Statistical Process Control Approach to Reduce the Bullwhip Effect

by

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ABSTRACT

The bullwhip effect is a pervasive problem in multi echelon supply chains that results in inefficient production operations and higher inventory levels. The causes of the bullwhip effect are well understood in industry and academia. Quantitative and qualitative solutions to attenuate this effect have been proposed in various research studies. In this research a quantitative solution in the form of a Statistical Process Control (SPC) based inventory management system is proposed that reduces the bullwhip effect while reducing inventory without compromising service level requirements for a variety of products. The strength of this methodology is in its effectiveness in reducing bullwhip for fast moving products in the mature phase of their lifecycles where improving production efficiency and lowering inventory investment are critical. However, fill rate issues are observed for slow moving products and therefore, the methodology is not recommended for such products. Finally, the application of this methodology to reduce the bullwhip effect is illustrated for a product family of a medical devices company. The results for the different classes of products in this family are discussed.

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This one's for Belu.

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1 INTRODUCTION

The increase in demand variability as one moves upstream in the supply chain, i.e., the 'Bullwhip effect', causes inefficient use of resources and higher supply chain costs. These costs are due to higher inventory being stocked and transported to meet the target customer service levels. Strategies to reduce the bullwhip effect and thus inventory levels include information sharing, channel alignment and improvement of operational efficiencies that are used by industries with varying degrees of success. The continued focus on improving efficiencies and reducing costs leads researchers to explore newer concepts and different techniques to improve supply chains. This research applies the Statistical Process Control (SPC) principles, primarily used to monitor process variations in manufacturing, to the field of supply chain management. Specifically, the SPC principles shall be applied to develop an inventory management technique and assess its impact in reducing the Bullwhip effect.

This thesis addresses the question as to whether the principles of SPC can be applied to better manage inventory held at a distribution center and level load the upstream production facility in order to reduce the bullwhip effect and lower the overall supply chain costs.

SPC is used extensively in manufacturing. By establishing an upper and lower bound on a process, such as manufacturing, one can determine if the process is within or outside of normal operating conditions. This research examines if the inventory management technique based on SPC principles can be used within the replenishment cycle. This would entail establishing a statistically valid range by defining upper and lower control limits instead of having standard point replenishment. The thought is that this will allow us to dampen the over-reactions that can cause the bullwhip effect. Using research and modeling, the project would demonstrate how the principles of SPC can impact the inventory at a distribution center and the inventory replenishment planning at the manufacturing facility. As a part of the research case study, the SPC approach shall be compared against current inventory and replenishment policies at a medical device company.

In the following chapter, literature pertinent to the bullwhip effect, its causes and resolution strategies are reviewed. Chapter 3 introduces Statistical Process Control concepts which are applied to develop an inventory management methodology in Chapter 4. The methodology is demonstrated for a medical devices company in Chapter 5 and the results are discussed. Finally, in Chapter 6, the strengths and limitations of this SPC-based inventory management system are summarized and recommendations for further research are made.

2 BACKGROUND AND PREVIOUS WORK

Metters (1997) describes a typical supply chain for creation and sale of goods that involves distinct echelons operating in a serial time-line (Figure 2-1). In such a typical supply chain suppliers provide raw materials to manufacturers, who process the raw materials into finished goods and then provide the finished goods to wholesalers who combine products from a number of manufacturers for sale to retailers, who then sell the product to the consumer. In addition to the physical flow of goods downstream in the chain, there is an information flow that proceeds upstream. The retailer has direct contact with the ultimate consumer who is at the end of the supply chain. The demand seen by wholesalers consists of orders from retailers, rather than consumers, and so on for upstream entities of the supply chain. The goods and information flows in a supply chain can be shown in Figure 2-1.

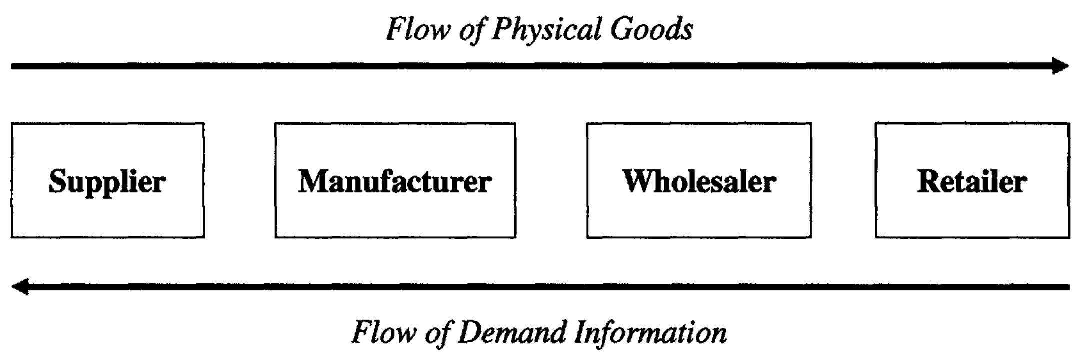

Figure 2-1: Goods and information flows in a supply chain

2.1 Bullwhip Effect

Demand variability increases as we move up the supply chain. This phenomenon is called the Bullwhip Effect. Lee, Padmanabhan and Whang (1997a) state that the

Bullwhip effect exists when the orders to the supplier tend to have larger variance than sales to the buyer (i.e. demand distortion). Also, this distortion propagates upstream in an amplified form (i.e. variance amplification). The costs for this variability are - inefficient use of production and warehouse resources, higher transportation costs, and high inventory costs (Silver, Pyke and Peterson, 1998). In order to reduce such costs, companies make efforts to improve supply chain management aimed by reducing the bullwhip effect.

Lee, Padmanabhan, and Whang (1997b) identify four rational factors that create the bullwhip effect.

1 *Demand signal processing:* In case the demand increases, firms order more in anticipation of further increases, thereby communicating an artificially higher level of demand.

2 *The rationing game:* To obviate possible shortages, firms order more than the actual forecast in anticipation of receiving a larger share of the items in short supply.

3 *Order batching*: Fixed costs at one location lead to batching of orders

4 *Manufacturer price variations:* Volume based discounts encourage bulk orders.

Lee et al (1997b) further suggest information sharing amongst supply chain partners, channel alignment and improving operational efficiencies as the three broad strategies to reduce the bullwhip effect in the supply chain. Their framework, which includes the different types of initiatives that can be made by the supply chain members, is shown in Table 2-1.

Causes of Bullwhip	Information Sharing	Channel Alignment	Operational Efficiency
Demand Forecast Update	• Understanding system dynamics • Use Point-of-Sale (POS) data • Electronic Data Interchange (EDI) • Internet • Computer assisted Ordering (CAO)	• Vendor Managed Inventory • Discount for information sharing • Consumer direct	• Lead time reduction • Echelon-based inventory control
Order Batching	• EDI • Internet ordering	• Discount for truck-load assortment • Delivery appointments • Consolidation • Logistics outsourcing	• Reduction in fixed cost of ordering by EDI or electronic commerce • CAO
Price Fluctuations		• Continuous replenishment program (CRP) • Everyday low Cost (EDLC)	• Everyday low Price (EDLP) • Activity based costing (ABC)
Order Gaming	• Sharing sales, capacity and inventory data	• Allocation based on past sales	

Table 2-1: A Framework for Supply Chain Coordination Initiatives

Disney and Towill (2003) also cite the above-mentioned causes of bullwhip effect – *demand signal processing* or demand amplification upstream in a supply chain, called the 'Forrester effect' and *order batching*, also known as the 'Burbidge effect'. The 'Burbidge effect' refers to the practice of placing orders up the supply chain in batches to gain economies of scale in set-up activities. Rationing or gaming is called the 'Houlihan effect'. The 'Houlihan effect' recognizes that customers overload their schedules or orders due to possibilities of shortages or missed deliveries in supply chain, resulting in excess demand on the production system that leads to more unreliable deliveries.

2.1.1 Quantifying the Bullwhip Effect

Chen, Drezner, Ryan and Simchi-Levi (2000), quantify the bullwhip effect in terms of the variance of the orders placed by the retailer to the manufacturer relative to the variance of the demand faced by the retailer. They consider a simple two echelon supply chain comprising a single manufacturer and a single retailer, having an order up-to policy. They quantify the Bullwhip in terms of the order and demand variances (Equation 2-1).

$$\text{Bullwhip} = \frac{\sigma_q^2}{\sigma_D^2}. \qquad \text{Equation (2-1)}$$

where σ_q^2 denotes variance of orders q, and σ_D^2 is the variance of demand D.

Disney, Towill and van de Velde (2004), also use this metric as a measure of Bullwhip, and give it the name 'Variance Ratio'. They further state that Variance Ratio > 1 results in a bullwhip; Variance Ratio < 1 results in order smoothing; and Variance Ratio = 1 may result in a ''pass-on-orders'' policy, where the production pattern exactly follows the demand pattern. This metric can be applied to a single ordering decision or echelon in a supply chain (Disney and Towill, 2003) or across many echelons in the supply chain (Dejonckheere et al, 2004). In the case where Variance ratio < 1 or order smoothing, a firm might not be able to meet its customer service levels when faced with increase in demand due to variability. In case the firm produces large quantities to smooth its production, the inventory levels would increase due to larger safety stocks in the supply chain to meet the same customer service levels.

Another metric to measure the Bullwhip Effect has been in terms of the coefficient of variation of orders and coefficient of variation of demand. Xiong and Helo, (2006) measure the extent of bullwhip effect in a supply chain as the quotient of the coefficient of variation of demand generated by this echelon and the coefficient of variation of demand received by this echelon. The variation of demand at a certain echelon is defined as the standard deviation of the demand divided by the average demand during a certain time interval. This is calculated for both incoming and outgoing demand at the echelon in the chain. This has been represented by Xiang and Helo as:

$$\text{Bullwhip } (\omega) = \frac{CV_{out}}{CV_{in}} = \frac{\sigma_{out}}{D_{out}} \div \frac{\sigma_{in}}{D_{in}},$$

$$\omega = \left(\frac{\sigma_{out}}{\sigma_{in}}\right)\left(\frac{D_{in}}{D_{out}}\right). \qquad \text{Equation (2-2)}$$

Metters (1997) uses the term 'variance/mean ratio' for quantifying the Bullwhip Effect. The 'variance/mean ratio' is a similar metric as used by Xiong and Helo (2006). The 'variance/mean ratio' can be written as:

$$\text{Bullwhip 'variance/mean ratio'} = \frac{\sigma^2{}_{out}}{D_{out}} \div \frac{\sigma^2{}_{in}}{D_{in}}$$

$$= \left(\frac{\sigma_{out}}{\sigma_{in}}\right)\left(\frac{\sigma_{out}}{\sigma_{in}}\right)\left(\frac{D_{in}}{D_{out}}\right)$$

$$= \left(\frac{\sigma_{out}}{\sigma_{in}}\right)\frac{CV_{out}}{CV_{in}} = \left(\frac{\sigma_{out}}{\sigma_{in}}\right)\omega \qquad \text{Equation (2-3)}$$

From Equation 2-3, it is seen that the 'variance/ mean ratio' used by Metters is equivalent to the bullwhip metric used by Xiang and Helo multiplied by the ratio of standard deviations of outgoing and incoming demand.

For the purpose of this research, the bullwhip metric used by Xiang and Helo shall be followed. The variance and standard deviation are absolute measures of dispersion, depending upon the units of measurement. Coefficient of variation is a relative measure of dispersion and is a pure number independent of units of measurement. It is therefore more suitable for comparing the variability of two distributions (Gupta, 1992). The variability of the distribution of outgoing demand can be compared with the distribution of incoming demand at an echelon and the distribution with a lower coefficient of variation is considered to be less variable (or more homogenous) than the other.

2.1.2 Recent studies to reduce the bullwhip effect

Significant research has been done in the last ten years on the topic of reducing the bullwhip effect using the above mentioned framework. The research ranged from evaluating the information sharing strategy, to vendor managed inventory as a channel alignment strategy, and to improving operational efficiencies using various approaches like fuzzy sets theory, non-linear goal programming, integrated production-inventory models etc. An overview of some of the recent research studies on the subject is as under.

Information sharing strategies

Lee, Padmanabhan, and Whang (1997b) state that the information transferred in the form of orders tends to be distorted and can misguide upstream members in their inventory and production decisions. Information sharing strategies to reduce the bullwhip effect has been one of the most researched topics in the area of bullwhip effect. Lau et al (2003) researches the impacts of sharing information on the supply chain dynamics, and reviews recent representative papers since 1996. Their review shows that the benefits of information sharing are significant in reducing the bullwhip effect as supply chain entities can make better decisions on ordering, capacity allocation and production/material planning for optimizing supply chain dynamics.

Wong et al (2007) compares actual bullwhip effects provided by retailers who shared downstream demand information and retailers who did not share demand information in a three-echelon toy supply chain, A reduction in bullwhip effect and an improvement of the fill rate was observed for retailers who shared downstream demand information, to plan premature replenishment, and update forecast, even without coordination between the toy manufacturer and the retailers.

Croson and Donohue (2006) studies the bullwhip effect from a behavioral perspective in the context of a simple, serial, supply chain subject to information lags and stochastic demand. They conduct two experiments on different sets of participants to find that the bullwhip effect still exists when normal operational causes (e.g. batching etc.) are removed. This was explained to some extent by evidence that decision-makers consistently underweight the supply line when making order decisions. In the second experiment, they found that the bullwhip, and the underlying tendency of underweighting, remains when information on inventory levels is shared. However, the information sharing helps somewhat to alleviate the bullwhip effect.

However, the trade-off between the costs of technology for information sharing and the value generated by such investments, and lack of discipline in complying to the collaborative process by supply chain partners, need to be addressed while adopting information-sharing as the prime strategy to reduce the bullwhip effect. Lau et al (2003) state that "information sharing, may not be beneficial to some supply chain entities due to high adoption cost of joining the inter-organizational information system, unreliable and imprecise information (Swaminathan et al., 1997; Cohen, 2000), and different operational condition of each firm (Dong and Xu, 2001)."

Channel Alignment Strategies

Disney and Towill (2003) use a simulation model to compare the bullwhip effect in a vendor managed inventory (VMI) supply chain with those of a traditional 'serially-linked' supply chain. The model considers each of the four important sources of the bullwhip effect in turn. The analysis shows that with VMI implementation two sources of the bullwhip effect may be completely eliminated, i.e. rationing and gaming or the Houlihan effect, and the order batching effect or the Burbidge effect. VMI is also significantly better at responding to rogue changes in demand due to the promotion effect or to price induced variations. However, the effect of VMI on demand signal processing introduced bullwhip or the Forrester effect not clear. They state that VMI offers a significant opportunity to reduce the bullwhip effect in real-world supply chains.

Operational efficiency strategies

Some research studies applied fuzzy sets theory in managing inventory strategies. The most recent is of Xiong and Helo (2006). They cite Carlsson and Fuller (2001), who proposed a fuzzy logic approach to reduce the bullwhip effect, and is used in the paper industry. Xiong and Helo (2006) propose a multi-echelon fuzzy inventory model to counteract the demand fluctuation in supply demand networks. By using a simulation model, their research shows that the proposed multi-echelon fuzzy inventory model provides can reduce the bullwhip effect with lower inventory levels and costs.

Dhahri and Chabchoub (2007) propose use of nonlinear goal programming models as a decision making aid, by using preference functions based on a statistical chronological series analysis (Box and Jenkins method) in order to construct the different models for demand, stock level, and the order quantity. They further propose integration of the decision maker preference in the demand forecast and inventory management processes. Though results have not been encouraging, they have suggested the possibility to integrate various statistical tools and mathematical models in a decision support system for reduction in the inventory due to the bullwhip effect.

Boute et al (2007) suggest an integrated production and inventory model to dampen upstream demand variability in the supply chain. They consider a two-echelon supply chain, where the retailer would propagate demand variability often in amplified form. The manufacturer, however, prefers to smooth production, and thus he prefers a smooth order pattern from the retailer. At first sight, a decrease in order variability comes at the cost of an increased variance of the retailer's inventory levels, inflating the retailer's safety stock requirements. However, integrating the impact of the retailer's order decision on the manufacturer's production leads to new insights. A smooth order pattern generates shorter and less variable (production/replenishment) lead times, introducing a compensating effect on the retailer's safety stock. It is shown in this research that by including the impact of the order decision on lead times, the order pattern can be smoothed to a considerable extent without increasing stock levels.

This thesis focuses on echelon based inventory control as a means to reduce the bullwhip effect. The application of Statistical Process Control, a concept widely used in the manufacturing environment especially for Quality Control, shall be explored to inventory management policies and a simulation model shall be used to assess the impact of such an approach on the bullwhip effect.

2.2 Statistical Process Control applications

Statistical quality control (SQC) dates back to the 1930s originating from the work of Walter Shewart of Bell Telephone Laboratories. His student, W.Edwards Deming, taught quality control in Japan, thereby igniting the Japanese quality revolution. (Namhias, 2005). SQC generally focuses on manufacturing quality, as measured by conformance to specifications. "The ultimate objective of SQC is the systematic reduction of variability in quality measures. The three major classes of tools used in SQC are Acceptance Sampling, Statistical Process Control (SPC), and Design of Experiments" (Hopp and Spearman, 2000). In acceptance sampling, products are inspected to determine whether they conform to specifications; while in SPC, processes are continuously monitored with respect to mean and variability of performance to determine whether the process is in control or has gone out of control. In Design of Experiments, causes of quality problems are traced through specifically targeted experiments by varying controllable variable to determine their effect on quality measures.

SPC has been primarily used in the manufacturing environment. However, the possibility of using the SPC approach outside the manufacturing environment has also been explored by various industries. For example, Jiang et al (2007) uses a SPC framework to identify changes in business activity monitoring in telecommunications industry for tracking diversified customer behaviors, to establish successful customer loyalty programs for churn prevention and fraud detection. Health-care organizations use SPC and six sigma to determine important elements of the healthcare experience to consumers, and to monitor and reduce errors (e.g. medical errors, wait times, errors from high-risk medications, turnaround time for pharmacy orders etc.) and their associated costs (Camille James, 2006).

SPC has not been used much in the supply chain environment. This research aims to extend the SPC technique beyond the manufacturing processes by applying it to the inventory control area of supply chain management. By investigating its impact on the level of bullwhip, this research fits into the strategy to improve operational efficiencies for reducing the bullwhip effect.

A review of the literature indicates that SPC approach has been tried in inventory management by the industry only to a very limited extent. SPC tools for specific forecast periods were used by General Electric's aircraft engine division planners between 1993 and 1995, to reduce their aircraft engines parts inventory by 25% (Beck, 1999). The demand forecast and inventory levels were monitored using control charts by comparing statistic being monitored with applicable control limits, and placing any outlier on the exception list for review. The use of specific forecast periods brought focus to apply the SPC tools only where the application was successful during simulations.

Simulation studies using the SPC approach for management have been conducted by Pfohl et al (1999) and Lee and Wu (2006). Pfohl et al (1999) uses data from 3M Medical products for twelve European warehouses over a six month period. Decision rules were set up for demand and inventory control charts to track changes in demand and inventory. The average inventory levels reduced by 20% to 65%, but there were an increase in back-orders for some products. However, the study was limited to examine the effects on inventory levels by using the SPC approach.

Lee and Wu (2006) examine the bullwhip effect caused by order batching and researched the traditional inventory replenishment method (event-triggered and time-triggered inventory policies) and the SPC based replenishment method for a two echelon supply chain. By using simulation, they find that the SPC method outperforms the traditional methods in the categories of average inventory levels and, and in the number of back-orders when the fill rate is 99%. However, at lower fill rate of 95%, the SPC method reduces the back-orders but leads to higher inventory levels and increasing variation in inventory levels.

SPC based inventory management systems have not focused on reducing the bullwhip effect in supply chains. This thesis uses SPC to develop an inventory management system as an operational strategy to control the amplification of demand variability up the supply chain. In the following chapter, principles of statistical process control are introduced that are later leveraged to develop the SPC based inventory management system.

3 STATISTICAL PROCESS CONTROL (SPC)

Statistical process control offers a graphical means of monitoring a process in real-time using control charts. Process variables are typically assumed to have an underlying normal distribution. In the control charts, the process mean is represented by a center line and the process standard deviation is captured in the upper and lower control limits. A process is said to be in statistical control if it is within the control limits. Trends in the process variable can be monitored real-time to identify deviations in the variable from the historically calibrated state. The process supervisor can identify anomalies in a process by monitoring the control charts. By conducting root-cause analysis, the assignable causes for variation can be identified and resolved.

The Central Limit Theorem forms the basis of most control charts. The Central Limit Theorem states that the distribution of the sum of independently and identically distributed random variables approaches the Normal distribution as the number of terms in the sum increases. In this light, the probability of having an observation of a process variable outside controls limits can be easily determined. The probability associated with finding an observation of a process variable that lies outside the 3σ or -3σ control limits on a control chart is less than 0.0026 or roughly less than 3 chances in 1,000. The probabilities associated with finding observations outside 1σ, 2σ and 3σ control limits are shown in Figure 3-1.

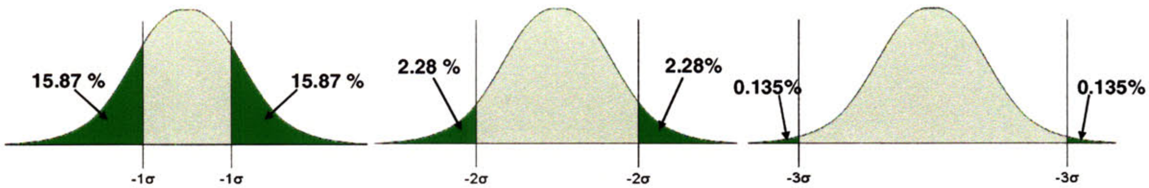

Figure 3-1: Probabilities associated with 1σ, 2σ and 3σ control limits for a normal distribution

Since events involving observations that lie outside the 3σ control limits are very rare, a single event could signal an anomaly in the process that requires immediate attention. However, events involving observations that lie between the 2σ and 3σ control limits would require more than a single event to warrant remedial action. Events that involve observations which lie between the center line and 1σ and between 1σ and 2σ are higher probability events and therefore, require more a compelling case in terms of the number of outliers to drive remedial action.

A process supervisor reacts to shifts and drifts in a process variable based on the nature of the process involved. However, in general, a process supervisor would be expected to investigate the causes for process deviation in the following manner:

1. Identify causes for the change in the process variable characteristics
2. Determine if the causes are endogenous to the system or exogenous
3. Effect remedial action for endogenous causes within the control of the process supervisor
4. Coordinate with agents responsible for exogenous causes to mitigate operational risk

3.1 SPC for the inventory management process

Inventory management is an emerging application for statistical process control in which the key components of demand and inventory can be represented by control charts. This requires the interpretation of demand and inventory to be the two tightly integrated process variables for the inventory control process. Control charts can be created for each of these variables based their historical characteristics. It will be demonstrated in subsequent sections that the control charts for demand drives the control chart of inventory. Once the control charts are created, subsequent observations of these variables can be plotted on these charts to detect any situations where the variables are out of control.

The use of SPC to manage demand and inventory has advantages over traditional inventory management systems. Changes in demand characteristics, such as, the mean and standard deviation measures of demand, drive inventory levels in the inventory planning system. If these demand characteristics are updated frequently in the inventory control system, the stocking requirements will also change resulting in system nervousness. This would result in high production costs and inventory issues of deficits or excesses. Therefore, it is critical to understanding whether the changes in demand characteristics of a product are significant or not from an inventory planning perspective before the inventory planning system is updated.

The following are the main advantages of using SPC in inventory management.

1. Develop greater understanding of the demand and inventory process variables and recognize acceptable limits of operation
2. Determine when the demand and inventory process variables are out of control
3. Reduce nervousness in the ordering behavior by ensuring that changes are made to the inventory system only when the process variables are out of control. This

significantly attenuate the bullwhip effect while maintaining service levels

The above advantages hold special merit for products that have low to moderate volatility in demand (coefficient of variation of weekly demand less than 0.5). If minor variations in the demand characteristics are passed on to the inventory planning system, the fluctuations in the production orders can adversely impact the efficiency of the production plant and result in greater production costs.

Once the changes in demand characteristics are transmitted to the inventory control system, fluctuations in the order quantities to the production plant can be reduced significantly by using economic order quantities (EOQ). However, the use of EOQ requires that the demand be known with certainty and stay relatively constant throughout the year which may not hold for many products. In such situations, the inventory chart can be used to identify the fixed order quantity for the given state of demand. The derivation of the fixed order quantity will be addressed in following sections. The fixed order quantity obtained from the inventory chart for the given characteristics of demand can reduce the production order variability and help level-load the production plant. The predictability of a fixed order quantity for a given state of demand encourages habit forming behavior in the production line and improves the efficiency of the production process.

3.2 Control charts

Monitoring demand characteristics is directly relevant to inventory planning. A change in mean demand results in changes in cycle inventory. A change in the standard deviation of demand results in changes in safety stock levels. Therefore, demand needs to be controlled for changes in both the mean and standard deviation. A wide variety of control charts are used in statistical process control depending on the nature of the process to which the technique is applied. All control charts have a center line which represents the mean of the process variable and upper and lower control limits that represent +/- 3 standard deviations (Figure 3-2).

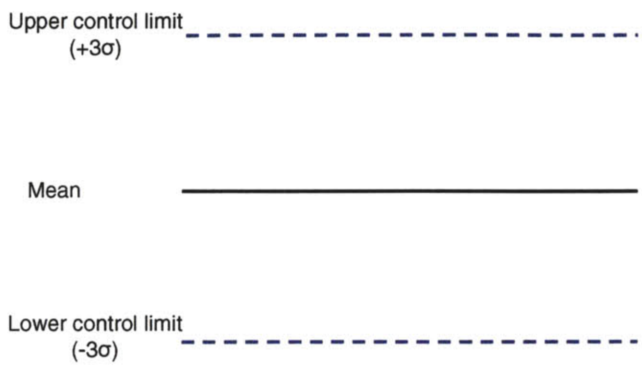

Figure 3-2: A typical control chart

Control charts are primarily of two types – one that controls for variation in processes mean and one that controls for variation in processes dispersion. It is customary in SPC to monitor the charts related to the dispersion of a process prior to monitoring the charts related to the process mean. This is because a change in the dispersion of a process has a direct impact on the chart that controls for the process mean since the control limits around the process mean are calculated based on the process dispersion. However, changes in the process mean do not require recalibration of the control limits of the chart for process dispersion. The three most widely used charts that lend themselves easily to controlling demand are $\overline{R}$ chart, σ chart and $\overline{X}$ chart. These are described briefly below.

3.2.1 $\overline{R}$ Chart

An $\overline{R}$ Chart controls for variation in process range. The popularity of the $\overline{R}$ chart is historic. Prior to the advances in computation speed, $\overline{R}$ charts provided simplicity in estimating the dispersion of a process. The range of a sample can be calculated with less computational effort than the standard deviation. The average of the range of the samples can be converted into an estimate of the standard deviation using the statistical relationship between the mean range for data from a normal distribution and the standard deviation of that distribution.

The center line of the chart can be constructed by calculating the range, R_i, for each sample. The average of the sample ranges gives the center line, $\overline{R}$. The upper and lower control limits (UCL and LCL) for the chart can be computed using the factors of D_3 and D_4 from the standard SPC/SQC table for $\overline{R}$ Charts (Appendix A). The values D_3 and D_4 are calculated based on the assumption of normal distribution and can be determined for a given number of samples. The estimated standard deviation can be computed from $\overline{R}$ by using the value d_2 also from the standard SPC tables.

$$\overline{R} = \frac{1}{n}\sum_{1}^{n} Ri \qquad LCL = D_3\,\overline{R} \qquad UCL = D_4\,\overline{R} \qquad \hat{\sigma} = \frac{\overline{R}}{d_2}$$

$\overline{R}$ charts are best suited for samples sizes up to 10. For larger samples, the $\overline{R}$ statistic becomes a poor estimator of the sample standard deviation. For these sample sizes the σ chart become a more appropriate chart for controlling process dispersion.

3.2.2 σ Chart

The σ Chart controls for variation in process standard deviation. Computing the standard deviation of samples has historically been computationally more intensive. However, the level of effort to generate σ Chart has been greatly eased with the development of higher computing capabilities.

The center line of the chart can be constructed by calculating the standard deviation, s_i, for each sample. The average of the sample standard deviations gives the center line, $\bar{s}$ and represents the estimated standard deviation for the process. The upper and lower control limits (UCL and LCL) for the chart can be computed using the values of B_3 and B_4 from the standard SPC/SQC table (Appendix A).

$$\bar{s} = \frac{1}{n}\sum_{1}^{n} s_i \qquad \text{LCL} = B_3\,\bar{s} \qquad \text{UCL} = B_4\,\bar{s}$$

σ Charts are best suited for larger number of samples of 10 and greater. For sample sizes between 6 and 10 the accuracy of using the $\overline{R}$ Chart decrease to less than 90% compared to the σ Chart decreases, making the σ Chart a better suited candidate (Jack Prins, 2003).

For the purposes of this research, the σ Chart is chosen over the $\overline{R}$ Chart to control for standard deviation of demand due to the larger sample sizes (6 to 12) used in the methodology. The details of this methodology are provided in the next chapter.

3.2.3 $\overline{X}$ Chart

The $\overline{X}$ Chart controls for variation in process mean. The center line of the chart can be constructed by calculating the mean, X_i, for each sample. The average of the sample means gives the center line, $\overline{X}$. The upper and lower control limits (UCL and LCL) for the chart can be computed using the values of either the estimated standard deviation from the $\overline{R}$ Chart or the average sample standard deviation from the σ Chart.

$$\overline{X} = \frac{1}{n}\sum_{1}^{n} X_i$$

$LCL = \overline{X} + 3.\overline{s}$ $\quad$ $UCL = \overline{X} - 3.\overline{s}$ $\quad$ for σ Chart

where, $\overline{s}$ represents the average of the sample standard deviations from the σChart

$LCL = \overline{X} + 3.\hat{\sigma}$ $\quad$ $UCL = \overline{X} - 3.\hat{\sigma}$ $\quad$ for $\overline{R}$ Chart

where, $\hat{\sigma}$ represents the estimate of the standard deviation from the $\overline{R}$ Chart

3.3 Outlier Rules

For the purposes of this document, two types of statistically out of control situations are defined. A *shift* in a process refers to the a situation in which the process is deemed out of control due to observations on a control chart that are consistently beyond the 2σ control limits above and below the center line. This includes observations that are 2σ to ∞ and -2σ to -∞ on the control chart. A *drift* in a process refers to the situation in which the process is deemed out of control due to consecutive observations on a control chart that are between the center line and +2σ and between the center line and -2σ. Essentially, a shift is a higher magnitude event than a drift. A shift may represent the addition or removal of a customer. On the other hand, a drift may represent a steady growth in demand from an existing customer. Shifts and drifts are shown in the figure 3-3 below.

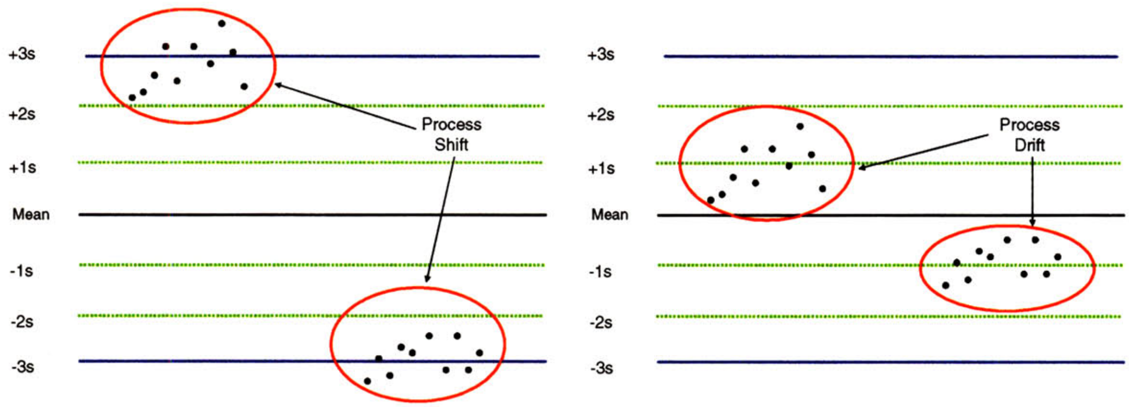

Figure 3-3: Shifts and drifts in a process

Detection of shifts and drifts in a process in SPC is based on outlier rules. Outlier rules defined the number of observations that need to fall outside specified control limits to be considered evidence of a shift or a drift in the process. Outlier rules are usually designed such that fewer outlier observations are needed to signal a shift than that for a drift. This is fundamentally due to the fact that events involving a shift in a process are rarer those involving a drift in that process. The outlier rules are defined specific to applications since these are influenced by the nature of the process variables. The outlier rules for the demand and inventory charts will be defined in the next chapter.

4 SPC INVENTORY MANAGEMENT METHODOLOGY

4.1 Introduction

The inventory management methodology developed as part of this research applies statistical process control techniques to traditional inventory management. In particular, a basic inventory policy with periodic review, (R, S) policy, is enhanced with control charts for managing demand and inventory levels to adapt the reorder point and order quantities to reduce the bullwhip effect while simultaneously maintaining or reducing inventory levels without any degradation in service levels.

In the (R, S) policy, for an inventory review period of R and a order lead time of L, the order up to level, S can be computed as below.

$$S = X_L + k\sigma_{L+R} \qquad \text{Equation (4-1)}$$

where, X_L is the demand over the lead time and $k\sigma_{L+R}$ represents the safety stock. Here, k is the safety stock factor and σ_{L+R} is the standard deviation of demand over the sum of the review period and lead time

The order quantity in this policy is determined as the difference between the order up to level and the inventory level at a review period. The order quantity will change from period to period if the demand is non-stationary. Further, the order up to level, S, may itself need to be monitored and recomputed if the demand characteristics change. From Equation 4-1, it is clear that a change in the demand characteristics will impact either the cycle stock or safety stock requirements or both. Company typically periodically review trends in demand and update the mean and standard deviation of demand in their inventory systems.

The (R, S) policy described above is modified by incorporating SPC techniques to create an integrated demand and inventory planning system that monitors and responds to shifts and drifts in demand characteristics and inventory levels. Consider a demand process which operates in three states over time (Figure 4-1). In state 1, the demand operates with a particular mean and standard deviation. In state 2, the standard deviation increases due to higher volatility in demand. However, the mean demand remains steady at the level at state 1. Finally, in state 3, the mean demand shifts to a new level and the standard deviation decreases. The inventory control chart shows the operating limits of the inventory process for each state of demand. From state 1 to state 2, it is seen that the safety stock represented by the minimum inventory level increases due to the increase in the standard deviation of the demand process. Similarly, the maximum inventory level also shifts by the increase in the safety stock. The difference between the maximum inventory level and the minimum inventory level represents the fixed order quantity of the system. Clearly, the fixed order quantity remains unchanged from state 1 to state 2 and is purely a function of the mean demand. From state 2 to state 3, the mean demand increases and the standard deviation of demand decreases. The inventory chart responds by reducing the safety stock and increasing the fixed order quantity.

The ability of the system to monitor changes in demand characteristics and to respond to such changes by recalculating the inventory and ordering requirements is explained in detail in the following sections.

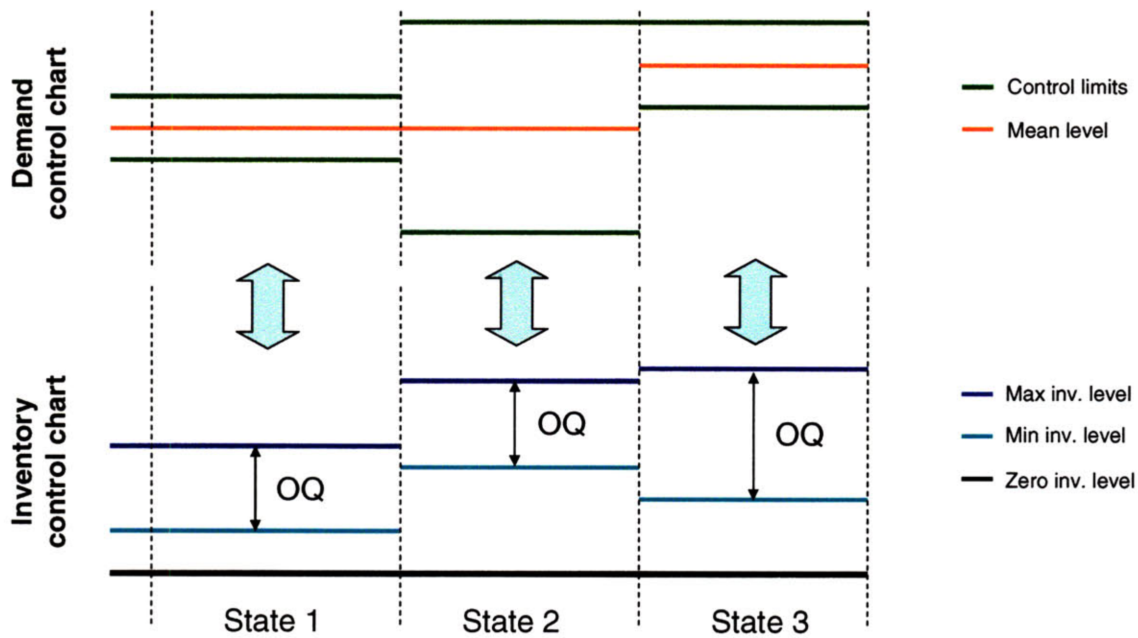

Figure 4-1: Relationship between demand and inventory control charts

4.2 Demand Control Charts

As described previously in Chapter 3, demand involves two important parameters– mean and standard deviation. The $\overline{X}$ chart is used to control for the variation in mean demand and the σ chart is used to control for the variation in the standard deviation of demand. The $\overline{X}$ chart and σ charts for demand are constructed using the historical mean and standard deviation. Using historical demand for this purpose ensures that the control charts are a good representation of the current state of demand. For the purposes of this research, weekly demand is considered for the $\overline{X}$ and σ charts.

Assume that 20 weeks of demand are used to construct the first estimate of the $\overline{X}$ chart and σ charts. The total demand and standard deviation for each of the 20 weeks can be computed. The average of the 20 weekly demand and standard deviation represent the center lines on the $\overline{X}$ chart and σ charts, respectively. The control limits are then drawn around the center lines using the formulas below.

$$\bar{s} = \frac{1}{n}\sum_{1}^{n} s_i \qquad \text{LCL3} = B_3\,\bar{s} \qquad \text{UCL3} = B_4\,\bar{s} \qquad \text{for } \sigma \text{ Chart} \qquad \text{Eqn 1}$$

$$\bar{X} = \frac{1}{n}\sum_{1}^{n} X_i \qquad \text{LCL3} = \bar{X} - 3.\bar{s} \qquad \text{UCL3} = \bar{X} + 3.\bar{s} \qquad \text{for } \bar{X} \text{ Chart} \qquad \text{Eqn 2}$$

In addition to the outer control limits, intermediate controls limits are also drawn around the center line that are 1 and 2 standard deviations above and below the center line. The intermediate control limits for the $\bar{X}$ chart can be constructed by constructing UCL2 = $\bar{X} + 2.\bar{s}$, LCL2 = $\bar{X} - 2.\bar{s}$, UCL1 = $\bar{X} + 1.\bar{s}$ and LCL1 = $\bar{X} - 1.\bar{s}$ (Figure 4-2). However, for the σ Chart, LCL2 is calculated to be such that it is a third of the distance between the center line and LCL3 from LCL3 in the direction of the center line. LCL1 is midway between LCL2 and the center line (Figure 4-3).

Figure 4-2: Intermediate limits in $\bar{X}$ control chart

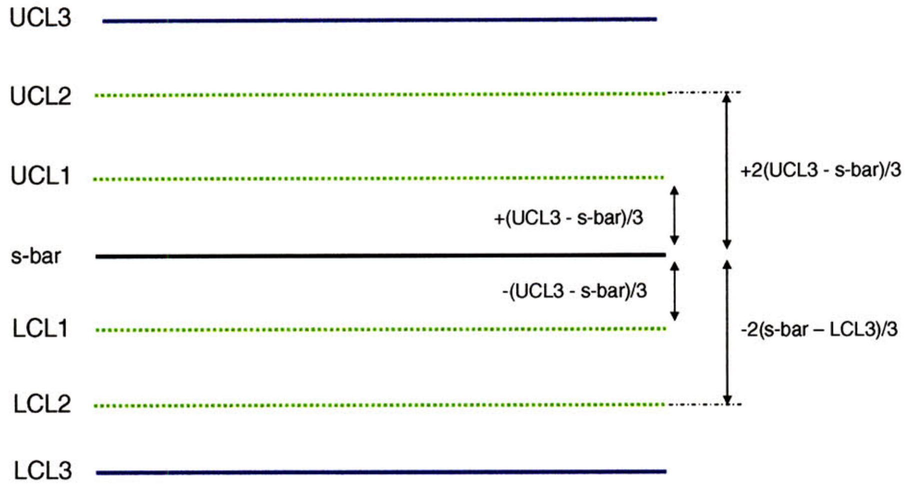

Figure 4-3: Intermediate limits in σ control chart

Once the demand control charts are created, outlier rules are defined. The outlier rules ensure that sufficient data exists to signal a change in the process characteristics. The following is an example of a set of outlier rules where an out of control event is signaled:

1. If a single observation exists above +3 σ limits
2. If two consecutive observations exist above the +2 σ limits
3. If six consecutive observations exist above the +1 σ limits
4. If twelve consecutive observations exist above the center line

Similar rules can also be defined for observations that lie below the center line.

Now, new observations of mean demand and standard deviation of demand are collected. Consider a window of 10 weeks over which observations are collected. The weekly demand over the most recent 10 weeks would be treated as the current week's mean demand observation. Also, the weekly standard deviation of the most recent 10 weeks would be treated as the current week's standard deviation observation. Once this data is compiled, the standard deviation observation is first entered on the σ Chart. If the observation satisfies an outlier rule, the σ Chart indicates an out of control process. The process supervisor is alerted and the system recommends that the center line of the σ chart, $\bar{s}$, be adjusted. The adjustment may be calculated by averaging a specified number of the most recent observations. The center line and the corresponding control limits of the σ chart are now updated.

The σ chart directly impacts the control limits of the $\bar{X}$ chart. If the σ chart changes, so will the $\bar{X}$ chart. The $\bar{X}$ chart is updated to reflect the appropriate control limits. Now the mean demand observation for the current week is entered on the $\bar{X}$ chart and tested for the outlier rules for the $\bar{X}$ chart. If the outlier rules are satisfied, the $\bar{X}$ chart signals an out of control event. The process supervisor is alerted and the system recommends that the center line of the $\bar{X}$ chart, $\bar{\bar{X}}$, be adjusted. The adjustment may be calculated by averaging a specified number of the most recent observations. The center line and the corresponding control limits of the $\bar{X}$ chart are now updated.

In the event that none of the outlier rules are satisfied, the $\bar{X}$ chart and σ chart are maintained as per their current state.

4.3 Inventory Control Charts

Once the demand control charts are constructed, the inventory control chart can be derived. The inventory control chart recommended in this research is the $\bar{X}$ chart and is constructed for the average minimum inventory level (safety stock level). The minimum inventory level can be calculated from the standard deviation of demand for a given level of service. For a product with standard deviation of demand σ with a lead time L, review period R and a customer service safety factor k on the (R, S) policy, the safety stock can be computed as follows.

$$SS = k\sigma_{L+R}$$

where, σ_{L+R} is the standard deviation of demand over L + R

The safety stock represents the average minimum level of inventory as the center line of the $\overline{X}$ chart. To determine the control limits around the center line, the standard deviation of the inventory around safety stock first needs to be calculated. Consider figure 4-4 that shows a sample plot of inventory over a period in time with a characteristic saw tooth profile using the (R, S) policy. The steep vertical lines represent inflow of inventory while the lines with the slopes indicate the consumption of inventory at some rate. In each cycle, the inventory hits a lowest point which on an average is the safety stock. The lowest point inventory in a cycle can be expressed as follows:

$$I^- = S - D_{L+R}$$

where, D_{L+R} is the demand over L + R and S is the reorder point for the policy

Since the S is a constant for a given state of demand, the variability in I^- is the variability in the demand over the lead time and review period. Intuitively, this is the same standard deviation used in the computation of the safety stock. With this information, the inventory control chart can be constructed (Figure 4-4). The chart is constructed only with +1 σ_{L+R} and -1 σ_{L+R} control limits.

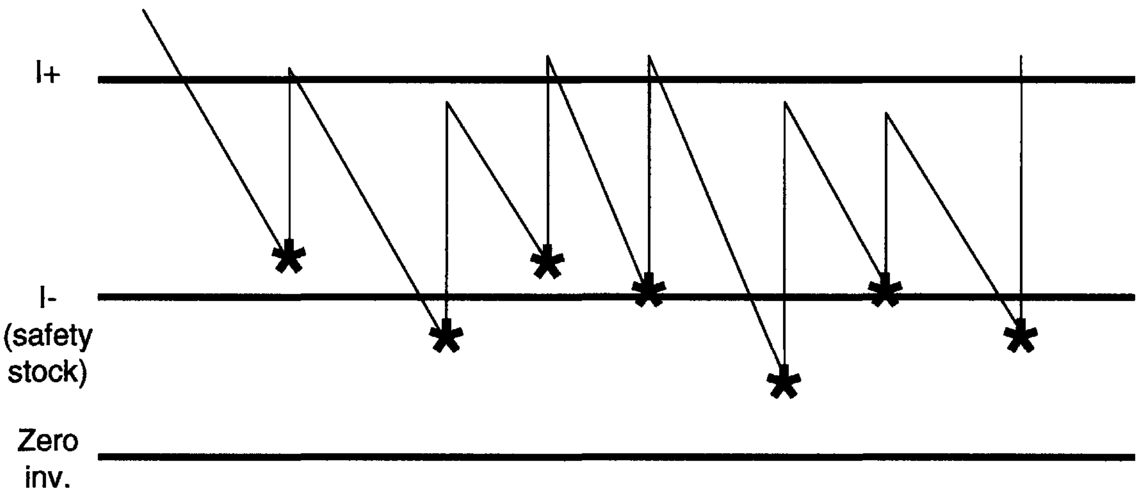

Figure 4-4: Inventory control chart

The inventory control chart is used as a feedback correction mechanism to augment orders placed by the SPC inventory system if the inventory falls below the lower control limit on the chart. The ability of the system to anticipate potential shortages of inventory improves over all fill rates as compared to the standard (R, S) policy. The inventory chart only includes the minimum inventory level to ensure that the required customer service is met. The need for the maximum inventory level is obviated by the intelligence of the system to compute a fixed order quantity described in the following section.

It is important to note here that since the inventory control chart is derived from the parameters on the demand control charts, a change in the demand control chart will cascade changes into the inventory control chart.

4.4 Fixed Order Quantity

For a production plant, achieving a fixed quantity of supply that meets customer demand is beneficial for various reasons. The pattern of fixed production quantities result in a habit forming schedule for the plant. The schedule of a fixed production quantity for a product improves efficiencies of production with implications to lowering the cost of production. However, the recommended fixed order quantity for a state of demand must be such that it does not negatively impact inventory investment and the fill rate. The methodology assumes that the fill rate is an adequate measure of customer satisfaction.

The SPC inventory management system has the ability to determine a fixed order quantity for a given level of demand based on the characteristics of the (R, S) policy. The average maximum inventory during each ordering cycle is expressed as below.

$$I^{+} = S - D_L$$

where, D_L is the demand over lead time and S is the reorder point for the policy

The fixed order quantity (Figure 4-5) for a given demand is shown below.

$$Fixed\ Order\ Qty = I^{+} - I^{-}$$

The fixed order quantity is equivalent to the demand over review period for a given state of demand on the control chart.

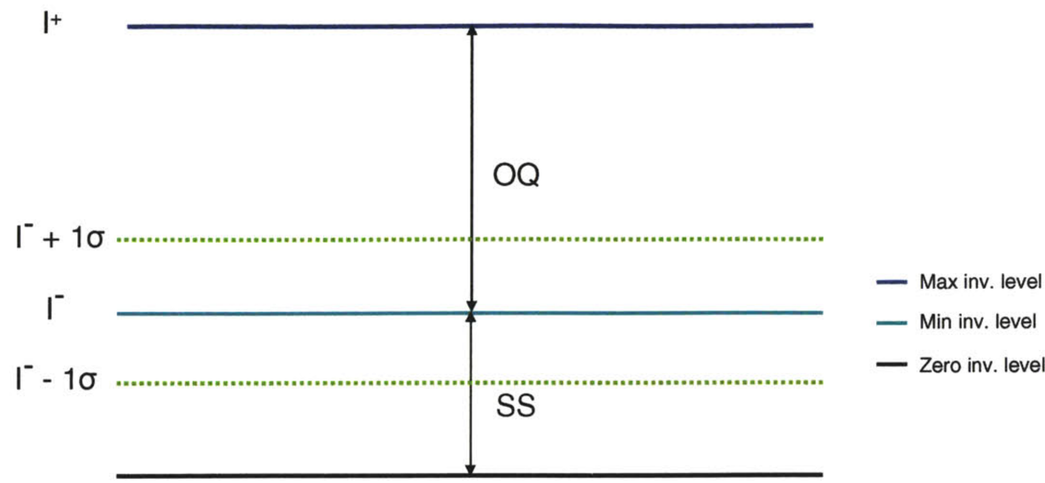

Figure 4-5: Determination of the fixed order quantity on the inventory control chart

If the (R, S) policy is simulated over a long duration, the average order quantity placed by the (R, S) system will correspond to the fixed order quantity calculated above. However, the order placed by the system will vary period over period. The fixed order quantity calculated above may lead to excess and deficit inventory conditions in the short term. The condition of deficit inventory would be the critical to business since this represents lost sales or back orders with potential drop in customer service. However, when the fixed order quantity logic is coupled with the inventory chart capability to mitigate potential shortages by augmenting the order quantity, the quality of the plans the system generates is improved. Further, the system places an order up to the new reorder point for the first period when the reorder point changes.

It is important to note here that since the average maximum inventory level (I^+) and the average minimum inventory level (I^-) are derived from the parameters on the demand control charts, a change in the demand control chart will cascade changes into the inventory control chart and the estimate of the average maximum inventory level. This will produce a change in the recommended fixed order quantity of the system.

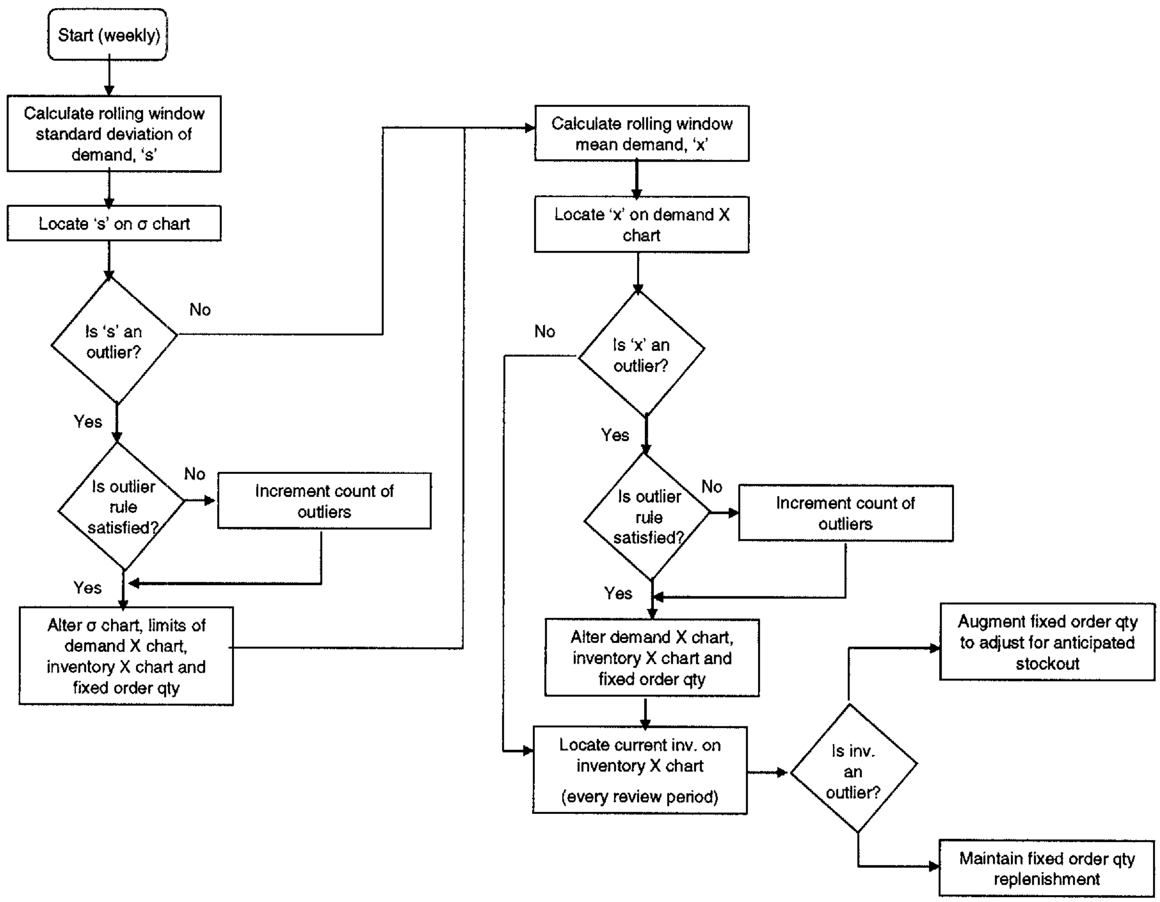

Figure 4-6: SPC based inventory management system flow chart

The flow chart presented in Figure 4-6 represents the robust framework for controlling inventory developed in this section. Changes are made to inventory and replenishment parameters only when statistically significant demand changes occur. In this manner, demand signal processing effects based on speculation is minimized and the system responsiveness is based on a set of well-defined rules. In the following section, the methodology is applied to a product family of a medical devices company.

5 CASE STUDY: MEDICAL DEVICES COMPANY

Medical Devices Company is one of the world's leading developer and manufacturer of breakthrough products for interventional medicine, minimally invasive computer-based imaging, and electrophysiology for fighting disease. It sells products in markets worldwide and is headquartered in the United States of America. Medical Devices Company has five business units/divisions: cardiovascular disease management, peripheral vascular and obstructive disease management, neurovascular management, electrophysiology and medical sensor technology, and biologics delivery. The major product families of the cardiovascular disease management division are Drug-eluting stents, Guidewires, Cardiovascular catheters, Dilatation catheters, Sheath Introducers, Biopsy forceps and Diagnostic Guidewires. This thesis studies the cardiovascular catheter product family.

5.1 Cardiovascular catheters

The American Heart Association defines 'cardiac catheterization' as 'the process of examining the heart by guiding a thin tube (catheter) into a vein or artery and passing it into the heart and into the coronary arteries.' These tubes are called 'cardiovascular catheters'. The cardiovascular catheters manufactured by Medical Devices Company are tubes with stainless steel braiding, PTFE liner and a blended nylon outer coat. The cardiovascular catheter is specified by both outer diameter (OD) and inner diameter (ID), the length of the tip and shape of the catheter.

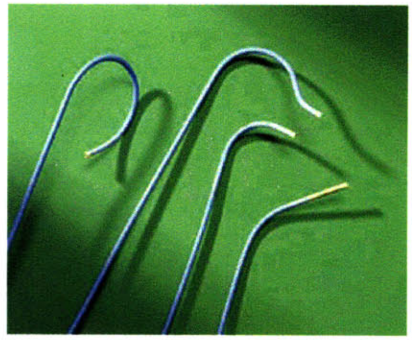

Figure 5-1: Photograph showing cardiovascular catheters

The cardiovascular catheter product family is in the mature phase of the product life cycle. Though this product family has more than 500 Stock keeping units (SKU), the case study shall be limited to those SKUs which fulfill the criterion that at least 20 weeks of demand (orders placed by hospitals to the distribution center) history is available. This has resulted in our analysis being limited to 397 SKUs.

5.2 Supply Chain for cardiovascular catheters

The supply chain for cardiovascular catheters is described in Figure 5-2. The cardiovascular catheters are manufactured at a single location. After manufacturing, the products are shipped to sterilization centers which hold inventory in the form of work-in-progress (WIP). For the product to be sold in the Americas (comprising of United States of America, Canada, Mexico, Latin America) catheters are transported to 'Location B' for sterilization; and for the product to be sold in Europe, Asia, Africa and Australia, the catheters are transported to 'Location D'. Subsequent to sterilization of the catheters, the catheters are sent to the distribution centers which dispatch the products to customers as per the demand. While the 'Location B' sterilization center sends the products to the

'Location C' distribution center, the 'Location D' Sterilization center sends the products to the Location E distribution center.

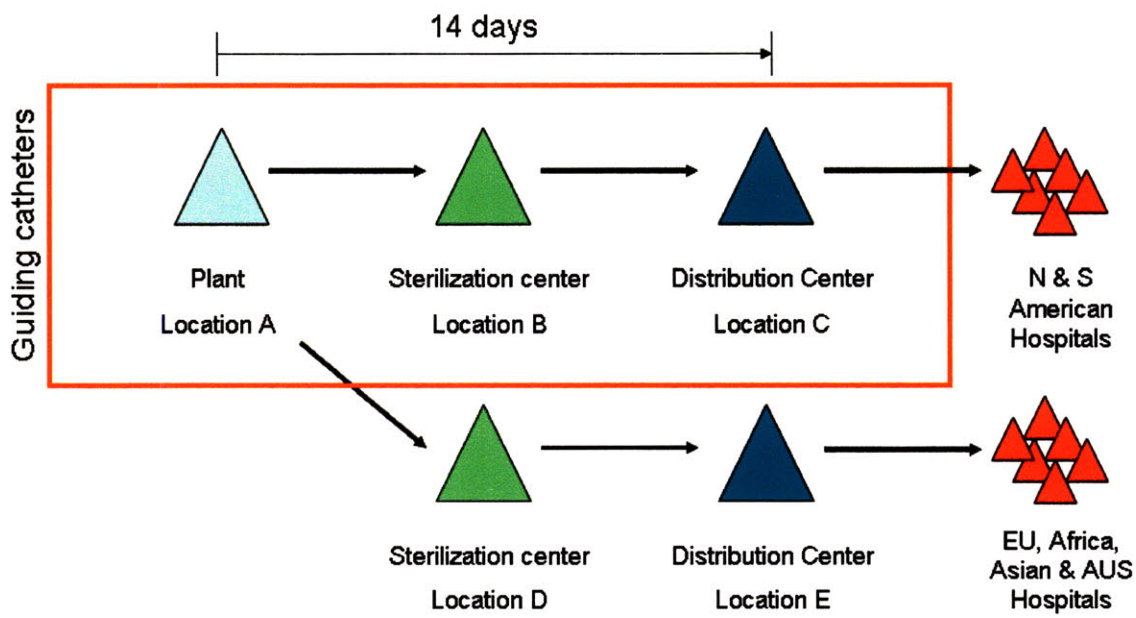

Figure 5-2: Supply Chain for the cardiovascular catheters.

The red box in Figure 5-2 defines the scope of the case study, which is limited to the Manufacturer–Distribution Center echelon of the supply chain. The sterilization center is considered part of manufacturing and the inventory at the sterilization center is considered work-in-progress. This simplifies to a two echelon supply chain with orders being placed by the distribution center to the manufacturer. This study focuses on the 'Location A' plant to 'Location C' distribution center.

5.2.1 Manufacturing

The manufacturing facility in 'Location A' manufactures five product families including the cardiovascular catheters for Division 1 of the Medical Devices Company.

All the 397 SKUs of the cardiovascular catheter family are manufactured in this company owned facility having multiple production lines. The production lines are segregated on basis of equipment for producing a particular mix of SKUs. Some production lines are similar and interchangeable. The average weekly demand of SKUs on these lines is 81 units and consists of a mix of all categories of SKUs. They manufacture a product mix of 358 SKUs. A particular single line manufactures a product mix of 39 SKUs. This line manufactures low volume SKUs and the average weekly demand of SKUs produced is 30 units. The production plan is based on the inventory levels at the Location B distribution center which transmits demand and inventory balance information to the manufacturing plant. Based on this information, lot-sizing of the various SKUs is decided for the manufacturing process.

5.2.2 Transportation

Transportation of product is done by trucks for intra-continental requirements and by air for inter-continental requirements. The transportation times within the manufacturer-sterilization plant-distribution center echelon of the supply chain are:

Transportation from Plant to Sterilization Center	**Location A to Location B**	**Location A to Location D**
Transportation Time	1 day	7 days
Transportation mode	Truck	Airplane
Transportation from Sterilization Center to Distribution Center	**Location B to Location C**	**Location D to Location E**
Transportation Time	2-3 days	<1 day
Transportation mode	Truck	Truck

Table 5-1: Transportation times in the manufacturer-distribution center echelon of cardiovascular catheter supply chain

The transportation time for the ‘Location C’ distribution center from the manufacturing plant is 3-4 days, whereas the transportation time for the ‘Location E’ distribution center is between 7-8 days. The former uses only truck, whereas the latter uses both trucks and air.

The transportation time from ‘Location A’ plant to the ‘Location B’ distribution center, which serves demand of North and South America, accounts for 21% of the total lead time of 14 days. This case study considers the transportation time as deterministic and does not focus upon the possibility of reduction in transportation or lead times.

5.2.3 Inventory Replenishment at Distribution Center:

The inventory and replenishment policy is defined at the SKU level. The demand for a SKU, in terms of orders from hospitals, is received daily by the distribution center. The demand is met by the inventory available at the distribution center. The inventory balance is reviewed on a periodic basis by information sharing between the ‘Location A’ manufacturing plant and the ‘Location C’ distribution center. If no stock is available at the distribution center to meet the demand on a particular day, a back-order is generated, and information is transmitted to the plant during the following periodic review. The inventory replenishment is done using an (R,S) policy with a three day review period for critical SKUs and five day review period for other SKUs. The 'criticality' of the SKU is decided by the Medical Devices Company management depending upon various considerations. For simplicity, this case study assumes a three day review period for all SKUs.

The (R,S) policy has been modified by the Medical Devices Company management due to the constraints of destructive testing to be performed on each lot of SKUs manufactured, and taking into account cost, consolidation and batching constraints. As a result of these considerations, whenever the inventory level at distribution center goes below the order-up to level, a minimum order quantity is placed on the plant for manufacture. In case, the order quantity is not sufficient to make the inventory reach the order-up to level, a quantity equal to a multiple(s) of a constant quantity is added to the order quantity, so that the inventory reaches at least the order-up to level. Due to these considerations, the order-up to level does not remain fixed, but has variability around it. Due to the considerations, the inventory level overshoots the order-up to level resulting in an amplification of order quantity, which increases the inventory levels.

In order to assess the impact of SPC approach on bullwhip effect as compared to the (R,S) policy, in this case study, the researchers have removed this additional amplification of order quantity by not considering the minimum order and multiplier constraints. The order up-to level has been considered fixed for each SKU and has been computed on basis of the demand history.

The lead time for fulfilling the demand placed by the 'Location C' distribution center to the plant is 14 days. It is considered to be deterministic for this case study.

5.3 Demand Characteristics of Cardiovascular catheters

Hospitals in North and South America place orders on the 'Location C' distribution center for various SKUs of the cardiovascular catheter product family. Based on data pertaining to the period between November 2005 and October 2006, one can estimate the annual demand for the cardiovascular catheter family to be 1,506,100 (1.5

million) units, assuming a year of 50 weeks for this study. The data shows wide variation in the units of various SKUs supplied by the distribution center to the hospitals.

Categories of SKUs

The SKUs can be classified on the basis of volume (units) sold into very high volume, high volume, medium volume, small volume and very small volume SKUs. Their distribution is indicated in table 5-2.

Category of SKUs	**Range of average annual demand (units)**	**Number of SKUs**	**Number of units sold annually**
Very Small Volume	0-1,000	223	80,876
Small Volume	1,000-5,000	118	267,056
Medium Volume	5,000-15,000	39	323,449
High Volume	15,000-40,000	13	350,668
Very High Volume	40,000-200,000	4	484,145
		Total	**1,506,193**

Table 5-2: Distribution of SKUs on the basis of volume sold

The very small volume SKUs account for 56% of the total number of SKUs, while small volume SKUs account for 30% of the total number of SKUs. Together, these categories account for 86% of the total number of SKUs but only 23% of total volume. The break-up is shown in Figure 5-3.

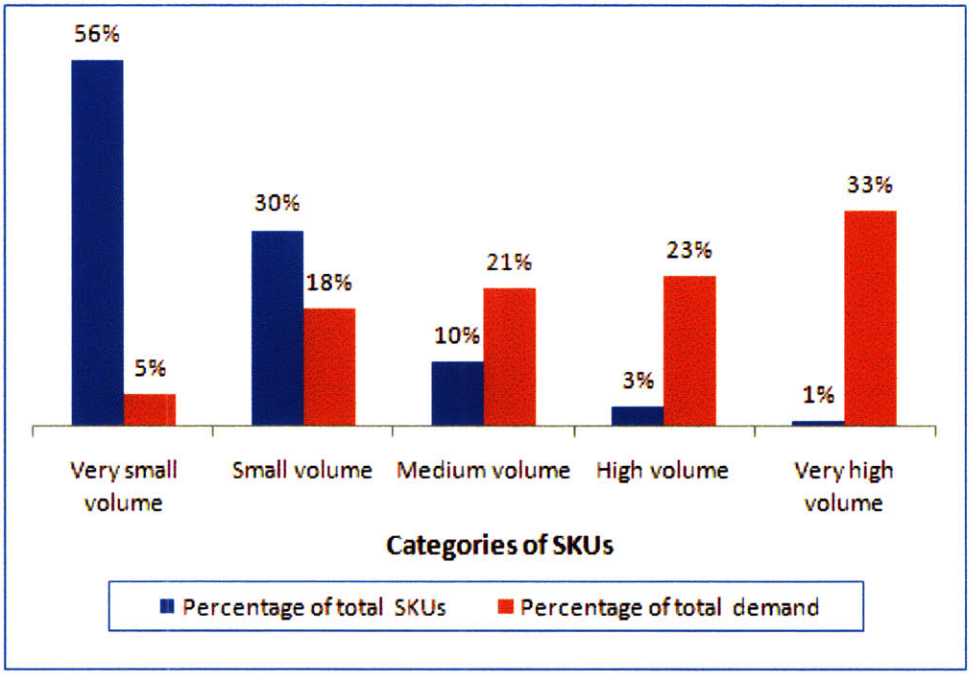

Figure 5-3: Break-up of the cardiovascular catheter family by categories of SKU

The very large volume SKUs account for 33% of the total demand of cardiovascular catheters, while large volume SKUs and medium volume SKUs account for 23% and 21% of the total demand of cardiovascular catheters respectively. The break-up of annual demand is shown in Figure 5-3.

From the above tables and charts, Pareto analysis can be done to see that the very high volume, high volume and medium volume SKUs, though fewer in number (14% of total number of SKUs) account for majority (77%) of demand of the cardiovascular catheter family. The Pareto curve for the cardiovascular catheter family is shown in Figure 5-4.

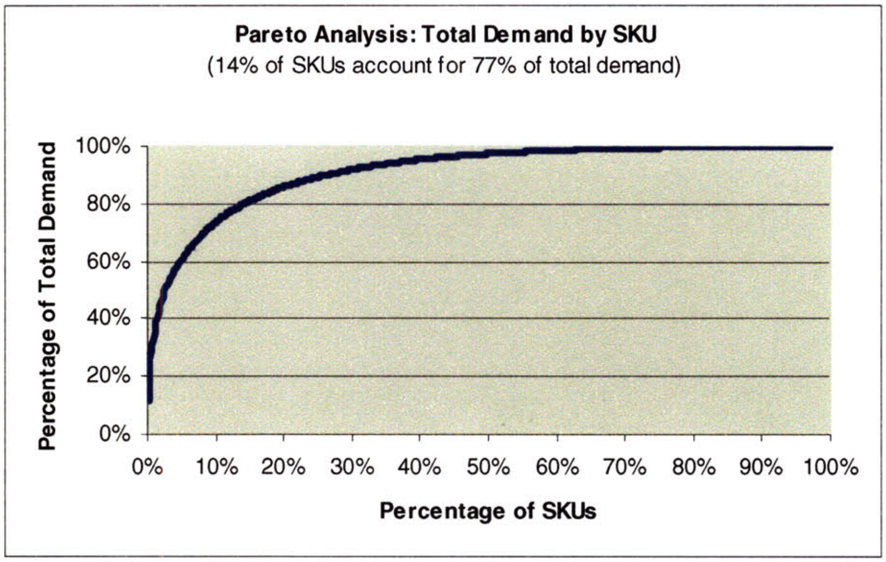

Figure 5-4: Pareto Analysis for the total demand of SKUs

Further, the very high volume and high volume SKUs, though only 4% of total number of SKUs account for 55% of the total demand for cardiovascular catheters.

Variability of demand of SKUs

The demand variability characteristics of the different categories of SKUs are listed in Table 5-3 as under:

Category of SKUs	Coefficient of Variation (CV) of Demand at Distribution Center				
	Minimum	Average	Median	Maximum	Range
Very Small Volume	0.36	0.96	0.92	2.52	2.16
Small Volume	0.21	0.51	0.49	1.41	1.20
Medium Volume	0.19	0.42	0.40	0.68	0.49
High Volume	0.21	0.41	0.38	0.72	0.51
Very High Volume	0.30	0.32	0.31	0.38	0.08

Table 5-3: Demand variability characteristics of different categories of SKUs

The demand variability, measured in terms of coefficient of variation, is the least for the very high volume SKUs on considering the average and median coefficient of variation. It is seen that the average and median coefficient of variation increases as one moves from very high volume to the very small volume SKUs. The maximum coefficient of variation increases from 0.38 to 2.52 and the range of coefficient of variation increases from 0.08 to 2.16 as one moves from very high volume category to very small category. The very small volume SKUs, which are 223 in number and comprise 56% of the total SKUs, exhibit a very high variability. However, these SKUs constitute only a 5% of total volumes of cardiovascular catheters sold.

Demand pattern at distribution center within a week

The demand pattern for the very large volume SKUs shows a wide variation within the week. It is seen that most of the demand, in form of orders from hospitals, for these SKUs are placed on the distribution center on Mondays (around 65%) followed by Tuesdays (around 15%). The remaining 20% orders come on the rest of the days of the week. The demand pattern for the distribution center can be seen from the figure 5-5. It is evident from the figure that a large number of hospitals have a system of placing orders during the earlier part of the week, most frequently on Mondays.

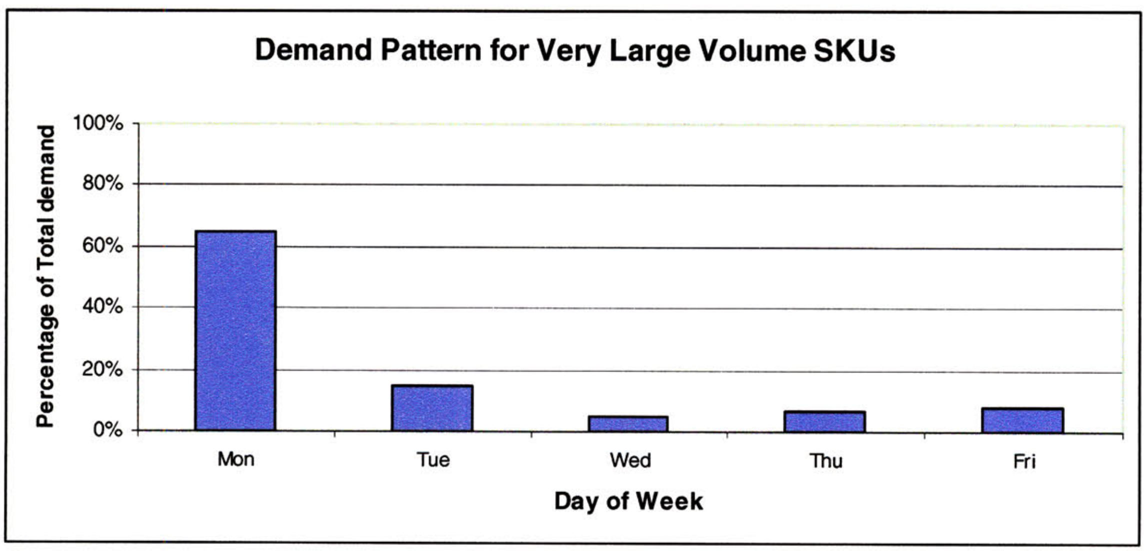

Figure 5-5: Demand pattern for very large volume SKUs

The effect of variation in daily ordering pattern for the very large volume SKU category has been by mitigated by aggregating the demand at weekly level for the study.

5.4 Bullwhip Effect in the cardiovascular catheter supply chain

The present level of bullwhip on overall basis, for the cardiovascular catheter family using a simple average is 1.17. However, given the wide variation in the demand of the individual SKUs, it is considered more appropriate to use the weighted average, using volume sold as the basis for measuring overall bullwhip effect. The level of bullwhip for the cardiovascular catheter family, using the weighted average, is 1.44.

The level of bullwhip in the various categories of SKUs is listed in figure 5-6.

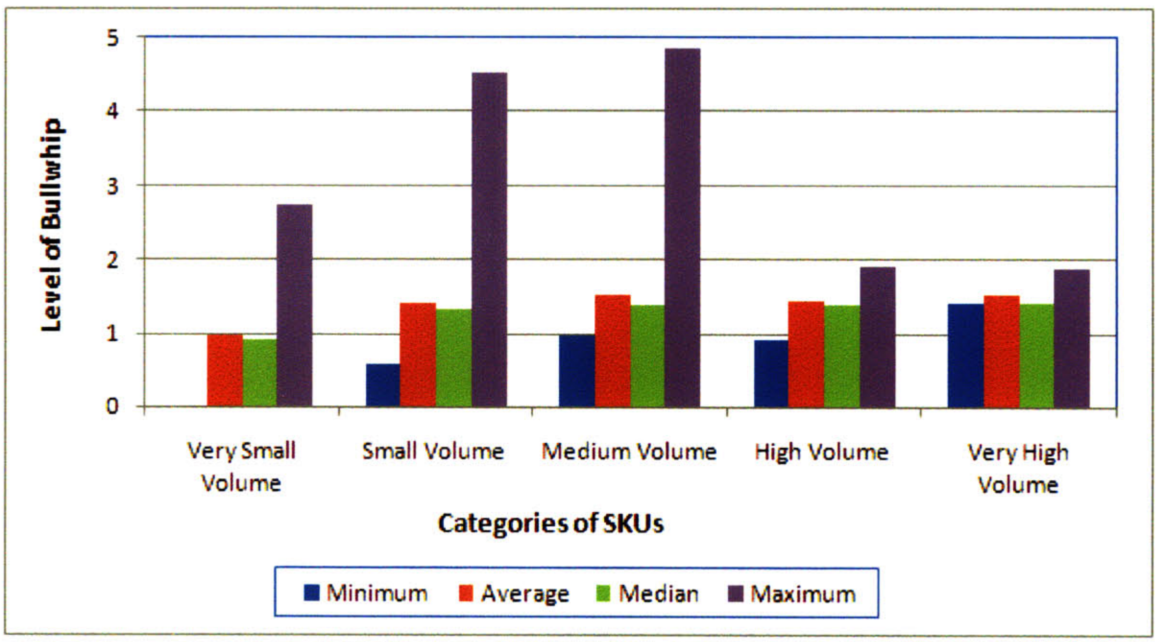

Figure 5-6: Level of Bullwhip in various categories of SKUs

The table indicates that, on an average, the present (R, S) policy results in bullwhip or amplification of demand variability amongst all SKU categories except the very small volume SKUs. All the very high volume SKUs have a significant level of bullwhip, the level ranging from 1.40 to 1.87. All the other categories have some SKUs which have production smoothing (bullwhip level< 1), but the maximum level of bullwhip in these categories is much higher than the very high volume category.

The level of production smoothing amongst various categories of SKUs is given in Table 5-4. The table indicates that the very small volume SKUs have a high percentage of SKUs, which already have production smoothing with the existing (R, S) policy. The high volume, medium volume and small volume categories of SKUs have a moderate number of SKUs which have production smoothing with the (R, S) policy. However, the very high volume category of SKUs do not have any production smoothing with the present (R, S) policy.

Category of SKUs	**Total # SKUs**	**# SKUs having Level of Bullwhip <1**	
		# SKUs	**Percentage**
Very Small Volume	223	131	59%
Small Volume	118	13	11%
Medium Volume	39	1	3%
High Volume	13	1	8%
Very High Volume	4	0	0%
All categories	**397**	**146**	**37%**

Table 5-4: SKUs in various categories having production smoothing

5.5 ABC Classification of SKUs

The ABC analysis is a framework used in multi-product inventory systems where the trade-off between the cost of controlling the system and the potential benefits that accrue from that control are assessed by firms (Namhias, 2005). Based on the Pareto analysis, the SKUs can be classified into 'A', 'B' and 'C' class items. The very high volume and high volume categories are considered to be 'A' class items, the medium and small categories are considered to be 'B' class items and the very small volume SKUs are considered to be 'C' class items. The volume, variability and bullwhip characteristics of

'A', 'B', and 'C' classes of items of the cardiovascular catheter family can be summarized in the table 5-5, as under:

Characteristics	**'A' Class items**	**'B' Class items**	**'C' Class items**
Percentage of total SKUs	4%	40%	56%
Percentage of total demand	56%	39%	5%
Variability [1]	Low	Low-Medium	High
% of SKUs with Production Smoothing [2]	Low	Medium	High
Overall Level of Bullwhip [3]	Medium	Medium	Low

Table 5-5: Characteristics of 'A', 'B', and 'C' classes of items of the cardiovascular catheter family

Assuming the demand to be an indicator of the revenue earned by a particular class of items, one can infer that the 'A' class items constitute the largest source of revenue, and have the highest inventory levels. A significant reduction in bullwhip for these items without increase in inventory stocks while meeting customer service levels, would result in the highest gains to the company. The 'B' class items form 40% of total SKUs and account for 39% of revenues. From the production point of view, the 'A' and 'B' Class items are manufactured under the batch processing. The small volume category of the 'C' Class items comprise of the small volume category manufactured under batch processing and the very small volume category under the 'make-to-order' environment.

5.6 Application of SPC Methodology

The effectiveness of the SPC methodology in reducing the bullwhip effect is tested on the guiding catheters family of products using historical data for simulation

[1] For this case study, variability is considered low if average CV is less than 0.5, medium if average CV is between 0.5 and 0.75, and high if average CV is more than 0.75

[2] For this case study, % of SKUs with Production Smoothing is considered low if % is less than 10%, medium if % is between 10% and 50% and High if % is more than 50%.

purposes. The most recent 22 weeks of historical sales is treated as future demand information for the simulation. Six weeks of historical sales prior to this is treated as history for the simulation and used to calibrate the demand control charts. The results of the methodology are compared with those of a simple (R, S) policy for the same sales data for the following metrics:

1. Coefficient of variation of order quantities
2. Average inventory
3. Fill rate

The application of the SPC methodology is demonstrated for SKU 123. The characteristics of SKU 123 are presented below.

Average demand (historical), $\overline{X}$	= 3,522
Standard deviation of demand (historical), $\bar{s}$	= 1,081
Beginning on-hand inventory	= 15,776
Customer service level	= 99%
Lead time	= 14 days
Review period	= 3 days

The characteristics above are calibrated at a weekly level. The average demand is determined by calculating the average weekly demand for each week over the 6 weeks of history. The average of the average weekly demand is termed the average demand (historical) in the table above. The standard deviation of demand (historical) is computed in a similar manner. The weekly standard deviations over the 6 weeks of history are averaged to obtain 1,081 units. For the purposes of simulation, the beginning on hand

[3] For this case study, overall level of Bullwhip is considered low if average Bullwhip is less than 1.25, medium if Bullwhip is between 1.25 and 1.75 and high if bullwhip is more than 1.75

inventory for the first day of the simulation run is set as the order up to level of the (R, S) policy. The order up to level is calculated using the formula, $S = X_{R+L} + k\sigma_{R+L} = 15{,}776$.

A moving window of 6 weeks of most recent history is consistently used throughout the simulation. The set of outlier rules for this simulation are listed below.

Rule 1: If a single observation exists above +3 σ limits

Rule 2: If two consecutive observations exist above the +2 σ limits

Rule 3: If three consecutive observations exist above the +1 σ limits

Rule 4: If six consecutive observations exist above the center line

These rules are identical to the observations below the center line with respect to the lower control limits. These rules apply to the $\overline{X}$ chart and σ chart. When any of the outlier rules are triggered, in either of these charts, the most recent 4 weeks of observations is averaged to determine the new location for the center line for the specific chart.

Constructing the σ chart for the demand process

The values of B_3 and B_4 are obtained from Appendix A for a sample size of twelve.

Center line = standard deviation of demand (historical), $\bar{s} = 1{,}081$

Lower control limit (LCL) = $B_3\,\bar{s} = 383$ [$B_3 = 0.080$ from Appendix A]

Upper control limit (UCL) = $B_4\,\bar{s} = 2{,}129$ [$B_4 = 1.970$ from Appendix A]

Constructing the $\overline{X}$ chart for the demand process

The center line of the σ chart represents the standard deviation of the $\overline{X}$ chart.

Center line = Average demand (historical), $\overline{X} = 3{,}522$

Lower control limit (LCL) = $\overline{X} - 3.\bar{s} = 280$

Upper control limit (UCL) = $\overline{X} + 3.\bar{s} = 6765$

Monitoring the demand control charts

Every week the average of the weekly sales and standard deviation of the most recent 6 weeks are computed. The averaged weekly standard deviation of demand is first plotted on the σ chart and checked for outlier rules. If outlier observations exist, the center line and control limits of the chart are updated. The control limits around the $\overline{X}$ chart is also updated in such a case. The twelve week averaged weekly demand is now plotted on the $\overline{X}$ chart and tested for the outlier rules. If the outlier observations exist, the center line of the $\overline{X}$ chart is updated. The $\overline{X}$ chart and σ chart for SKU 123 over a 22-week simulation run are shown in Figures 5-7 and 5-8.

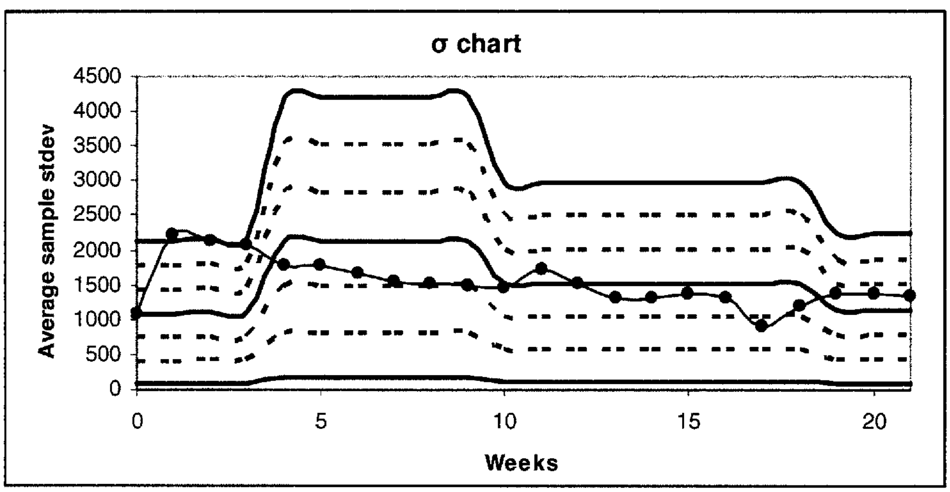

Figure 5-7: σ chart for SKU 123

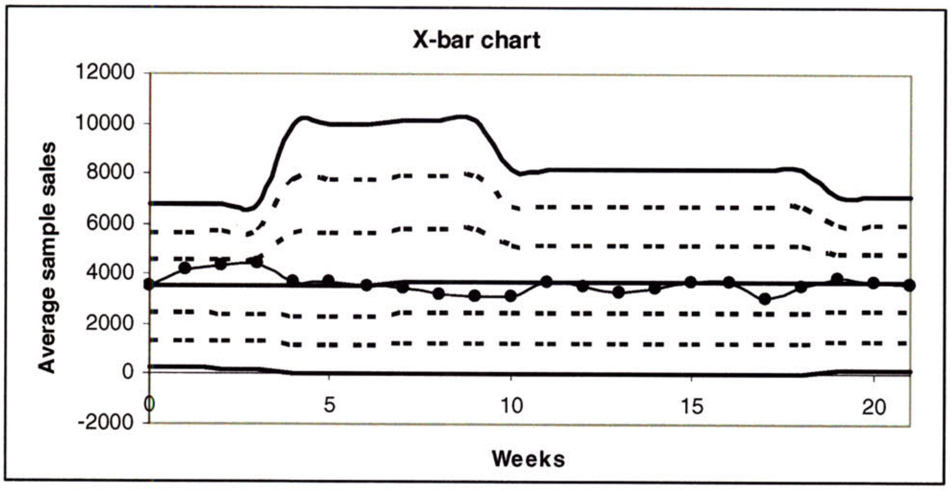

Figure 5-8: $\overline{X}$ chart for SKU 123

It can be observed from Figure 1 that at weeks 4, 10 and 19 the σ chart displays changes in the standard deviation. This results in updates in the control limits of the $\overline{X}$ chart. The center line of the $\overline{X}$ chart changes once in week 7. Where the standard deviation (from the σ chart) changes, the safety stock requirement changes and where the mean demand (from the $\overline{X}$ chart) changes, the cycle inventory changes. This is shown in the inventory control chart (Figure 5-9).

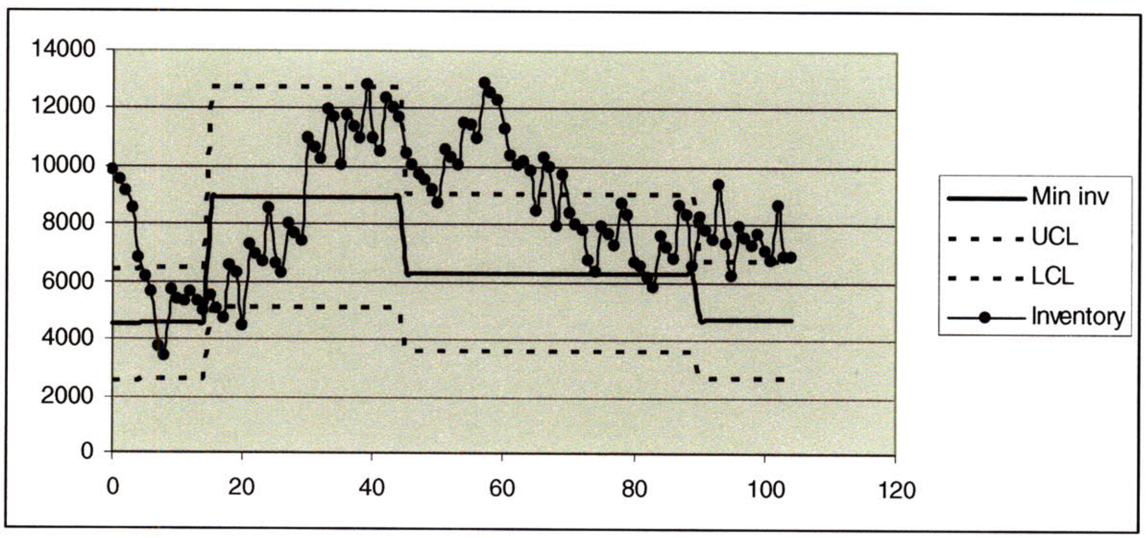

Figure 5-9: Inventory chart for SKU 123

The upper and lower control limits represents one standard deviation (σ_{R+L}) distance from the center line. An adjustment order is not recommended by the inventory control chart even though the inventory on two occasions drops below the lower control limit. This is because the events do not occur on a review period. The order quantities generated by the SPC methodology is plotted alongside that generated by the (R, S) methodology (Figure 5-10). The coefficient of variation of the order quantities generated by the SPC methodology is 16% compared to 42% for the (R, S) methodology suggesting an improvement in production level loading. Additionally, the average inventory also decreases by 7% using the SPC methodology.

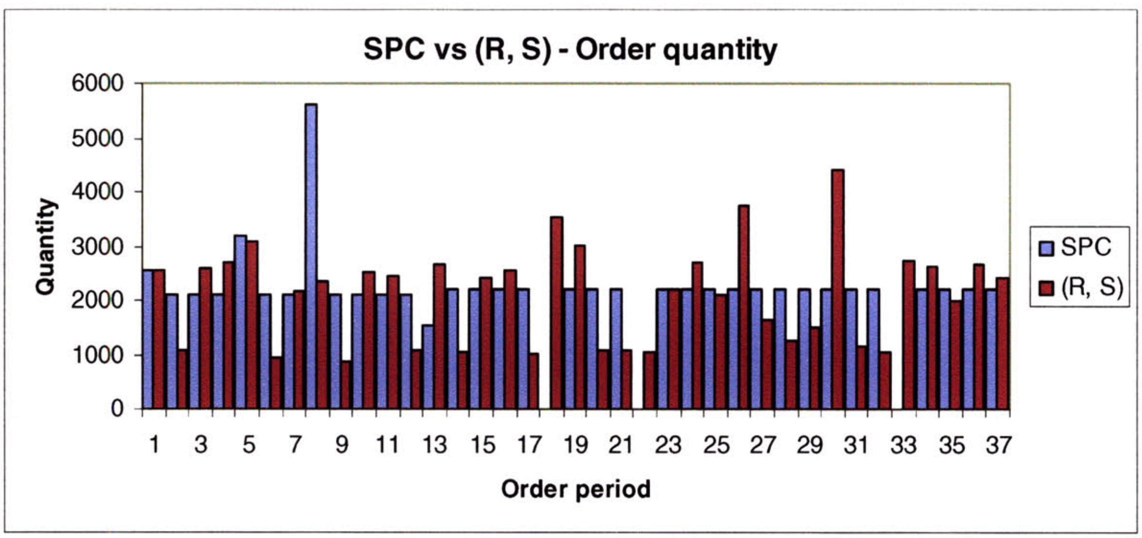

Figure 5-10: Comparison of order quantities generated by SPC methodology to (R, S) policy

5.7 Results

The SPC based inventory management system leads to the significant reduction in the level of bullwhip. On an overall basis for the cardio-vascular product family, the level of bullwhip reduced by 32% from 1.17 to 0.85. By using weighted averages on the basis of demand, the level of bullwhip reduces by 61% from 1.44 to 0.56. The SPC based inventory management system results in production smoothing since level of bullwhip is less than 1. Along with the reduction in bullwhip, the SPC system also results in overall inventory reduction of 12.4%. The results obtained for the 'A', 'B' and 'C' class items are detailed as under.

'A' Class items

The SPC based inventory management technique leads to a significant reduction in level of bullwhip for all the seventeen 'A' class SKUs, as given in the table 5-6.

Minimum reduction in Bullwhip	Average reduction in Bullwhip	Median reduction in Bullwhip	Maximum reduction in Bullwhip
46.8%	74.5%	76.8%	98.9%

Table 5-6: Reduction in levels of Bullwhip for 'A' class items with SPC based inventory system compared to the present (R,S) inventory system

This reduction in bullwhip effect was accompanied by reduction in average inventory levels for all the SKUs, as given in table 5-7.

Minimum reduction in average inventory levels	Average reduction in average inventory levels	Median reduction in average inventory levels	Maximum reduction in average inventory levels
0.4%	11.8%	10.7%	33.2%

Table 5-7: Reduction in average inventory levels for 'A' class items with SPC based inventory system compared to the present (R,S) inventory system

All of the seventeen 'A' class SKUs had 100% customer service level under the present inventory management system, which is more than the target customer service level of 99%. With the SPC based inventory system, all SKUs meet the 99% target service level. All except one SKU continue to show 100% customer service level; the exception showing a reduction in customer service level to 98.62% together with a 17.8% reduction in average inventory level and 46.8% reduction in the level of bullwhip. It is felt that the benefits of level-loading the plant, due to reduction in bullwhip, to such a great extent with substantial reduction in average inventory level might outweigh the costs of the service level falling short of the target of 99% by a marginal 0.38%. The SPC based inventory system showed superior results than the present (R,S) inventory replenishment system by reducing the level of bullwhip on an average by 74.5% along with reduction in average inventory by 11.8%.

B Class items

The SPC based inventory management technique leads to a significant reduction in level of bullwhip for all the thirty nine 'B' class SKUs, as given in the table 5-8.

Minimum reduction in Bullwhip	**Average reduction in Bullwhip**	**Median reduction in Bullwhip**	**Maximum reduction in Bullwhip**
13.3%	62.9%	64.7%	98.5%

Table 5-8: Reduction in levels of Bullwhip for 'B' class items with SPC based inventory system compared to the present (R,S) inventory system

It is seen that the reduction in levels of bullwhip for 'B' class items is less than the reduction in bullwhip for the 'A' class items. This reduction in bullwhip effect was also accompanied by reduction in average inventory levels for all the SKUs except one, as given in table 5-9.

Minimum reduction in average inventory levels	Average reduction in average inventory levels	Median reduction in average inventory levels	Maximum reduction in average inventory levels
-2.1% (increase)	13.4%	12.6%	39.1%

Table 5-9: Reduction in average inventory levels for 'B' class items with SPC based inventory system compared to the present (R,S) inventory system

Though there has been a marginal increase in inventory level for one SKU, all the other SKUs have shown reduction in average inventory level. It is further seen that even though the percentage reduction in bullwhip for 'B' class SKUs is lower than the reduction seen for 'A' class SKUs, the average, median and maximum reduction in average inventory levels is higher for 'B' class SKUs. The results do not indicate a trend in the customer service levels. Six SKUs have less than 99% customer service level with the present (R,S) inventory system, while seven SKUs have less than 99% customer service level with the SPC based inventory system.

C Class items

The 'C' Class items showed mixed results with the SPC based system. The results are shown in table 5-10.

Bullwhip for SKUs with SPC based system	Number of SKUs	Percentage
SKUs showing reduction in bullwhip level	225	66.2%
SKUs showing increase in bullwhip level	115	33.8%
Total 'C' Class SKUs	340	110%

Table 5-10: Bullwhip level for C Class SKUs with SPC based system

It is seen that the only two-thirds of the C Class SKUs have shown a reduction in the level of bullwhip with the SPC based system and for one-third of the SKUs, the bullwhip has increased. This contrasts with the 'A' and 'B' class SKUs where all the

SKUs had shown a reduction in the bullwhip level. The increase in bullwhip amongst 'C' Class SKUs is more pronounced in the very small volume SKU category as compared to the small volume SKU category. This can be seen from the category-wise break-up (Table 5-11).

SKUs showing increase in Bullwhip level	**Number of SKUs**	**Percentage**
Small Volume category	16	13.9%
Very small volume category	99	86.1%
Total 'C' Class SKUs	115	100%

Table 5-11: Break up of SKUs with increase in Bullwhip with SPC based system

The very small category, which has average weekly demand between 0-20 units, has 223 SKUs. It can further be seen in the very small volume SKU category that as the average weekly demand becomes lower, the proportion of SKUs showing increase in bullwhip levels becomes higher (Table 5-12).

Average Weekly Demand	**0-5**	**5-10**	**10-15**	**15-20**
Number of SKUs	94	65	41	23
Number of SKUs showing increase in bullwhip	59	26	9	5
Proportion of SKUs showing increase in bullwhip	62.8%	40%	22%	21.7%

Table 5-12: Very small category SKUs showing an increase in bullwhip level

Average inventory reduction in this category was 48.5% causing customer service levels to become much lower than the target level. The average inventory reduction in the small volume category was 16.4%. However the average inventory reduction in the very small volume category was 65.6%. It is felt that by applying SPC inventory technique to C Class items, particularly to very small volume category, situations of increasing bullwhip take place. Large inventory reduction also occurs, causing problems of inability to meet the target customer service level.

The SPC based inventory management system reduces bullwhip for the product family by 61% using averages weighted by demand. This is accompanied by 12.4% reduction in inventory for the family. There is significant reduction in bullwhip for 'A' and 'B' class items with associated inventory savings while maintaining service levels. However, for 'C' class items, customer service issues arise. Those 'C' class items having very low demand (0 to 20 units per week) show an increase in bullwhip in addition to fill rates issues.

6. CONCLUSIONS

Statistical Process Control is a powerful technique that can be applied to enhance existing inventory management policies to attenuate the bullwhip effect while reducing inventory investments and maintaining customer service levels. Specifically, the methodology has been employed in conjunction with a simple periodic review inventory policy – (R, S) policy. The SPC concepts can be easily adapted to benchmark and monitor demand and inventory processes and provide a visual representation of these processes. The methodology developed in this thesis can be configured to manage inventory for products with a variety of demand characteristics.

The SPC methodology lends itself particularly well to Class A and B products that have reasonably stable demand and where production level loading can provide significant cost savings. For Class C products with sporadic demand characteristics, the authors recommend a make-to-order policy. In such a scenario, Class C products will not offer any practical benefits to production level loading. However, if there are a significant number of Class C products to disrupt production efficiencies, these products can be bundled into a single virtual product. The order quantity for the virtual product can be planned using the SPC methodology to ensure production level loading. The order quantities can then be allocated to actual products by ratio of their respective demand.

Special orders often arise in ordering systems which can affect the demand characteristics of a product. For Class C products, special orders may appear more frequently. These orders make the demand for these products very difficult to forecast. Applying the SPC methodology to managing inventory for such products will result in a very nervous stocking policy, potentially causing fill rate issues. This can be observed in the case study involving the Medical Devices Company. Fulfillment of special orders must to be managed separately and not mixed with cycle stock planning. This insulates the demand control charts from reacting to assignable causes of demand variability that are known in advance.

SPC determines changes in demand characteristics with a time lag resulting in short term deficit or excess in inventory. The impact of the lag in responding to significant changes in demand characteristics due to known events such as the addition or loss of a customer can have tremendous immediate inventory implications. This issue can be mitigated by updating the existing demand control charts with the expectation of demand stemming from the planned event. Over time, the system will adjust the demand control charts to reflect the actual nature of the demand characteristics due to this event.

The authors recognize that the methodology developed in this thesis has been restricted to items in the mature phase of their lifecycles with a set of predefined outlier rules. Additional research is required to explore the sensitivity of this methodology to changes in the outlier rules and its effectiveness in the launch and declines phases of the product lifecycle. To overcome the limitations of the methodology in addressing 'C' class items, the demand for these items can be aggregated to create a virtual item that can be accommodated by the methodology. However, this strategy has not been tested. Further, since the methodology has been applied only to medical devices, the extension of this methodology into products in other industries needs to be investigated.

BIBLIOGRAPHY

1. Beck, J. (1999). Statistical process control and selectable forecast calendars reduce GE Aircraft Engines' parts inventory. *Production and Inventory Management Journal* , *40* (3), 5.

2. Boute, R. N., Disney, S. M., Lambrecht, M. R., & Houdt, B. V. (2007). An integrated production and inventory model to dampen upstream demand variability in the supply chain. *European Journal of Operational Research* , *178* (1).

3. *Cardiac Catheterization.* (2007). Retrieved March 2007, from American Heart Association: http://www.americanheart.org/presenter.jhtml?identifier=4491

4. Chen, F., Drezner, Z., Ryan, J. K., & Simchi-Levi, D. (2000). Quantifying the bullwhip effect in a simple supply chain: The impact of forecasting, lead times, and information. *Management Science* , *46* (3), 8.

5. Croson, R., & Donohue, K. (2006). Behavioral causes of the bullwhip effect and the observed value of inventory information. *Management Science* , *52* (3), 14.

6. Dejonckheere, J., Disney, S. M., Lambrecht, M. R., & Towill, D. R. (2004). The impact of information enrichment on the bullwhip effect in supply chains: a control engineering perspective. *European Journal of Operational Research* , *153* (4), 23.

7. Dhahri, I., & Chabchoub, H. (2007). Nonlinear goal programming models quantifying the bullwhip effect in supply chain based on ARIMA parameters. *European Journal of Operational Research* , *177* (3).

8. Disney, S. M., & Towill, D. R. (2003). The effect of vendor managed inventory (VMI) dynamics on the bullwhip effect in supply chains. *International Journal of Production Economics* , *85* (2), 16.

9. Disney, S. M., Towill, D. R., & Velde, W. v. (2004). Variance amplification and the golden ration in production and inventory control. *International Journal of Production Economics , 90* (3).

10. Gupta, S. C. (1992). *Fundamentals of statistics* (5th ed.). Mumbai: Himalaya Publishing House.

11. Hopp, W., & Spearman, M. (2000). *Factory physics* (2nd ed.). McGraw-Hill.

12. Huang, G. Q., Lau, J. S., & Mak, K. L. (2002). Web-based simulation portal for investigating impacts of sharing production information on supply chain dynamics from the perspective of inventory allocation. *Integrated Manufacturing Systems , 13* (5), 14.

13. James, C. (2006, January). Manufacturing's prescription for improving health care quality. *Health Care Strategic Management , 24* (1), p. 1.

14. Jiang, W., Au, T., & Tsui, K.-L. (2007). A statistical process control approach to business activity monitoring. *IIE Transactions , 39* (3), 15.

15. Lee, H. L., Padmanabhan, V., & Whang, S. (1997). Information distortion in a supply chain: the bullwhip effect. *Management Science , 43* (4), 13.

16. Lee, H. L., Padmanabhan, V., & Whang, S. (1997). The bullwhip effect in supply chains. *Sloan Management Review , 38* (3).

17. Lee, H. T., & Wu, J. C. (2006). A study on inventory replenishment policies in a two-echelon supply chain system. *Computers and Industrial Engineering , 51* (2).

18. Metters, R. (1997). Quantifying the bullwhip effect in supply chains. *Journal of Operations Management , 15* (2), 12.

19. Namhias, S. (2005). *Production and operations analysis* (5th ed.). McGraw-Hill.

20. Pfohl, H.-C., Cullmann, O., & Stolzle, W. (1999). Inventory management with statistical process control: simulation and evaluation. *Journal of Business Logistics*, *20* (1), 20.

21. Prins, J. (2003, June). *Shewhart X-bar and R and S Control Charts.* Retrieved March 2007, from NIST/SEMATECH e-Handbook of Statistical Methods: http://www.itl.nist.gov/div898/handbook/pmc/section3/pmc321.htm

22. Silver, E. A., Pyke, D. F., & Peterson, R. (1998). *Inventory Management and Production Planning and Scheduling* (3rd ed.). Wiley.

23. Wong, C. Y., El-Beheiry, M. M., Johansen, J., & Hvolby, H.-H. (2007). The implications of information sharing on bullwhip effects in a toy supply chain. *Internation Journal of Risk Assessment and Management*, *7* (1).

24. Xiong, G., & Helo, P. (2006). An application of cost effective fuzzy inventory controller to counteract demand fluctuation caused by bullwhip effect. *International Journal of Production Research*, *44* (24), 16.

APPENDIX A

Factors useful in the construction of control charts

Sample size	σ chart Factors for control limits		Range chart Factors for control limits	
n	B3	B4	D3	D4
2	0.000	3.267	0.000	3.267
3	0.000	2.566	0.000	2.575
4	0.000	2.266	0.000	2.282
5	0.000	2.089	0.000	2.115
6	0.080	1.970	0.000	2.004
7	0.118	1.882	0.076	1.924
8	0.185	1.815	0.136	1.864
9	0.239	1.761	0.184	1.816
10	0.284	1.716	0.223	1.777
11	0.321	1.679	0.256	1.744
12	0.354	1.646	0.284	1.716
13	0.382	1.618	0.308	1.692
14	0.406	1.594	0.329	1.671
15	0.428	1.572	0.348	1.652