. This can be done with only a small constant factor of further inefficiency and makes it possible to assume

* We have since realized (Jan. 1975) that this application to reference count implementations is misleading since the simulation of cells simulates the cycles as well. Instead of having a lot of inaccessible cycles of cells which are not garbage collected, there are now as many accessible locations containing information about what was in that cycle. These locations will never be accessed or garbage collected.

that the new program will not require more garbage collection than the old one did (independent of how that garbage collection will be implemented). Thus there is no additional inefficiency not covered by the bounds given.

The implications of this theorem for goals such as readable code are less direct. The fact that elimination is possible without loss of generality is reassuring, but the bound on loss of efficiency is not directly applicable. The Elimination of Side Effects Theorem cannot be applied directly to arbitrary programming styles for the following reason. If one goal in eliminating cells is to make programs more understandable, programming with cells and then simply simulating their use to technically eliminate them would be of no use. The overall structure of the program would remain unchanged and it will not be of greater clarity. It is possible that the "most understandable" style, when discovered, will turn out to require such complete and possibly redundant rewriting and restructuring of code that it will be considerably more efficient or even considerably less efficient than the bound calculated in the theorem.

However, we will show below how to apply the method of the theorem to standard cases that are generally encountered in programming practice to cause only fairly reasonable loss of efficiency. The styles illustrated in the following section are all based on the same principle as the procedure used in the elimination of cells. That is, the information that would be stored in a cell is instead passed around to those who might need it. However, it is usually passed in such a way that extra garbage collection becomes necessary. Thus there is a trade-off between efficiency gained by using reference count garbage collection, and inefficiency due to generation of extra garbage.

One further point should be made before going on to programming styles. It concerns the restriction of this result to systems without parallelism. The procedure for eliminating cells depends on the existence of exactly one activator so that each event is guaranteed to have the current map when it occurs. The procedure will not work in the presence of more than one activator. One difficulty is in simulating the non-determinism which could occur due to the presence of both parallelism and cells.

Programming Styles which do not use Cells

Note that the Elimination of Cells Theorem applies to actor systems with explicit access to transmissions as opposed to the λ-calculus where there is explicit access only to the message part of a transmission. Thus it applies to systems with iteration, generators, and co-routines [e.g. systems as powerful as SIMULA-67 [Birtwistle et al, 1974] which use quasi-parallelism but not real parallelism]. Each of the above control structures has its own programming style which does not involve the use of cells and which is related to the use of continuations as in [Mazurkiewicz, 1971; Fischer, 1972]. We illustrate these styles and the trade-offs involved using simple examples below.

Iteration and Side-effects

Iterative control structures can be realized in the λ-calculus and thus without cells. We will give a simple example to illustrate how this is done:

```
(label iterative-factorial
   (λ [n a]
      (if (n = 1)
         then a
         else (iterative-factorial (n - 1) (n * a)))))
```

Of course the above definition is also a *recursive* definition of **iterative-factorial** in the sense that it is defined *in terms of itself*. However, in terms of the actor semantics it is not a *recursive* definition in the sense of ALGOL-like languages. I.e., it does not use up arbitrarily large amounts of intermediate working storage in order to do the computation. Using the actor semantics the following version of factorial is *recursive* in both senses of *recursive*:

```
(label recursive-factorial
   (λ [n]
      (if (n = 1)
         then 1
         else (n * (factorial (n - 1))))))
```

The reason that storage increases is that the number of actors currently pointed to ("*known about*") continues to increase. Actors which are pointed to are not eligible for garbage collection. The fact that **iterative-factorial** only requires a bounded amount of working storage can be demonstrated by analysis of the computations of **iterative-factorial**. In the course of the computation several new actors will be created but only a bounded number of these actors will be known about at any given instant. This establishes an upper bound on the amount of temporary storage needed. In the case of iteration, it is just as efficient to program without the use of cells as it is to use cells and the programs seem more readable without cells.

Generators With and Without Cells

We shall demonstrate how to define generators with and without using cells by defining generators that produce the squares 1, 4, 9, 16, etc. in sequence on demand using a device called streams by Peter Landin [1965]. We will develop two implementations that use the same interface with the polymorphic operator **next** which requests the next element. The interface is defined in such a way that it will work with streams with side-effects [Balzer, 1971; Krutar, 1971; and Mitchell, 1971] as well as with functional streams [Landin, 1965]. The polymorphic operator **next** which makes this happen works by immediately turning around and passing the buck to **the-supply** using the message

(next: (else-complain-to: the-complaint-dept))

to politely ask the-supply for its next and to complain to the-complaint-dept if it doesn't have a next.

A stream-sequence consists of two components: a first which is the next element in the stream, and a rest which is another stream that holds the rest of the elements after the first. Below we define a stream that produces integers which are squares up to some limit. It produces the next larger square each time it is asked for it, and returns the square along with a stream for generating more squares.

```
To construct a stream of squares,
  receive a sequence of
    a limit of how many squares to produce
    and a count from which to begin producing squares.
   Then return an actor which,
    when asked either to produce the next element
    or to complain to a complaint-dept if it doesn't have one,
      checks the count to see
        if it is greater than the limit
          and if so complains to the complaint-dept.
        Otherwise it
          returns a stream-sequence which has
            the square of the current count as its next element
            and rest which is a stream of squares
              with the same limit
              but whose count is one greater than the input count.
```

Below we define a polymorphic operator print-elements which prints the elements of any stream which it is given. For example

```
(print-elements (cons-stream-for-squares (limit: 3) (count: 1)))
```

will print 1, 4, 9, and then print "done."

```
To print-elements of a stream
  receive a stream.
    Ask the stream for its next.
      If it has a next,
        receive the stream-sequence and bind its first and rest.
          Print the first of the stream-sequence.
          Then print-elements of the rest of the stream-sequence.
      If the stream has no next,
        print "done."
```

Below we define a port [a stream with side effects] that produces square integers up to some limit. It produces the next larger square each time it is asked for its next, returning itself [with its behavior modified by a side effect] as the "rest."

```
To construct a port for squares,
  receive a sequence of
    a limit of how many squares to produce
    and a count from which to begin producing squares.
   Let current-count be a cell that initially contains the initial count.
    Label the actor whose behavior is defined below as the-port.
      Return the-port which is an actor which,
        when asked either to produce the next
        or to complain to a complaint-dept if it doesn't have one,
          checks the contents of current-count to see
            if it is greater than the limit,
              and if so complains to the complaint-dept.
            Otherwise it
              increments the contents of the current count.
              Then returns a stream-sequence which has
                as its next element
                  the square of
                    the contents of current-count
                and rest which is the-port.
```

Note that we have defined the port in such a way that the polymorphic operator **next** can also be used to obtain squares from it. Thus in many cases the decision of which of the two implementations of the square generator to use can often be deferred or even later reconsidered. For example

(print-elements (cons-port-for-squares (limit: 3) (count: 1)))

will also print 1, 4, 9, and then print "done." However, the behavior of **cons-stream-for-squares** is not exactly the same as that of **cons-port-for-squares**. Once a port for squares has been printed then it is "totally used up" and has no more useful behavior. There is no "backup" capability built into ports. The burden of providing such a capability if it is desired is placed upon the user of the port. Debugging a system where ports are used as the basic communication primitive can be painful because useful debugging activities such as printing the contents of a port sometimes have unfortunate side-effects.

In contrast, a stream for squares does not change its behavior as a result of being printed and can be used for other purposes at the same time such as searching for successive squares whose sum is an odd prime. Generators can always be implemented almiost as efficiently without the use of cells as by using cells except that avoiding the use of cells imposes more of a garbage collection problem. The extra garbage collection in the style of programming without cells stems from the fact that old generators disappear only when they are no longer being referenced.

Data Structures and Side-Effects

The style of programming used for generators is applicable to definition of actors which behave like a variety of data structures such as stacks, queues, sets, deques, and bags, making it possible to implement them either with or without side-effects. For example **(pop the-stack)** can return a value of the form

(next: the-top-of-the-stack (rest: a-stack-without-the-top-element))

The technique is also applicable to implementing control structures [such as iteration and co-routines] which can be implemented either with or without cells.

Co-routines and Side-effects

Co-routines are a control structure which are often used by programmers in artificial intelligence and systems programming. They have recently found increasing popularity [Hewitt, 1971; Bobrow-Wegbreit, 1972; McDermott and Sussman, 1972; Davies, 1972; Hewitt, Bishop, and Steiger, 1973] in knowledge-based applications as the foundation for a *dialogue-based* programming style. Early Artificial Intelligence programs were mainly organized as multi-pass heuristic programs consisting of a pass of information gathering, a pass of constraint analysis, and a pass of hypothesis formation. It is now generally recognized that multi-pass organizations of this kind are inflexible because it is often necessary for information to flow across these boundaries *in both directions* in a *dialogue* at all stages of the processing.

Co-routines were used by Conway [1963] to convert the separate lexical and syntactic passes of a two-pass compiler into a single pass compiler. Using cells and gotos, a co-routine control structure [in a language like PAL, GEDANKEN, or CONNIVER] can be built to do this in the following way. We assume that there are four cells known globally to both the lexical analyzer and the syntactic analyzer of the compiler:

$\text{message}_{\text{to-lexical}}$: cell for messages to lexical analyzer
$\text{message}_{\text{to-syntactic}}$: cell for messages to syntactic analyzer
$\text{resume}_{\text{lexical}}$: cell for resume-points of lexical analyzer
$\text{resume}_{\text{syntactic}}$: cell for resume-points of syntactic analyzer

To start the system execute the following:

```
Update the contents of resume_syntactic with starting-syntactic-resume-point
Call the procedure start-lexical-analysis with the-input-character-stream as its argument
```

Whenever the **a-lexical-procedure** of the lexical analyzer has found the next lexical item L then it can resume the syntax analyzer by executing the code in the box below:

```
Define a-lexical-procedure to be
  (λ argument-to-a-lexical-procedure
      ...
______________________________________________________________________________
|     Update the contents of $\text{resume}_{\text{lexical}}$ to be a-lexical-resume-point     |
|        which is defined below.                                              |
|     Then update the contents of $\text{message}_{\text{to-syntactic}}$ to be L               |
|     and then goto the contents of $\text{resume}_{\text{syntactic}}$.                        |
|   a-lexical-resume-point:                                                   |
|     Read the contents of $\text{message}_{\text{to-lexical}}$.                              |
______________________________________________________________________________
      ...
      Return $\underline{\text{value}}_{\text{lexical}}$ as the value of a-lexical-procedure.)
```

Note that within the lexical analyzer, a-lexical-procedure is an ordinary procedure; it is called with an ordinary procedure call and returns with an ordinary procedure return. The lexical analyzer need have no knowledge that a-lexical-procedure interacts with the syntax analyzer.

Whenever a-syntax-procedure of the syntactic analyzer needs more input it can resume the lexical analyzer by executing the code in the box below:

```
Define a-syntax-procedure to be
  (λ argument-to-a-syntax-procedure
      ...
______________________________________________________________________________
|     Update the contents of $\text{resume}_{\text{syntactic}}$ to be a-syntactic-resume-point |
|        which is defined below.                                              |
|     Update the contents of $\text{message}_{\text{to-lexical}}$ to be 'next                  |
|     and then goto the contents of $\text{resume}_{\text{lexical}}$.                          |
|   a-syntactic-resume-point:                                                 |
|     Read the contents of $\text{message}_{\text{to-syntactic}}$.                            |
______________________________________________________________________________
      ...
      Return $\underline{\text{value}}_{\text{syntactic}}$ as the value of a-syntax-procedure.)
```

Using transmissions we shall indicate how to eliminate the side effects involved in co-routining between the syntactic and lexical analyzers without severe loss of efficiency. Our scheme can be viewed as an extension to λ-calculus in which we assume that we have the ability to explicitly bind transmissions as opposed to only being able to bind the message in a continuation which is all that the λ-calculus allows. The scheme is very similar to the one used in the Elimination of Cells Theorem in that we will increase the size of all the messages passed around inside the lexical analyzer to include what would have been the contents of $\text{resume}_{\text{syntactic}}$ and increase the size of the messages passed in the syntax analyzer to include what would have been the contents of $\text{resume}_{\text{lexical}}$. The language PLANNER-73 has been designed for the direct realization of actor semantics and so has a syntax which allows access to transmissions when this is desirable. The advantage of explicitly binding transmissions is that it provides a powerful straightforward co-routine mechanism with simple rigorous semantics defined in terms of actors. The semantics of alternative co-routine mechanisms [such as Landin's J-operator] seem to be inextricably bound up with the detailed evaluation mechanisms of the language in which they are implemented [Reynolds, 1972].

Using transmissions to implement coroutines, the system is started by executing the following procedure call:

```
(start-lexical-analysis input-character-stream begin-syntactic-analysis)
```

Whenever a-lexical-procedure of the lexical analyzer has found the next lexical item L, then it can resume the syntax analyzer by executing the code in the box below:

```
Define a-lexical-procedure to be
  Receive-transmission (apply: [argument-to-lexical-procedure resume_syntactic]
                        (then-to: reply-to-lexical))
    ...
```

```
|    resume_syntactic is sent the transmission                                            |
|      (apply: L                                                                          |
|        (then-to:                                                                        |
|          receive-transmission (apply: message_to-lexical                                |
|                                (then-to: another-resume_syntactic))                     |
|            ...                                                                          |
|            reply-to-lexical is sent the message [value_lexical another-resume_syntactic])) |
```

Whenever a-syntax-procedure of the syntax analyzer needs more input from the lexical analyzer it can resume the syntax analyzer by executing the code in the box below:

```
Define a-syntax-procedure to be
  Receive-transmission (apply: [argument-to-syntax-procedure resume_lexical]
                        (then-to: reply-to-syntactic))
    ...
```

```
|    resume_lexical is sent the transmission                                              |
|      (apply: 'next                                                                      |
|        (then-to:                                                                        |
|          receive-transmission (apply: message_to-syntactic                              |
|                                (then-to: another-resume_lexical))                       |
|            ...                                                                          |
|            reply-to-syntactic is sent the message [value_syntactic another-resume_lexical]))|
```

Use of co-routines for the above application becomes more justifiable in cases where a *dialogue* between the lexical analyzer and the syntax analyzer is needed. This happens when more interesting messages than simply 'next need to be sent to the lexical analyzer by the syntax analyzer. An application which needs this kind of sophistication is an error correcting compiler where both lexical and syntactic errors occur in programs being compiled.

Should Cells be Eliminated?

There has been a shift away from heavy reliance on side-effects in formalisms for artificial intelligence. It is apparent in the development of the context mechanism in QA-4 [Rulifson, 1972], in the definition of PLANNER worlds in terms of actors [Hewitt, Bishop, and Steiger, 1973], and in the communication between PLANNER-73 plans in terms of message passing [Smith and Hewitt, 1974].

A programming style that avoids the use of cells seems very attractive if the programs are almost as efficient, since they seem to be more readable to many people. In the case of iterative programs, a programming style which does not use cells is just as efficient and imposes no additional garbage collection overhead. In the case of generators and co-routines, not using cells does circumvent pitfalls of using side-effects but imposes some additional garbage collection overhead.

We would like to re-emphasize that we do not advocate rewriting programs using the algorithm described in the proof of the Elimination of Side-effects Theorem. For the purposes of clarity and easy debugging of programs, such activity is just as pointless as systematically rewriting programs that use the **goto** construct by substituting **while** loops and boolean variables. Instead a style of programming that has certain definite advantages must be developed.

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