CHAPTER 9 LECTURE.

QUANTIFYING & MEASURING INVESTMENT PERFORMANCE: "RETURNS"

Returns

- RETURNS = PROFITS (IN THE INVESTMENT GAME)
- RETURNS = OBJECTIVE TO MAXIMIZE (CET.PAR.)
- RETURNS = WHAT YOU'VE GOT
  - WHAT YOU HAD TO BEGIN WITH, AS A PROPORTION OF WHAT YOU HAD TO BEGIN WITH.

QUANTITATIVE RETURN **MEASURES** NECESSARY TO:

- MEASURE PAST PERFORMANCE

  => "EX POST" OR HISTORICAL RETURNS;

- MEASURE EXPECTED FUTURE PERFORMANCE

  => "EX ANTE" OR EXPECTED RETURNS.

MANY DIFFERENT MATHEMATICAL DEFINITIONS OF "RETURNS"...

TWO MAJOR TYPES OF MATHEMATICAL RETURN DEFINITIONS:

1) PERIOD-BY-PERIOD RETURNS

2) MULTIPERIOD RETURN MEASURES

TYPE 1: PERIOD-BY-PERIOD RETURNS . . .

- AKA: **"PERIODIC"** RETURNS
- SIMPLE **"HOLDING PERIOD RETURN"** (HPR)
- MEASURES WHAT THE INVESTMENT GROWS TO WITHIN EACH SINGLE PERIOD OF TIME,
- ASSUMING ALL CASH FLOW (OR VALUATION) IS ONLY AT BEGINNING AND END OF THE PERIOD OF TIME (NO INTERMEDIATE CASH FLOWS).

TYPE 1: PERIOD-BY-PERIOD RETURNS (cont'd)

- RETURNS MEASURED SEPARATELY OVER EACH OF A SEQUENCE OF REGULAR AND CONSECUTIVE (RELATIVELY SHORT) PERIODS OF TIME.
- SUCH AS: DAILY, MONTHLY, QUARTERLY, OR ANNUAL RETURNS SERIES.
- E.G.: RETURN TO IBM STOCK IN: 1990, 1991, 1992, ...
- PERIODIC RETURNS CAN BE AVERAGED ACROSS TIME TO DETERMINE THE **"TIME-WEIGHTED"** MULTI-PERIOD RETURN.

TYPE 1: PERIOD-BY-PERIOD RETURNS (cont'd)

- **NOTE:** THE PERIODS USED TO DEFINE PERIODIC RETURNS SHOULD BE SHORT ENOUGH THAT THE ASSUMPTION OF NO INTERMEDIATE CASH FLOWS DOES NOT MATTER.

TYPE 2: MULTIPERIOD RETURN MEASURES

- PROBLEM: WHEN CASH FLOWS OCCUR AT MORE THAN TWO POINTS IN TIME, THERE IS NO SINGLE NUMBER WHICH UNAMBIGUOUSLY MEASURES THE RETURN ON THE INVESTMENT.

TYPE 2: MULTIPERIOD RETURN MEASURES (cont'd)

- NEVERTHELESS, MULTI-PERIOD RETURN MEASURES GIVE A SINGLE RETURN NUMBER (TYPICALLY QUOTED PER ANNUM) MEASURING THE INVESTMENT PERFORMANCE OF A LONG-TERM (MULTI-YEAR) INVESTMENT WHICH MAY HAVE CASH FLOWS AT INTERMEDIATE POINTS IN TIME THROUGHOUT THE "LIFE" OF THE INVESTMENT.

TYPE 2: MULTIPERIOD RETURN MEASURES (cont'd)

- THERE ARE MANY DIFFERENT MULTI-PERIOD RETURN MEASURES, BUT THE MOST FAMOUS AND WIDELY USED (BY FAR) IS:

THE **"INTERNAL RATE OF RETURN"** (IRR).

- THE IRR IS A **"DOLLAR-WEIGHTED"** RETURN BECAUSE IT REFLECTS THE EFFECT OF HAVING DIFFERENT AMOUNTS OF DOLLARS INVESTED AT DIFFERENT PERIODS IN TIME DURING THE OVERALL LIFETIME OF THE INVESTMENT.

ADVANTAGES OF PERIOD-BY-PERIOD (TIME-WEIGHTED) RETURNS:

1) ALLOW YOU TO TRACK PERFORMANCE OVER TIME, SEEING WHEN INVESTMENT IS DOING WELL AND WHEN POORLY.

ADVANTAGES OF PERIOD-BY-PERIOD (TIME-WEIGHTED) RETURNS (cont'd)

2) ALLOW YOU TO QUANTIFY RISK (VOLATILITY) AND CORRELATION (CO-MOVEMENT) WITH OTHER INVESTMENTS AND OTHER PHENOMENA.

ADVANTAGES OF PERIOD-BY-PERIOD (TIME-WEIGHTED) RETURNS (cont'd)

3) ARE FAIRER FOR JUDGING INVESTMENT PERFORMANCE WHEN THE INVESTMENT MANAGER DOES NOT HAVE CONTROL OVER THE TIMING OF CASH FLOW INTO OR OUT OF THE INVESTMENT FUND (E.G., A PENSION FUND).

ADVANTAGES OF MULTI-PERIOD RETURNS:

1) DO NOT REQUIRE KNOWLEDGE OF MARKET VALUES OF THE INVESTMENT ASSET AT INTERMEDIATE POINTS IN TIME (MAY BE DIFFICULT TO KNOW FOR REAL ESTATE).

ADVANTAGES OF MULTI-PERIOD RETURNS (cont'd)

2) GIVES A FAIRER (MORE COMPLETE) MEASURE OF INVESTMENT PERFORMANCE WHEN THE INVESTMENT MANAGER HAS CONTROL OVER THE TIMING AND AMOUNTS OF CASH FLOW INTO AND OUT OF THE INVESTMENT VEHICLE (E.G., PERHAPS SOME "SEPARATE ACCOUNTS" WHERE MGR HAS CONTROL OVER CAPITAL FLOW TIMING, OR A STAGED DEVELOPMENT PROJECT).

ADVANTAGES OF MULTI-PERIOD RETURNS (cont'd)

- **NOTE: BOTH HPRs AND IRRs ARE WIDELY USED IN REAL ESTATE INVESTMENT ANALYSIS**

PERIOD-BY-PERIOD RETURNS...

- **"TOTAL RETURN"** ("r"):

  $r_t=(CF_t+V_t-V_{t-1})/ V_{t-1}=((CF_t+V_t)/V_{t-1}) -1$

  where: CF_t= Cash Flow (net) in period "t"; V_t=Asset Value ("ex dividend") at end of period "t".

- **"INCOME RETURN"** ("y", AKA **"CURRENT YIELD"**, OR JUST **"YIELD"**):

  $y_t = CF_t / V_{t-1}$

- **"APPRECIATION RETURN"** ("g", AKA **"CAPITAL GAIN"**, OR "CAPITAL RETURN", OR **"GROWTH"**):

  $g_t = (V_t-V_{t-1}) / V_{t-1} = V_t / V_{t-1} - 1$

  NOTE: $r_t = y_t + g_t$

TOTAL RETURN IS MOST IMPORTANT:

- To convert y into g, reinvest the cash flow back into the asset.
- To convert g into y, sell part of the holding in the asset.

NOTE: This type of conversion is not so easy to do with most real estate investments as it is with investments in stocks and bonds.

EXAMPLE:

- PROPERTY VALUE AT END OF 1994:
  = $100,000
- PROPERTY NET RENT DURING 1995:
  = $10,000
- PROPERTY VALUE AT END OF 1995:
  = $101,000

WHAT IS 1995 r, g, y ?...

- y_{1995} = \$10,000/\$100,000 = 10%
- g_{1995} = (\$101,000 - \$100,000)/\$100,000 = 1%
- r_{1995} = 10% + 1% = 11%

A NOTE ON RETURN TERMINOLOGY

- "INCOME RETURN" - YIELD, CURRENT YIELD, DIVIDEND YIELD.
  - IS IT CASH FLOW BASED OR ACCRUAL INCOME BASED?
  - SIMILAR TO "CAP RATE".
  - IS A RESERVE FOR CAPITAL EXPENDITURES TAKEN OUT?
  - CI TYPICALLY 1% - 2% /YR OF V.
  - EXAMPLE: V=1000, NOI=100, CI=10:
    - y_t = (100-10)/1000 = 9%, “cap rate” = 100/1000 = 10%

"YIELD"

- CAN ALSO MEAN: "TOTAL YIELD", "YIELD TO MATURITY"
  - THESE ARE IRRs, WHICH ARE TOTAL RETURNS, NOT JUST INCOME.

"BASIS POINT"

= 1 / 100th PERCENT = .0001

CONTINUOUSLY COMPOUNDED RETURNS:

- THE <u>PER ANNUM</u> CONTINUOUSLY COMPOUNDED TOTAL RETURN IS:

$$r_t = \left(\mathrm{LN}(V_t + CF_t) - \mathrm{LN}(V_{t-1})\right)/Y \Leftrightarrow V_t + CF_t = V_{t-1} * \mathrm{EXP}(Yr_t)$$

WHERE "Y" IS THE NUMBER (OR FRACTION) OF YEARS BETWEEN TIME "t-1" AND "t".

EXAMPLE:

01/01/98 V = 1000

03/31/99 V = 1100 & CF = 50

PER ANNUM

r = (LN(1150) – LN(1000)) / 1.25

= 7.04752 – 6.90776

= 11.18%

"REAL" vs. "NOMINAL" RETURNS

NOMINAL RETURNS ARE THE "ORDINARY" RETURNS YOU NORMALLY SEE QUOTED OR EMPIRICALLY MEASURED. UNLESS IT IS EXPLICITLY STATED OTHERWISE, RETURNS ARE ALWAYS QUOTED AND MEASURED IN NOMINAL TERMS. **The NOMINAL Return is the Return in Current Dollars** (dollars of the time when the return is generated).

REAL RETURNS ARE NET OF INFLATION. **The REAL Return is the Return measured in constant purchasing power dollars** ("constant dollars").

Example:

Suppose INFLATION=5% in 1992 (i.e., need \$1.05 in 1992 to buy what \$1.00 purchased in 1991).

So: \$1.00 in "1992\$" = 1.00/1.05 = \$0.95 in "1991\$“

If r_t = Nominal Total Return, year t

i_t = Inflation, year t

R_t = Real Total Return, year t

Then: $R_t = (1+r_t)/(1+i_t) - 1 = r_t - (i_t + i_t R_t) \approx r_t - i_t$,

Thus: NOMINAL Return = REAL Return + Inflation Premium

Inflation Premium = $i_t + i_t R_t \approx I_t$

In the case of the current yield

$$(\text{Real } y_t)=(\text{Nominal } y_t)/(1+i_t) \approx (\text{Nominal } y_t)$$

EXAMPLE:

1991 PROPERTY VALUE = $100,000

1992 NET RENT = $10,000

1992 PROPERTY VALUE = $101,000

1992 INFLATION = 5%

WHAT IS THE REAL r, y, and g for 1992?

Answer:

Real g = (101,000/1.05)/100,000-1= -3.81% ≈ -4%
(versus Nominal g=+1%)

Real y = (10,000/1.05)/100,000 = +9.52% ≈ 10%
(versus Nominal y=10% exactly)

Real r = (111,000/1.05)/100,000-1=+5.71% ≈ 6%
(versus Nominal r = 11%)

= g + y =+9.52%+(-3.81%) ≈ 10% - 4%

RISK

INTUITIVE MEANING...

THE POSSIBILITY OF NOT MAKING THE EXPECTED RETURN:

$$r_t \neq E_{t-j}[r_t]$$

MEASURED BY THE RANGE OR STD.DEV. IN THE EX ANTE PROBABILITY DISTRIBUTION OF THE EX POST RETURN . . .

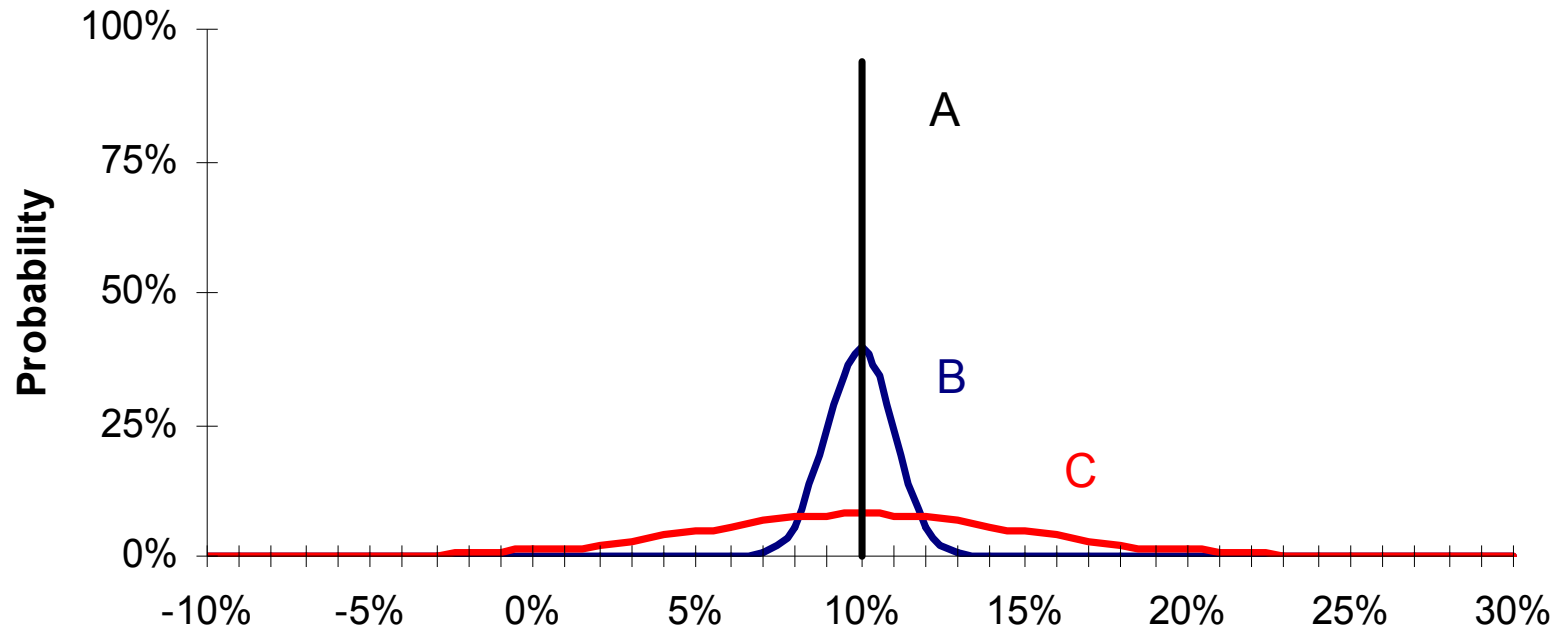


Figure 1

C RISKER THAN B. RISKIER THAN A. A RISKLESS.

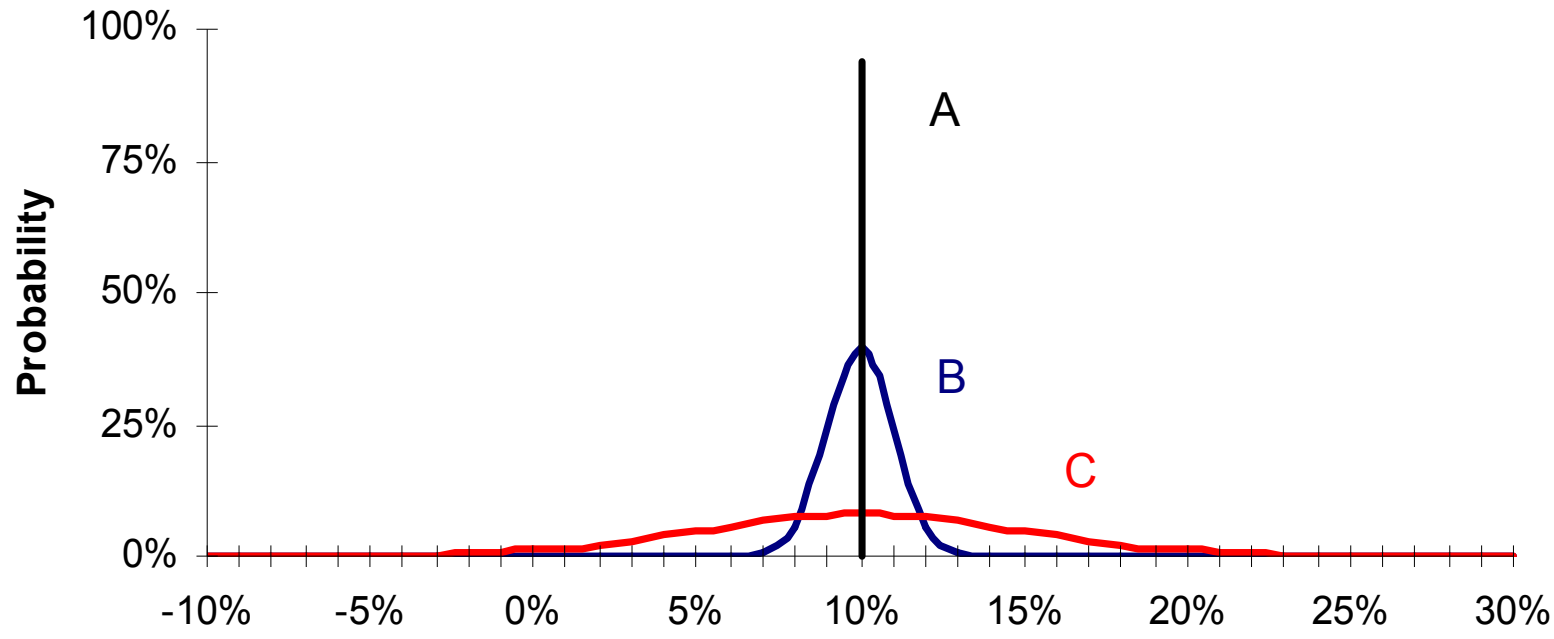


Figure 1

What is the expected return? . . .

EXAMPLE OF RETURN RISK QUANTIFICATION:

SUPPOSE 2 POSSIBLE FUTURE RETURN SCENARIOS. THE RETURN WILL EITHER BE:

+20%, WITH 50% PROBABILITY

OR:

-10%, WITH 50% PROBABILITY

"EXPECTED" (EX ANTE) RETURN

= (50% CHANCE)(+20%) + (50% CHANCE)(-10%)

= +5%

RISK (STD.DEV.) IN THE RETURN

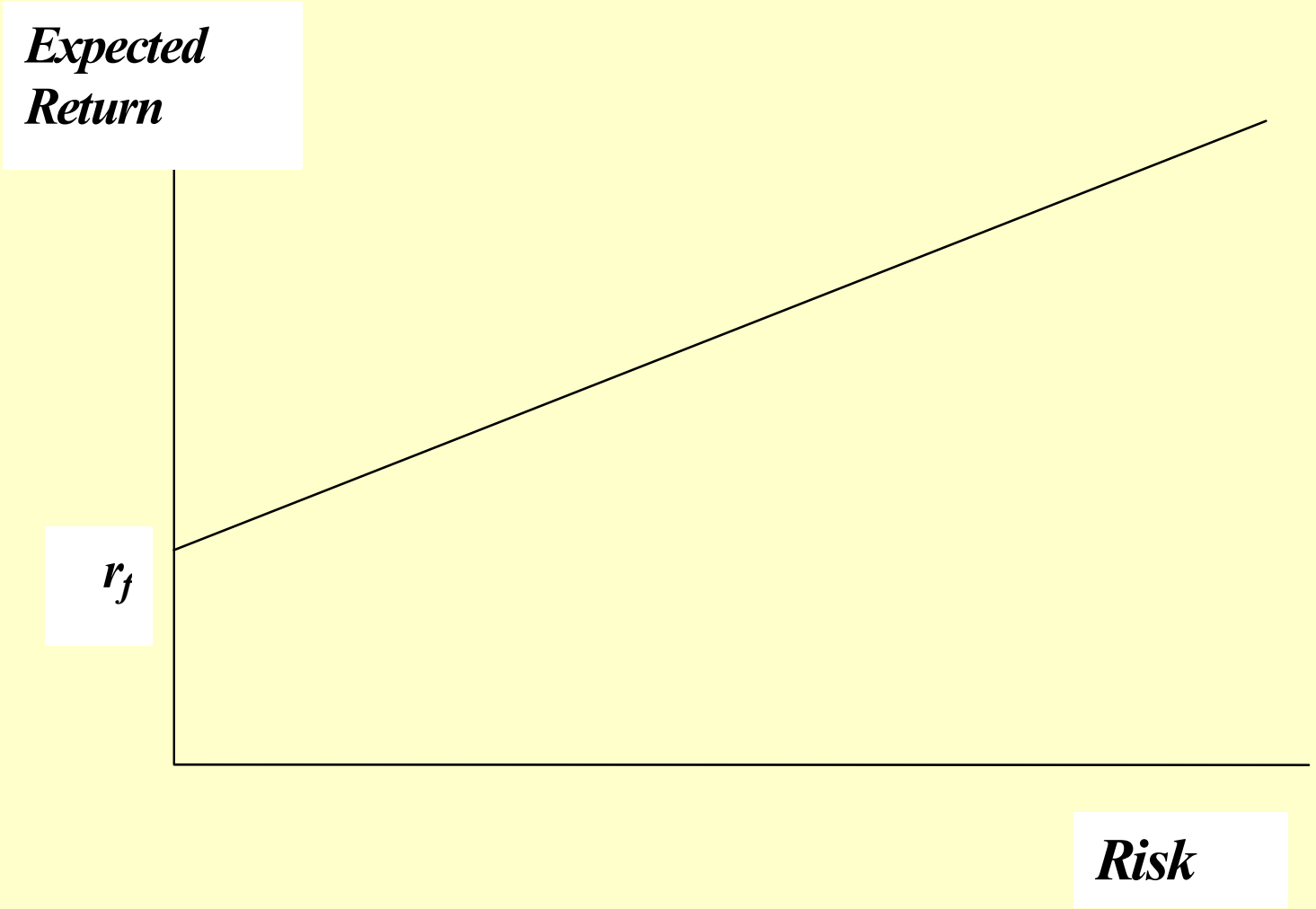
$= \text{SQRT}\{(0.5)(20-5)^2 + (0.5)(-10-5)^2\}$

= "15%

THE RISK/RETURN TRADEOFF..

INVESTORS DON'T LIKE RISK!

SO THE CAPITAL MARKETS COMPENSATE THEM BY PROVIDING HIGHER RETURNS (EX ANTE) ON MORE RISKY ASSETS . . .



RISK & RETURN:

TOTAL RETURN = RISKFREE RATE + RISK PREMIUM

$$r_t = r_{f,t} + RP_t$$

Risk Free Rate

RISKFREE RATE ($r_{f,t}$)

= Compensation for TIME

= "Time Value of Money"

≈ US Treasury Bill Return (For Real Estate, usually use Long Bond)

Risk Premium

RISK PREMIUM (RP_t):

EX ANTE: $E[RP_t]$

$= E[r_t] - r_{f,t}$

= *Compensation for* RISK

EX POST: RP_t

$= r_t - r_{f,t}$

= *Realization of* Risk ("Throw of Dice")

RELATION BETWEEN RISK & RETURN:

GREATER RISK <===> GREATER RISK PREMIUM

(THIS IS EX ANTE, OR ON AVG. EX POST, BUT NOT NECESSARILY IN ANY GIVEN YEAR OR ANY GIVEN INVESTMENT EX POST)

EXAMPLE OF RISK IN REAL ESTATE:

Current time is end of 1998.

PROPERTY "A" (**OFFICE**):

VALUE END 1998 = $100,000

POSSIBLE VALUES END 1999

$110,000 (50% PROB.)
$90,000 (50% PROB.)

STD.DEV. OF g_{99} = 10%

Example (cont'd)

PROPERTY "B" (**BOWLING ALLEY**):

VALUE END 1998 = $100,000

POSSIBLE VALUES END 1999

$120,000 (50% PROB.)

$80,000 (50% PROB.)

STD.DEV. OF g_{99} = 20%

Example (cont'd)

B IS MORE RISKY THAN A.

T-BILL RETURN = 7%

Example (cont'd)

A: Office Building

Known as of end 1998

- Value = $100,000
- Expected value end 99 = $100,000
- Expected net rent 99 = $11,000
- Ex ante risk premium = 11% - 7% = 4%

B: Bowling Alley

Known as of end 1998

- Value = $100,000
- Expected value end 99 = $100,000
- Expected net rent 99 = $15,000
- Ex ante risk premium = 15% - 7% = 8%

Example (cont'd) – Suppose the following occurred in 1999

A: Office Building

Not known until end 1999

- End 99 Value = $110,000
- 99 net rent = $11,000
- 99 Ex post risk premium = 21% - 7% = 14%
- ("The Dice Rolled Favorably")

B: Bowling Alley

Not known until end 1999

- End 99 Value = $80,000
- 99 net rent = $15,000
- 99 Ex post risk premium = -5% - 7% = -12%
- ("The Dice Rolled Unfavorably")

SUMMARY:

THREE USEFUL WAYS TO BREAK TOTAL RETURN INTO TWO COMPONENTS...

1) **TOTAL RETURN = CURRENT YIELD + GROWTH**

$$r = y + g$$

2) **TOTAL RETURN = RISKFREE RATE + RISK PREMIUM**

$$r = r_f + RP$$

3) **TOTAL RETURN = REAL RETURN + INFLATION PREMIUM**

$$r = R + (i+iR) \approx R + I$$

"TIME-WEIGHTED INVESTMENT". . .

SUPPOSE THERE ARE CFs AT INTERMEDIATE POINTS IN TIME WITHIN EACH “PERIOD” (E.G., MONTHLY CFs WITHIN QUARTERLY RETURN PERIODS).

THEN THE SIMPLE HPR FORMULAS ARE NO LONGER EXACTLY ACCURATE.

"TIME-WEIGHTED INVESTMENT". . .

A WIDELY USED SIMPLE ADJUSTMENT IS TO APPROXIMATE THE IRR OF THE PERIOD ASSUMING THE ASSET WAS BOUGHT AT THE BEGINNING OF THE PERIOD AND SOLD AT THE END, WITH OTHER CFs OCCURRING AT INTERMEDIATE POINTS WITHIN THE PERIOD.

THIS APPROXIMATION IS DONE BY SUBSTITUTING A “TIME-WEIGHTED” INVESTMENT IN THE DENOMINATOR INSTEAD OF THE SIMPLE BEGINNING-OF-PERIOD ASSET VALUE IN THE DENOMINATOR.

"TIME-WEIGHTED INVESTMENT". . .

$$r = \frac{EndVal - BegVal + \sum CF_i}{BegVal - \sum w_i CF_i}$$

where:

$\sum CF_i$ = sum of all net cash flows occurring in period t,

w_i = proportion of period t ***remaining*** at the time when net cash flow "i" was received by the investor.

(Note: cash flow from the investor to the investment is negative; cash flow from the investment to the investor is positive.)

EXAMPLE . . .

CF:	**Date:**
- 100	12/31/98
+ 10	01/31/99
+100	12/31/99

Simple HPR: (10 + 100-100) / 100
= 10 / 100
= 10.00%

TWD HPR: (10 + 100-100) / (100 – (11/12)10)
= 10 / 90.83
= 11.01%

IRR: = 11.00% . . .

EXAMPLE (cont'd)

$$0 = -100 + \frac{10}{1 + IRR/mo} + \sum_{j=2}^{11} \frac{0}{(1 + IRR/mo)^j} + \frac{100}{(1 + IRR/mo)^{12}} \quad \Rightarrow \quad IRR/mo = 0.87387\%$$

$$\Rightarrow \quad IRR/yr = (1.0087387)^{12} - 1 \quad = \quad 11.00\%$$

THE DEFINITION OF THE "NCREIF" PERIODIC RETURN FORMULA . . .

- THE MOST WIDELY USED INDEX OF PERIODIC RETURNS IN COMMERCIAL REAL ESTATE IN THE US IS THE "NCREIF PROPERTY INDEX" (NPI).
- NCREIF = "NATIONAL COUNCIL OF REAL ESTATE INVESTMENT FIDUCIARIES“
- “INSTITUTIONAL QUALITY R.E.”
- QUARTERLY INDEX OF TOTAL RETURNS
- PROPERTY-LEVEL
- APPRAISAL-BASED

NCREIF Formula

FORMULA INCLUDES A TIME-WEIGHTED INVESTMENT DENOMINATOR, ASSUMING:

ONE-THIRD OF THE QUARTERLY PROPERTY NOI IS RECEIVED AT THE ***END*** OF EACH CALENDAR MONTH;

PARTIAL SALES RECEIPTS MINUS CAPITAL IMPROVEMENT EXPENDITURES ARE RECEIVED ***MIDWAY*** THROUGH THE QUARTER...

$$r_{NPI} = \frac{EndVal - BegVal + (PS - CI) + NOI}{BegVal - (1/2)(PS - CI) - (1/3)NOI}$$

[Note: *(1/3)NOI = (2/3)(1/3)NOI+(1/3)(1/3)NOI+(0)(1/3)NOI*]

MULTI-PERIOD RETURNS…

SUPPOSE YOU WANT TO KNOW WHAT IS THE RETURN EARNED OVER A MULTI-PERIOD SPAN OF TIME, EXPRESSED AS A SINGLE AVERAGE ANNUAL RATE?...

YOU COULD COMPUTE THE **AVERAGE** OF THE **HPRs** ACROSS THAT SPAN OF TIME.

THIS WOULD BE A **"TIME-WEIGHTED"** AVERAGE RETURN.

MULTI-PERIOD RETURNS (cont'd)

IT WILL:

=> Weight a given rate of return more if it occurs over a longer interval or more frequently in the time sample.

=> Be independent of the magnitude of capital invested at each point in time; Not affected by the ***timing*** of capital flows into or out of the investment.

MULTI-PERIOD RETURNS (cont'd)

YOU CAN COMPUTE THIS AVERAGE USING EITHER THE **ARITHMETIC** OR **GEOMETRIC** MEAN...

Arithmetic average return over 1992-94:

$= (r_{92} + r_{93} + r_{94})/3$

Geometric average return over 1992-94:

$= [(1+r_{92})(1+r_{93})(1+r_{94})]^{(1/3)} - 1$

ARITHMETIC vs. GEOMETRIC MEAN…

Arithmetic Mean:

=>Always greater than geometric mean.

=>Superior statistical properties:

* Best "estimator" or "forecast" of "true" return.

=> Mean return components sum to the mean total return

=> Most widely used in forecasts & portfolio analysis.

ARITHMETIC vs. GEOMETRIC MEAN (cont'd)

Geometric Mean:

=> Reflects compounding ("chain-linking") of returns:

* Earning of "return on return".

=>Mean return components do not sum to mean total return

* Cross-product is left out.

=>Most widely used in performance evaluation.

ARITHMETIC vs. GEOMETRIC MEAN (cont'd)

The two are more similar:

- The less volatility in returns across time
- The more frequent the return interval

(Note: "continuously compounded" returns (log differences) side-steps around this issue. *(There is only <u>one</u> continuously-compounded mean annual rate: arithmetic & geometric distinctions do not exist)*.

TIME-WEIGHTED RETURNS: NUMERICAL EXAMPLES

An asset that pays no dividends . . .

Year:	***End of year asset value:***	***HPR:***
1992	$100,000	
1993	$110,000	(110,000 - 100,000) / 100,000 = **10.00%**
1994	$121,000	(121,000 - 110,000) / 110,000 = **10.00%**
1995	$136,730	(136,730 - 121,000) / 121,000 = **13.00%**

Three-year average annual return (1993-95):

Arithmetic mean:

$= (10.00 + 10.00 + 13.00) / 3$

$= 11.00\%$

Geometric mean:

$= (136{,}730 / 100{,}000)^{(1/3)} - 1$

$= ((1.1000)(1.1000)(1.1300))^{(1/3)} - 1$

$= 10.99\%$

Continuously compounded:

$= LN(136{,}730 / 100{,}000) / 3$

$= (LN(1.1)+LN(1.1)+LN(1.13)) / 3$

$= 10.47\%$

Another Example

Year	***End of year asset value:***	***HPR:***
1992	$100,000	
1993	$110,000	(110,000 - 100,000) / 100,000 = **10.00%**
1994	$124,300	(124,300 - 110,000) / 110,000 = **13.00%**
1995	$140,459	(140,459 - 124,300) / 124,300 = **13.00%**

Three-year average annual return (1993-95):

Arithmetic mean:

$= (10.00 + 13.00 + 13.00) / 3$

$= 12.00\%$

Geometric mean:

$= (140{,}459 / 100{,}000)^{(1/3)} - 1$

$= ((1.1000)(1.1300)(1.1300))^{(1/3)} - 1$

$= 11.99\%$

Continuously compounded:

$= LN(140{,}459 / 100{,}000) / 3$

$= (LN(1.1)+LN(1.13)+LN(1.13)) / 3$

$= 11.32\%$

Another Example

Year	*End of year asset value:*	*HPR:*
1992	$100,000	
1993	$110,000	(110,000 - 100,000) / 100,000 = **10.00%**
1994	$121,000	(121,000 - 110,000) / 110,000 = **10.00%**
1995	$133,100	(133,100 - 121,000) / 121,000 = **10.00%**

Three-year average annual return (1993-95):

Arithmetic mean:

$= (10.00 + 10.00 + 10.00) / 3$

$= 10.00\%$

Geometric mean:

$= (133{,}100 / 100{,}000)^{(1/3)} - 1$

$= ((1.1000)(1.1000)(1.1000))^{(1/3)} - 1$

$= 10.00\%$

Continuously comp'd:

$= LN(133{,}100 / 100{,}000) / 3$

$= (LN(1.1)+LN(1.1)+LN(1.1)) / 3 = 9.53\%$

ANOTHER MULTI-PERIOD RETURN MEASURE: The IRR...

CAN'T COMPUTE HPRs IF YOU DON'T KNOW ASSET VALUE AT INTERMEDIATE POINTS IN TIME (AS IN REAL ESTATE WITHOUT REGULAR APPRAISALS)

SO YOU CAN'T COMPUTE TIME-WEIGHTED AVERAGE RETURNS.

➔ *You need the **"IRR"**.*

IRR

SUPPOSE YOU WANT A RETURN MEASURE THAT REFLECTS THE EFFECT OF THE **TIMING** OF WHEN (INSIDE OF THE OVERALL TIME SPAN COVERED) THE INVESTOR HAS DECIDED TO PUT MORE CAPITAL INTO THE INVESTMENT AND/OR TAKE CAPITAL OUT OF THE INVESTMENT.

➔ *You need the **"IRR"**.*

IRR

FORMAL DEFINITION OF IRR

"**IRR**" (INTERNAL RATE OF RETURN) IS THAT *SINGLE* RATE THAT DISCOUNTS ALL THE NET CASH FLOWS OBTAINED FROM THE INVESTMENT TO A PRESENT VALUE EQUAL TO WHAT YOU PAID FOR THE INVESTMENT AT THE BEGINNING:

IRR

$$0 = CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_N}{(1+IRR)^N}$$

CF_t = Net Cash Flow to Investor in Period "t"

CF_0 is usually negative (capital outlay).

Note: CF_t is signed according to the convention:

⇨ cash flow from investor to investment is negative,

⇨ cash flow from investment to investor is positive.

Note also: Last cash flow (CF_N) includes two components:

➔ The last operating cash flow plus

➔ The (ex dividend) terminal value of the asset ("reversion").

WHAT IS THE IRR?...

A SINGLE ("BLENDED") INTEREST RATE, WHICH IF ALL THE CASH IN THE INVESTMENT EARNED THAT RATE ALL THE TIME IT IS IN THE INVESTMENT, THEN THE INVESTOR WOULD END UP WITH THE TERMINAL VALUE OF THE INVESTMENT (AFTER REMOVAL OF CASH TAKEN OUT DURING THE INVESTMENT):

WHAT IS THE IRR?...

THE IRR IS THAT RATE WHICH, IF THE INITIAL INVESTMENT PLUS ALL THE NET CASH FLOWS DURING THE HOLDING PERIOD EARNED THAT RATE CONTINUOUSLY FROM THE TIME OF THEIR OCCURRENCE DURING THE ENTIRE HOLDING PERIOD, THEN THE GEOMETRIC MEAN TIME-WTD HPR OVER THAT HOLDING PERIOD WOULD EQUAL THAT RATE.

WHAT IS THE IRR? (cont'd)

$$PV(1+IRR)^{N} - CF_1(1+IRR)^{(N-1)} - \Lambda \; - CF_{N-1}(1+IRR) = CF_N$$

where PV = -CF_0, the initial cash "deposit" in the "account" (outlay to purchase the investment).

IRR is **"internal"** because it includes only the returns earned on capital *while it is invested in the project*.

Once capital (i.e., cash) is withdrawn from the investment, it no longer influences the IRR.

This makes the IRR a **"dollar-weighted"** average return across time for the investment, because returns earned when more capital is in the investment will be weighted more heavily in determining the IRR.

THE IRR INCLUDES THE EFFECT OF:

1. THE INITIAL CASH YIELD RATE (INITIAL LEVEL OF CASH PAYOUT AS A FRACTION OF THE INITIAL INVESTMENT;
2. THE EFFECT OF CHANGE OVER TIME IN THE NET CASH FLOW LEVELS (E.G., GROWTH IN THE OPERATING CASH FLOW);
3. THE TERMINAL VALUE OF THE ASSET AT THE END OF THE INVESTMENT HORIZON (INCLUDING ANY NET CHANGE IN CAPITAL VALUE SINCE THE INITIAL INVESTMENT WAS MADE).

IRR

THE IRR IS THUS A ***TOTAL*** RETURN MEASURE (CURRENT YIELD PLUS GROWTH & GAIN).

NOTE ALSO:

THE IRR IS A ***CASH FLOW BASED*** RETURN MEASURE...

- DOES NOT DIFFERENTIATE BETWEEN "INVESTMENT" AND "RETURN ON OR RETURN OF INVESTMENT".
- INCLUDES THE EFFECT OF CAPITAL INVESTMENTS AFTER THE INITIAL OUTLAY.
- DISTINGUISHES CASH FLOWS ONLY BY THEIR DIRECTION: POSITIVE IF FROM INVESTMENT TO INVESTOR, NEGATIVE IF FROM INVESTOR TO INVESTMENT (ON SAME SIDE OF "=" SIGN).

IRR

In general, it is not possible to algebraically determine the IRR for any given set of cash flows. It is necessary to solve numerically for the IRR, in effect, solving the IRR equation by "trial & error". Calculators and computers do this automatically.

ADDITIONAL NOTES ON THE IRR . . .

TECHNICAL PROBLEMS:

➔ IRR MAY NOT EXIST OR NOT BE UNIQUE (OR GIVE MISLEADING RESULTS) WHEN CASH FLOW PATTERNS INCLUDE NEGATIVE CFs AFTER POSITIVE CFs.

➔ BEST TO USE NPV IN THESE CASES. (SOMETIMES “FMRR” IS USED.)

ADDITIONAL NOTES ON THE IRR . . .

THE IRR AND TIME-WEIGHTED RETURNS:

➔ IRR = TIME-WTD GEOMEAN HPR IF (AND ONLY IF) THERE ARE NO INTERMEDIATE CASH FLOWS (NO CASH PUT IN OR TAKEN OUT BETWEEN THE BEGINNING AND END OF THE INVESTMENT).

ADDITIONAL NOTES ON THE IRR . . .

THE IRR AND RETURN COMPONENTS:

➔ IRR IS A "TOTAL RETURN"

➔ IRR DOES NOT GENERALLY BREAK OUT EXACTLY INTO A SUM OF: **y + g**: INITIAL CASH YIELD + CAPITAL VALUE GROWTH COMPONENTS.

➔ DIFFERENCE BETWEEN THE IRR AND THE INITIAL CASH YIELD IS DUE TO A COMBINATION OF GROWTH IN THE OPERATING CASH FLOWS AND/OR GROWTH IN THE CAPITAL VALUE.

ADDITIONAL NOTES ON THE IRR . . .

➔ IF THE OPERATING CASH FLOWS GROW AT A CONSTANT RATE, AND IF THE ASSET VALUE REMAINS A CONSTANT MULTIPLE OF THE CURRENT OPERATING CASH FLOW, THEN THE IRR WILL INDEED EXACTLY EQUAL THE SUM OF THE INITIAL CASH YIELD RATE PLUS THE GROWTH RATE (IN BOTH THE CASH FLOWS AND THE ASSET CAPITAL VALUE), AND IN THIS CASE THE IRR WILL ALSO EXACTLY EQUAL BOTH THE ARITHMETIC AND GEOMETRIC TIME-WEIGHTED MEAN (CONSTANT PERIODIC RETURNS): $IRR_{t,t+N} = r_{t,t+N} = y_{t,t+N} + g_{t,t+N}$.

For example:

The cash flow stream below has IRR = 11%:

$$100 = \frac{10}{(1+IRR)} + \frac{(1+0.01)10}{(1+IRR)^2} + \mathrm{K} + \frac{(1+0.01)^{(N-1)}10 + \left((1+0.01)^N 10/(10/100)\right)}{(1+IRR)^N}$$

The PV is 100, with an initial cash yield of 10% (CF_1 = 10, y = 10/100 = 10%).

Each subsequent annual cash flow is 1% more than the previous, for N years (g = 1%).

At the end of the Nth year, the property is sold at a yield (on the up-coming year's cash flow) of 10%, same as the initial yield.

The IRR is 10% + 1% = 11%.

In general under these regularity conditions: IRR = y + g.

ADDITIONAL NOTES ON THE IRR . . .

THE IRR AND TERMINOLOGY:

➔ IRR OFTEN CALLED "TOTAL YIELD" (APPRAISAL)

➔ "YIELD TO MATURITY" (BONDS)

➔ EX-ANTE IRR = "GOING-IN IRR".

DOLLAR-WEIGHTED & TIME-WEIGHTED RETURNS:A NUMERICAL EXAMPLE . . .

"OPEN-END" (PUT) OR (CREF).

INVESTORS BUY AND SELL "UNITS" ON THE BASIS OF THE APPRAISED VALUE OF THE PROPERTIES IN THE FUND AT THE END OF EACH PERIOD.

SUPPOSE THE FUND DOESN'T PAY OUT ANY CASH, BUT REINVESTS ALL PROPERTY INCOME. CONSIDER 3 CONSECUTIVE PERIODS. . .

INVESTMENT PERIODIC RETURNS: HIGH, LOW, HIGH . . .

	1996	1997	1998	1999
YR END UNIT VALUE	$1000	$1100	$990	$1089
PERIODIC RETURN		**+10.00%**	**-10.00%**	**+10.00%**

GEOM MEAN TIME-WTD RETURN =

$$(1.089)^{(1/3)}-1 = 2.88\%$$

INVESTOR #1, "MR. SMART" (OR LUCKY): GOOD TIMING . . .

END OF YEAR:	1996	1997	1998	1999
UNITS BOUGHT	2			
UNITS SOLD		1		1
CASH FLOW	-$2000	+$1100	0	$1089

IRR = IRR(-2000,1100,0,1089) = 4.68%

INVESTOR #2, "MR. DUMB" (OR UNLUCKY): BAD TIMING . . .

END OF YEAR:	1996	1997	1998	1999
UNITS BOUGHT	1	1		
UNITS SOLD			1	1
CASH FLOW	-$1000	-$1100	+$990	$1089

IRR = IRR(-1000,-1100,990,1089) = -0.50%

Example (cont'd)

DOLLAR-WTD RETURN BEST FOR MEASURING INVESTOR PERFORMANCE IF INVESTOR CONTROLLED TIMING OF CAP. FLOW.

TIME-WTD RETURN BEST FOR MEASURING PERFORMANCE OF THE UNDERLYING INVESTMENT (IN THIS CASE THE PUT OR CREF), AND THEREFORE FOR MEASURING INVESTOR PERFORMANCE IF INVESTOR ONLY CONTROLS **WHAT** TO INVEST IN **BUT NOT WHEN**.