12.215 Modern Navigation

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Today's Class
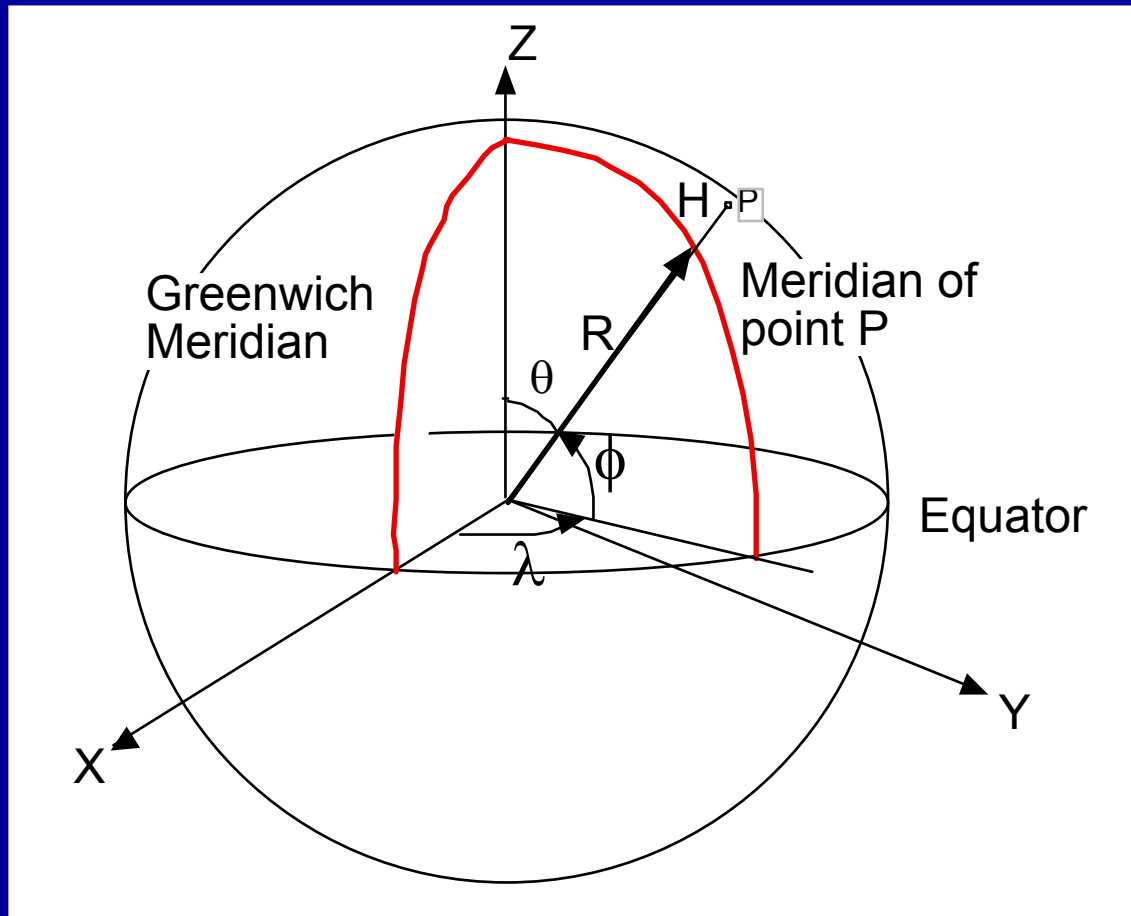# Latitude and Longitude

- Simple spherical definitions
- Geodetic definition: For an ellipsoid
- Astronomical definition: Based on direction of gravity
- Relationships between the types
- Coordinate systems to which systems are referred
- Temporal variations in systems

Simple Geocentric Latitude and Longitude

- The easiest form of latitude and longitude to understand is the spherical system:

- Latitude: Angle between the equatorial plane and the point. Symbol ϕ_c (in this class)

- Latitude is also the angle between the normal to the sphere and the equatorial plane

- Related term: co-latitude = 90°-latitude. Symbol θ_c (in this class). Angle from the Z-axis

- Longitude: Angle between the Greenwich meridian and meridian of the location. Symbol λ_c

Geocentric quantities

- Geocentric Latitude and Longitude
- Note: Vector to P is also normal to the sphere.

Geocentric relationship to XYZ

- One of the advantages of geocentric angles is that the relationship to XYZ is easy. R is taken to be radius of the sphere and H the height above this radius

$$\phi_c = \tan^{-1}(Z/\sqrt{X^2 + Y^2})$$
$$\lambda_c = \tan^{-1}(Y/X)$$
$$R + H_c = \sqrt{X^2 + Y^2 + Z^2}$$

$$X = (R + H_c)\cos\phi_c \cos\lambda_c$$
$$Y = (R + H_c)\cos\phi_c \sin\lambda_c$$
$$Z = (R + H_c)\sin\phi_c$$

Problem with Geocentric

- Geocentric measures are easy to work with but they have several serious problems
- The shape of the Earth is close to an bi-axial ellipsoid (i.e., an ellipse rotated around the Z-axis)
- The flattening of the ellipsoid is ~1/300 (1/298.257222101 is the defined value for the GPS ellipsoid WGS-84).
- Flattening is (a-b)/a where a is the semi-major axis and b is the semi-minor axis.
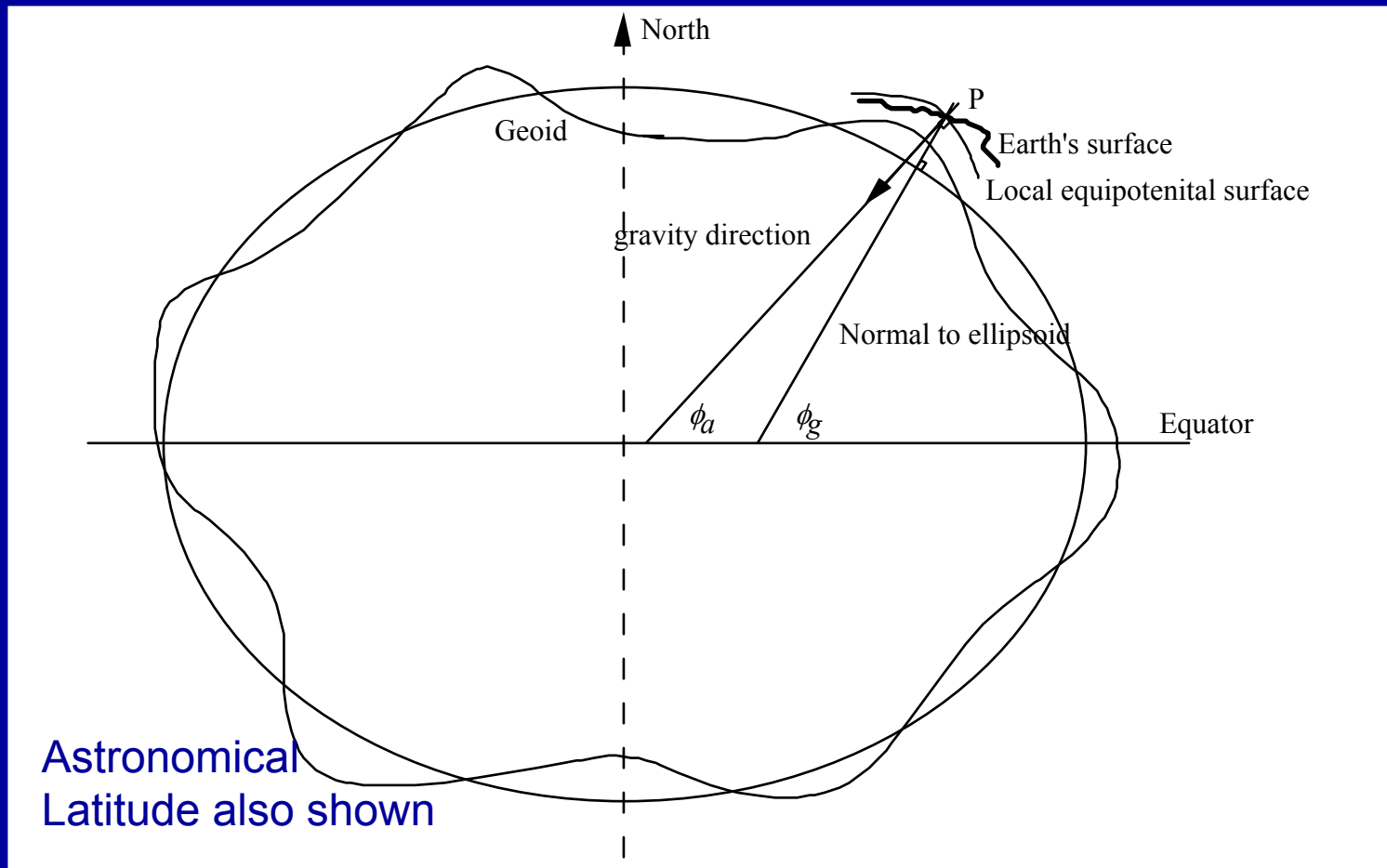- Since a=6378.137 km (WGS-84), a-b=21.384 km

Geocentric quantities

- If the radius of the Earth is taken as b (the smallest radius), then H_c for a site at sea-level on the equator would be 21km (compare with Mt. Everest 28,000feet~8.5km).

- Geocentric quantities are never used in any large scale maps and geocentric heights are never used.

- We discuss heights in more in next class and when we do spherical trigonometry we will use geocentric quantities.

Ellipsoidal quantities

- The most common latitude type seen is geodetic latitude which is defined as the angle between the normal to the ellipsoid and the equatorial plane. We denote with subscript g.

- Because the Earth is very close to a biaxial ellipsoid, geodetic longitude is the same as geocentric longitude (the deviation from circular in the equator is only a few hundred meters: Computed from the gravity field of the Earth).

Geodetic Latitude

North

Geoid

P

Earth's surface

Local equipotenital surface

gravity direction

Normal to ellipsoid

ϕ_a ϕ_g Equator

Astronomical
Latitude also shown

Relationship between ϕ_g and XYZ

- This conversion is more complex than for the spherical case.

$$X = (N + h_g)\cos(\phi_g)\cos(\lambda_g)$$

$$Y = (N + h_g)\cos(\phi_g)\sin(\lambda_g)$$

$$Z = [(1 - e^2)N + h_g]\sin(\phi_g)$$

where $e^2 = 2f - f^2$ and N (North-South radius of curvature) is

$$N^2 = a^2 / [1 - e^2\sin^2(\phi_g)]$$

Inverse relationship

- The inverse relationship between XYZ and geodetic latitude is more complex (mainly because to compute the radius of curvature, you need to know the latitude).

- A common scheme is iterative:

$$\mapsto N' = a/\sqrt{1 - e^2 \sin \phi'}$$

$$r' = \sqrt{X^2 + Y^2}[1 - e^2 N'/(N' + h')]$$

$$\phi' = \tan^{-1}(Z/r')$$

$$h' = \sqrt{X^2 + Y^2}/\cos \phi' - N' \ \text{ or } h' = Z/\sin \phi' - (1 - e^2)N'$$

iterate to $\mapsto$ until h' change is small

Coordinate Conversion: Cartesian (ECEF X, Y, Z) and Geodetic (Latitude, Longitude, and Height)

Direct Solution for Latitude, Longitude, and Height from X, Y, Z

This conversion is not exact and provides centimeter accuracy for heights < 1,000 km
(See Bowring, B. 1976. Transformation from spatial to geographical coordinates.
Survey Review, XXIII: pg. 323-327)

$$\phi = \operatorname{atan}\left(\frac{Z + e'^2 b \sin^3 \theta}{p - e^2 a \cos^3 \theta}\right)$$

$$\lambda = \operatorname{atan2}(Y, X)$$

$$h = \frac{p}{\cos(\phi)} - N(\phi)$$

where:

ϕ, λ, h = geodetic latitude, longitude, and height above ellipsoid

X, Y, Z = Earth Centered Earth Fixed Cartesian coordinates

and:

$$p = \sqrt{X^2 + Y^2} \qquad \theta = \operatorname{atan}\left(\frac{Za}{pb}\right) \qquad e'^2 = \frac{a^2 - b^2}{b^2}$$

$N(\phi) = a / \sqrt{1 - e^2 \sin^2 \phi}$ = radius of curvature in prime vertical

a = semi-major earth axis (ellipsoid equatorial radius)

b = semi-minor earth axis (ellipsoid polar radius)

$f = \dfrac{a - b}{a}$ = flattening

$e^2 = 2f - f^2$ = eccentricity squared

Closed form expression for small heights

From http://www.colorado.edu/geography/gcraft/notes/datum/gif/xyzllh.gif

Other items

- A discussion of geodetic datum and coordinate systems can be found at:
  http://www.colorado.edu/geography/gcraft/notes/datum/datum.html

- Geodetic longitude can be computed in that same way as for geocentric longitude

- Any book on geodesy will discuss these quanitities in more detail (also web searching on geodetic latitude will return many hits).

- The difference between astronomical and geodetic latitude and longitude is called "deflection of the vertical"

Astronomical latitude and longitude

- These have similar definitions to geodetic latitude and longitude except that the vector used is the direction of gravity and not the normal to the ellipsoid (see earlier figure).
- There is not direct relationship between XYZ and astronomical latitude and longitude because of the complex shape of the Earth's equipotential surface.
- In theory, multiple places could have the same astronomical latitude and longitude.
- As with the other measures, the values of depend on the directions of the XYZ coordinate axes.

Coordinate axes directions

- The origin of the XYZ system these days is near the center of mass of the Earth deduced from the gravity field determined from the orbits of geodetic satellites (especially the LAGEOS I and II satellites).

- The direction of Z-axis by convention is near the mean location of the rotation axis between 1900-1905. At the time, it was approximately aligned with the maximum moments of inertia of the Earth. (review:

http://dept.physics.upenn.edu/courses/gladney/mathphys/java/sect4/subsubsection4_1_4_2.html

Motion of rotation axis

- The rotation axis has moved about 10 m on average since 1900 (thought to be due to post-glacial rebound).

- It also moves in circle with a 10 m diameter with two strong periods: Annual due to atmospheric mass movements and 433-days which is a natural resonance frequency of an elastic rotating ellipsoid with a fluid core like the Earth.

Motion of rotation axis 1993-2001
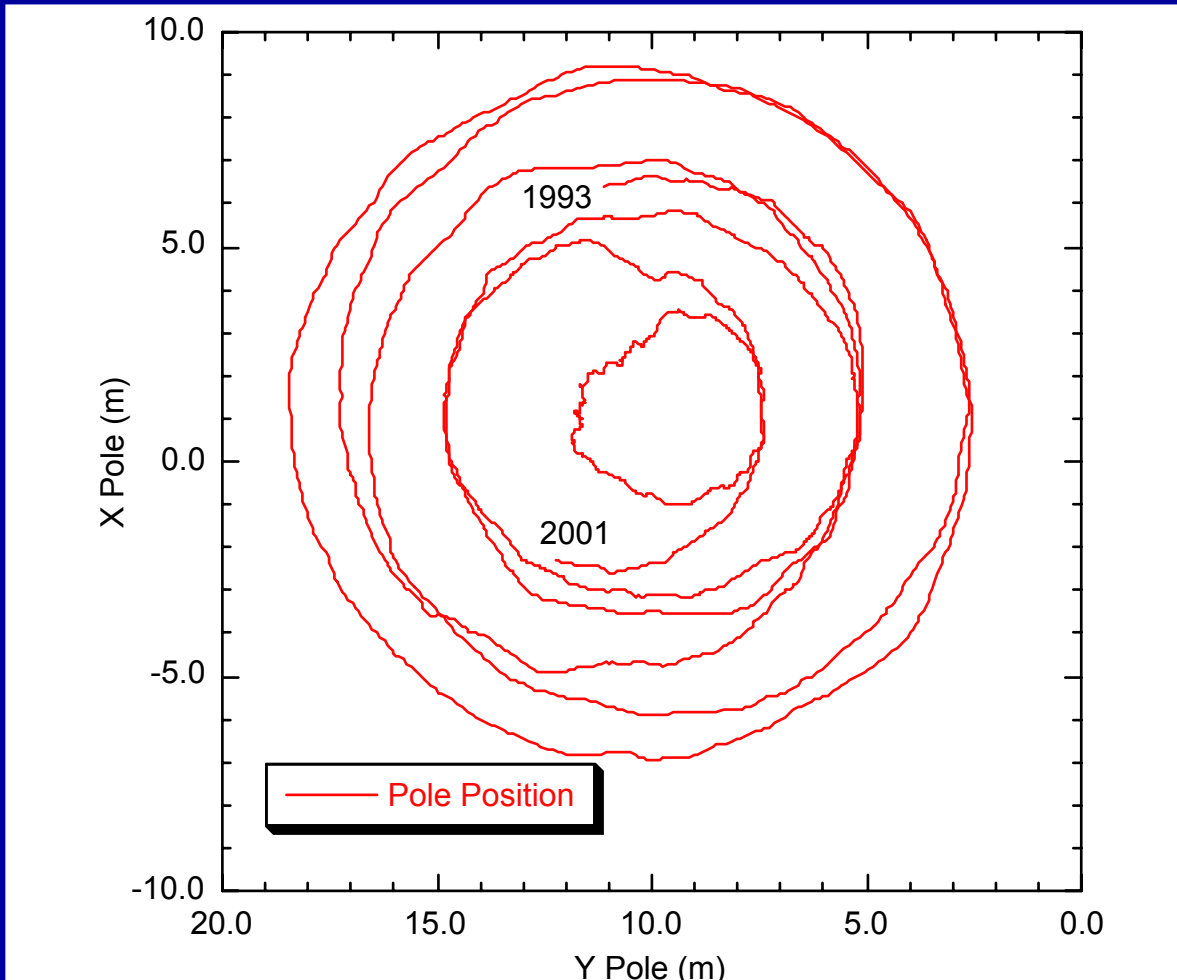
Note that the origin of this plot (0,0) is at the middle right hand edge

Rotation rate of the Earth

- To determine longitude we need to know the time difference between an event (such as a star crossing the meridian of a location) and the time that event occurs in Greenwich.

- One of the things that observatories such as Greenwich do, is measurements of celestial events and note the times they occur.

- With models of the changes in the rotation of the Earth in space, these measurements can be used to prediction when events will occur in the future.

Rotation rate of the Earth

- We examine these tabulations (called almanacs or ephemeredes) later in the course.
- One process that make these events non-predictable is changes in the rotation rate of the Earth.
- Measurements of the these changes are compiled and published by the International Earth Rotation Service (IERS: http://www.iers.org)
- We will examine these variations when we look at astronomical positioning.

Conventional definitions of coordinate systems

- Today we can use center of mass systems but in the past (prior to 1950) this was very difficult and so the origin and orientation of the axes was somewhat arbitrarily chosen.

- Different countries and at different times, the ellipsoidal parameters (a and b dimensions) were different

- The origin and orientation were set by adopting one location where the geodetic and astronomical latitude and longitude we set to be the same.

- For the US, this was Meades Ranch in mid-continent.

- The result of these arbitrary choices was that across country borders, geodetic coordinates could differ by several hundred meters. (Problematic when borders were defined by coordinates)

Summary

- The important things from this lecture are:
  - Definitions of different types of latitude and longitude
  - Mathematical relationships between these can XYZ coordinates
  - Definitions of Z axis of the Earth coordinates
  - Possible major differences in coordinates between countries and at different times (e.g., NAD-27 and NAD-83)
- Next lecture we will look at height systems.