12.215 Modern Navigation

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Review of last class

- Homework solutions for those that submitted should be handed back.
- Homework was broken into a number of small steps:
  - Determining the maximum observed angle to the sun and time this maximum occurred
  - Obtaining the mean index error
  - Computing maximum elevation to the sun
  - Computing the atmospheric bending correction
  - Computing the latitude
  - Computing the longitude

Today's class

- Map Projections:
  - Why projections are needed
  - Types of map projections
    - Classification by type of projection
    - Classification by characteristics of projection
  - Mathematics of map projections

Need for Map Projections

- Basic need is because the Earth's surface is curved and so it is not possible to represent on a flat surface with out some distortions
- Flat surfaces were needed so that people could carry maps with them (still a major use)
- With GPS, maps are now often represented in a computer in 3-D form or as ellipsoidal coordinates thus minimizing the distortion
- The amount of distortion depends on the area to be represented (over small areas the Earth is nearly flat).

Types of map projections

- Map projections are classified either by way the projection is made and the surface onto which it is projected or by the characteristics of the resultant projected maps.
- Some projection surfaces are planes, cones and cylinders (each of these surfaces can be un-wrapped into a flat surface)
- Some map projections are purely mathematical so that they can minimize distortions.
- We will deal (mathematically) with only projection from a spherical body. Most accurate map projections are projections from an ellipsoidal body.

Projection by characteristics

- The general characteristics of map projections are given by:
- **Conformality**: When the scale of a map at any point on the map is the same in any direction, the projection is conformal. Meridians (lines of longitude) and parallels (lines of latitude) intersect at right angles. Shape is preserved locally on conformal maps.
- **Distance:** A map is equidistant when it portrays distances from the center of the projection to any other place on the map.
- **Direction**: A map preserves direction when azimuths (angles from a point on a line to another point) are portrayed correctly in all directions.
- **Area:** When a map portrays areas over the entire map so that all mapped areas have the same proportional relationship to the areas on the Earth that they represent, the map is an equal-area map.

Scale characteristics

- **Scale:** Scale is the relationship between a distance portrayed on a map and the same distance on the Earth.
- A large scale map shows a small area with a large amount of detail (eg. 1:25000)
- A small scale map shows a large area with a small amount of detail (eg. 1:500000)
- The interpretation of the scale is 1:25000 is 1 unit on the map represents 25000 units on the Earth
- On many maps the scale changes across the map.
- Usually the scale is shown graphically somewhere on the map and if the scale varies across the map, the scale should indicate where it is applicable and the changes in scale across the map.

Large/small scale map

http://www.mapblast.com/

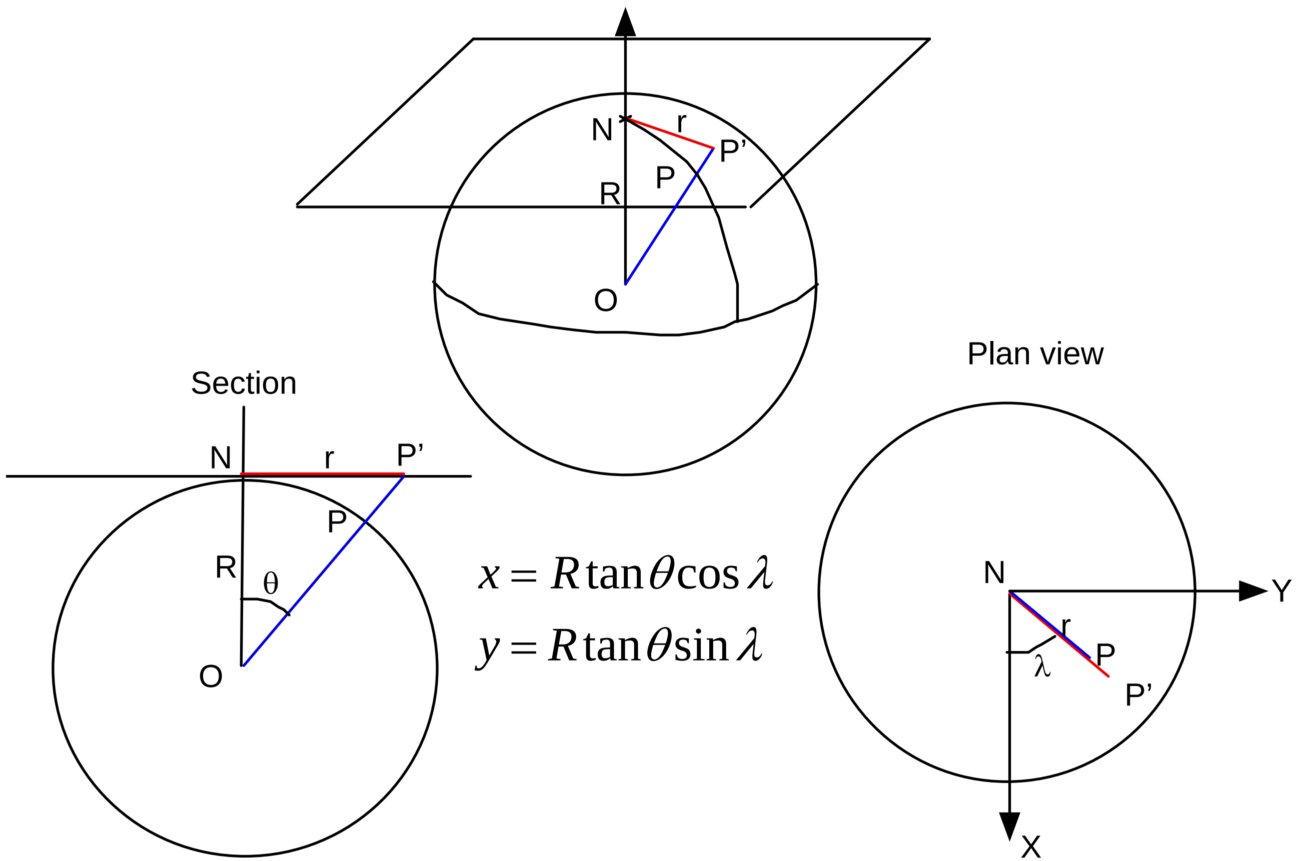
Projection type by surface

- Projections are often referred to by the type of surface that the projection is made on to.
- The three main surfaces are:
  - Plane (often referred to a Azimuthal Projections)
  - Cylindrical (Mercator is probably the most famous)
  - Conic projection
- The characteristics of the map are set by how the surface contacts the Earth (e.g., a Plane may be tangential to the surface or it may cut through the Earth at some depth.

General characteristics

- All projections can be written in a form that allows plane coordinates x and y to be written as functions of ϕ and λ: $x = f(\phi, \lambda)$ and $y = g(\phi, \lambda)$.
- The exact forms of the functions f and g depend on the projection. For the geometric projections from a sphere, these can be written as simple trigonometric functions as shown in the next few slides.
- More complicated projections can involve more complicated and sometime approximate formulas especially when ellipsoidal coordinates are projected (such as the Universal Transverse Mercator (UTM) projection which is used for many US maps
- On many maps UTM coordinates are given (also called grid coordinates) and GPS receivers can normally be set to output and interpret these types of coordinates.

Plane projection maps



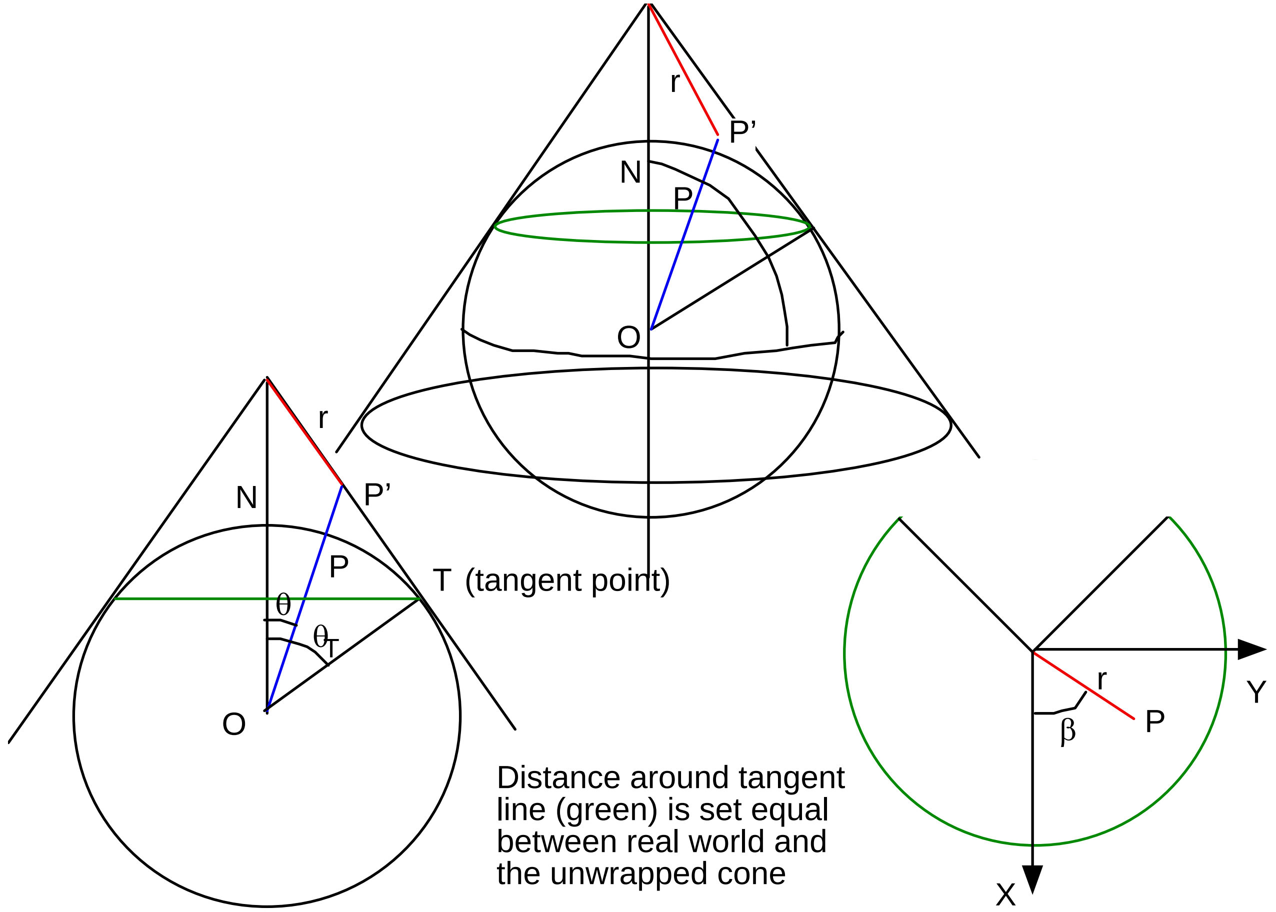
$$x = R \tan\theta \cos\lambda$$

$$y = R \tan\theta \sin\lambda$$

Conical Projection

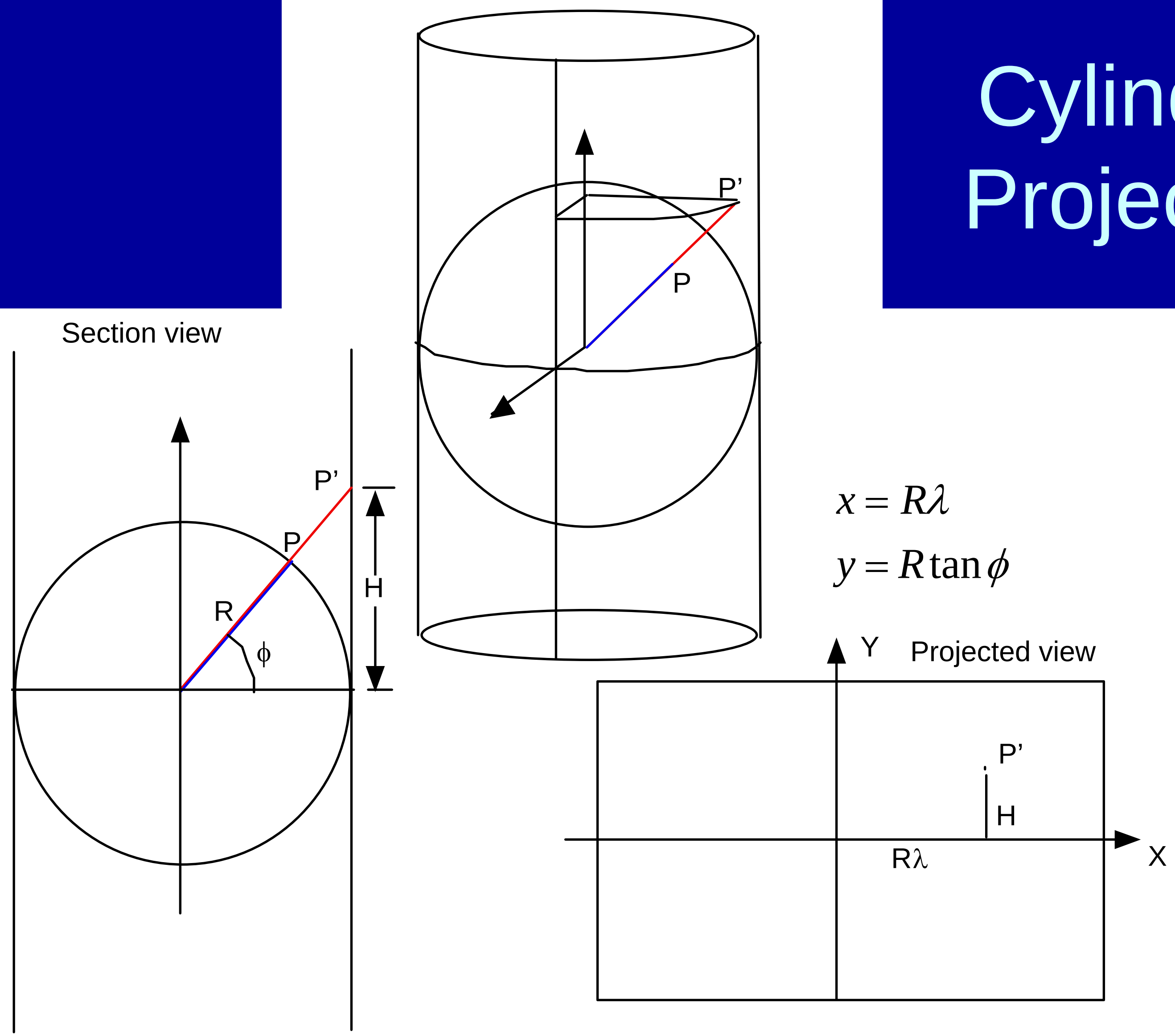
- The equations to solve the conical projection will be set as a homework exercise.
- In a conical projection, points are projected radially onto the cone. The cone is then “cut” and unwrapped to form the projection.
- In the case shown, the cone’s dimensions are set by specifying the co-latitude of the tangent point of the cone (θ_T). The distance around this part of the cone is set equal to the distance around the small circle on the Earth. This allows the relationship between longitude and the angle around the cut cone (β) to be determined.

Conical Projections



Distance around tangent line (green) is set equal between real world and the unwrapped cone

Cylindrical Projections

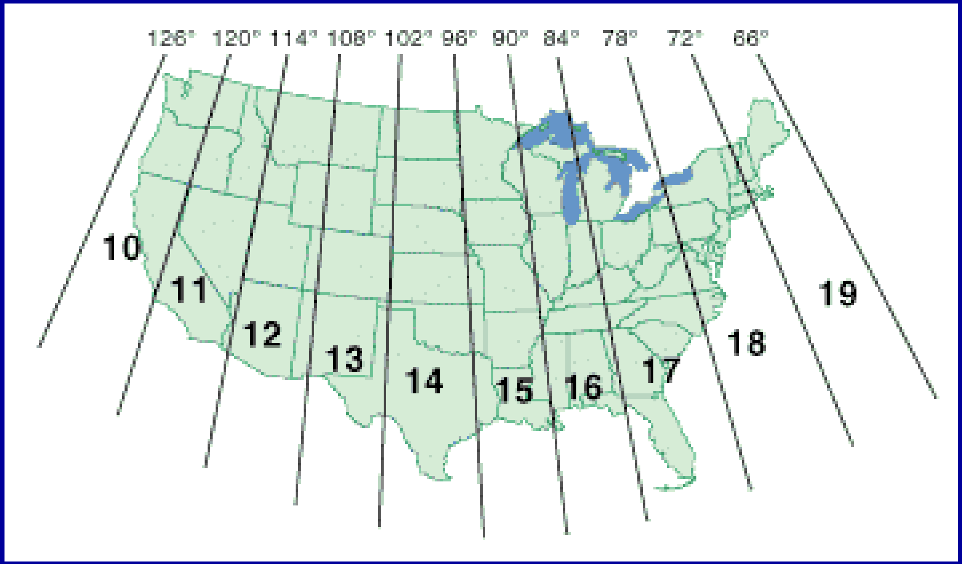


$$x = R\lambda$$

$$y = R\tan\phi$$

UTM coordinates

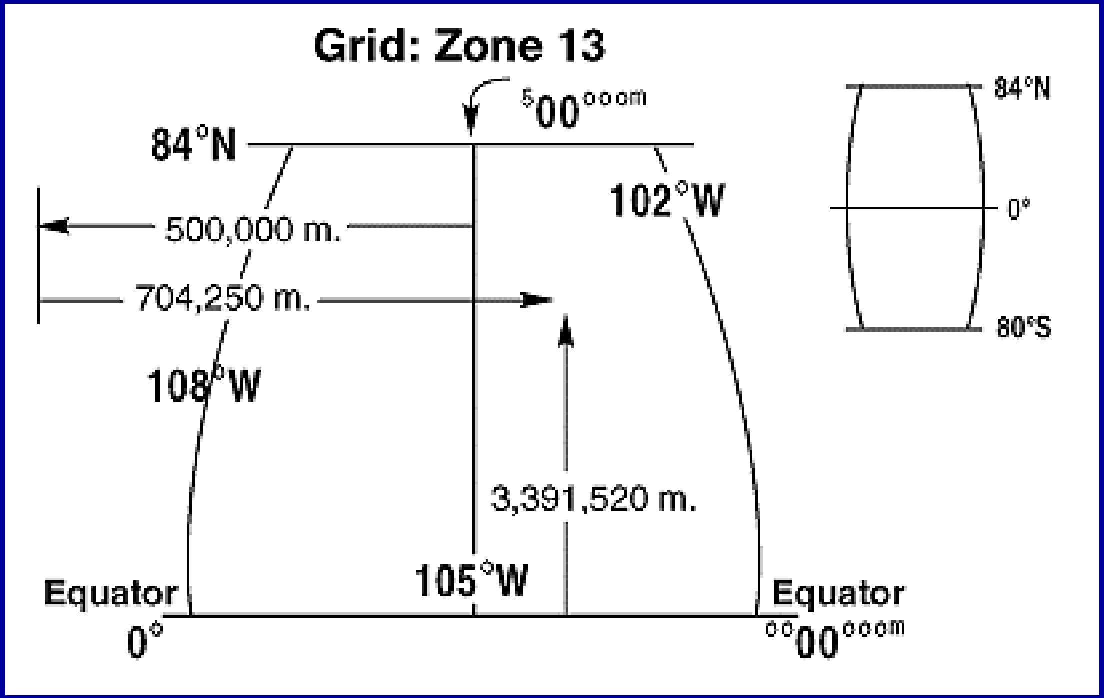
- The Universal Transverse Mercator (UTM) projection is most commonly used in the US (and many other mid-latitude to equatorial countries)
- This is an ellipsoidal projection that divides the world into numbered zones in longitude. For the US these zones are:



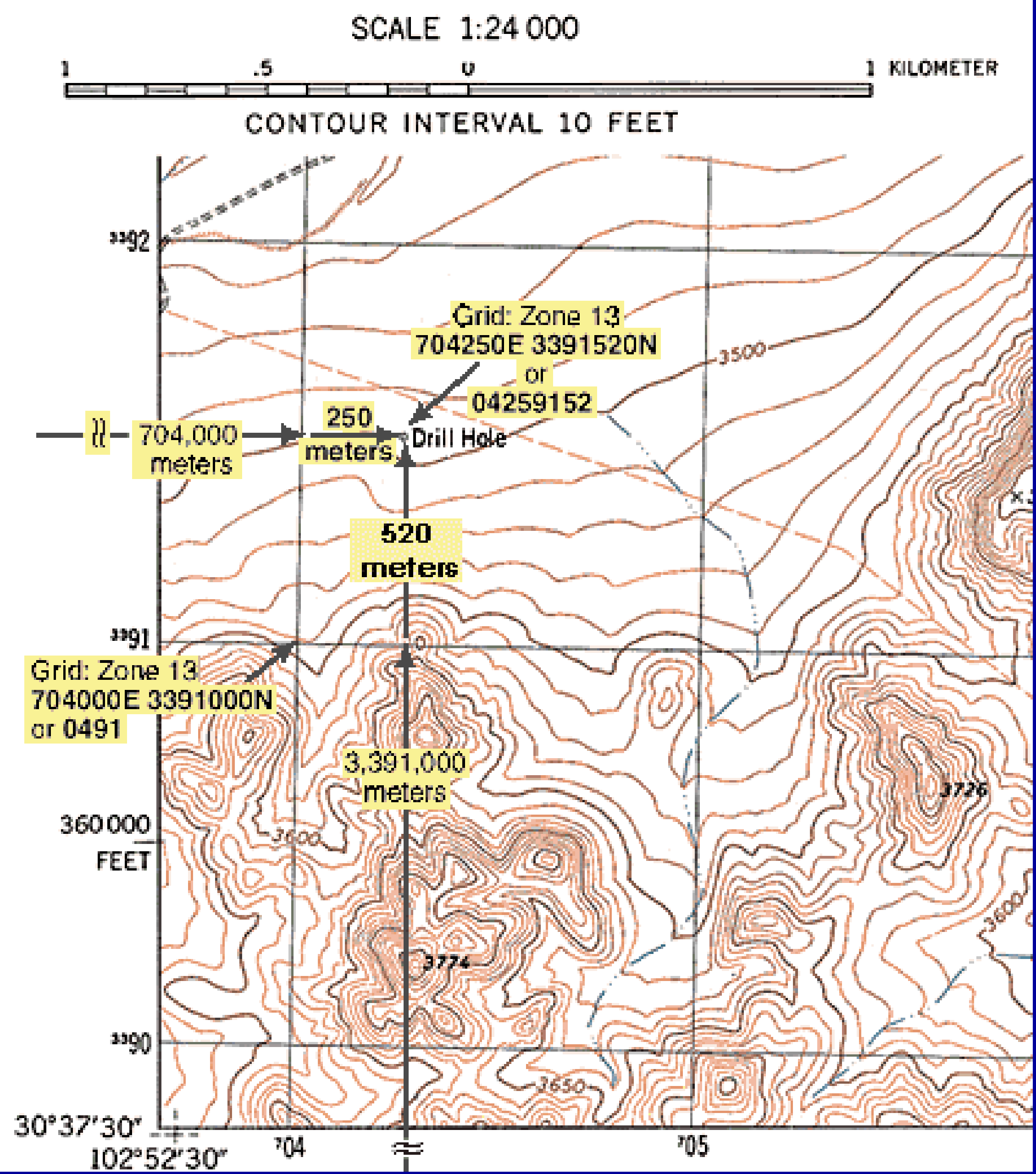
UTM coordinates

- Within each of the zones, the latitude and longitude difference from the central meridian is used to compute the UTM coordinates.

•These coordinates are given as Northing and Easting. (The east coordinates have 500,000 added so that they are not negative west of the central meridian)



Example of using UTM coordinates



Notes that go with previous figure

- UTM coordinate maps usually have notes that describe the projection in more detail
- Details given on datum (NAD-27 in this case)
- More details at http://www.maptools.com/UsingUTM/

5446 III SW
(SIERRA MADERA)

Mapped, edited, and published by the Geological Survey

Control by USGS and NOS/NOAA

Topography by photogrammetric methods from aerial photographs
taken 1974. Field checked 1975. Map edited 1980

Projection and 10,000-foot grid ticks: Texas
coordinate system, central zone (Lambert conformal conic)
1000-meter Universal Transverse Mercator grid, zone 13
1927 North American datum
To place on the predicted North American Datum 1983
move the projection lines 14 meters south and
41 meters east as shown by dashed corner ticks

Fine red dashed lines indicate selected fence lines

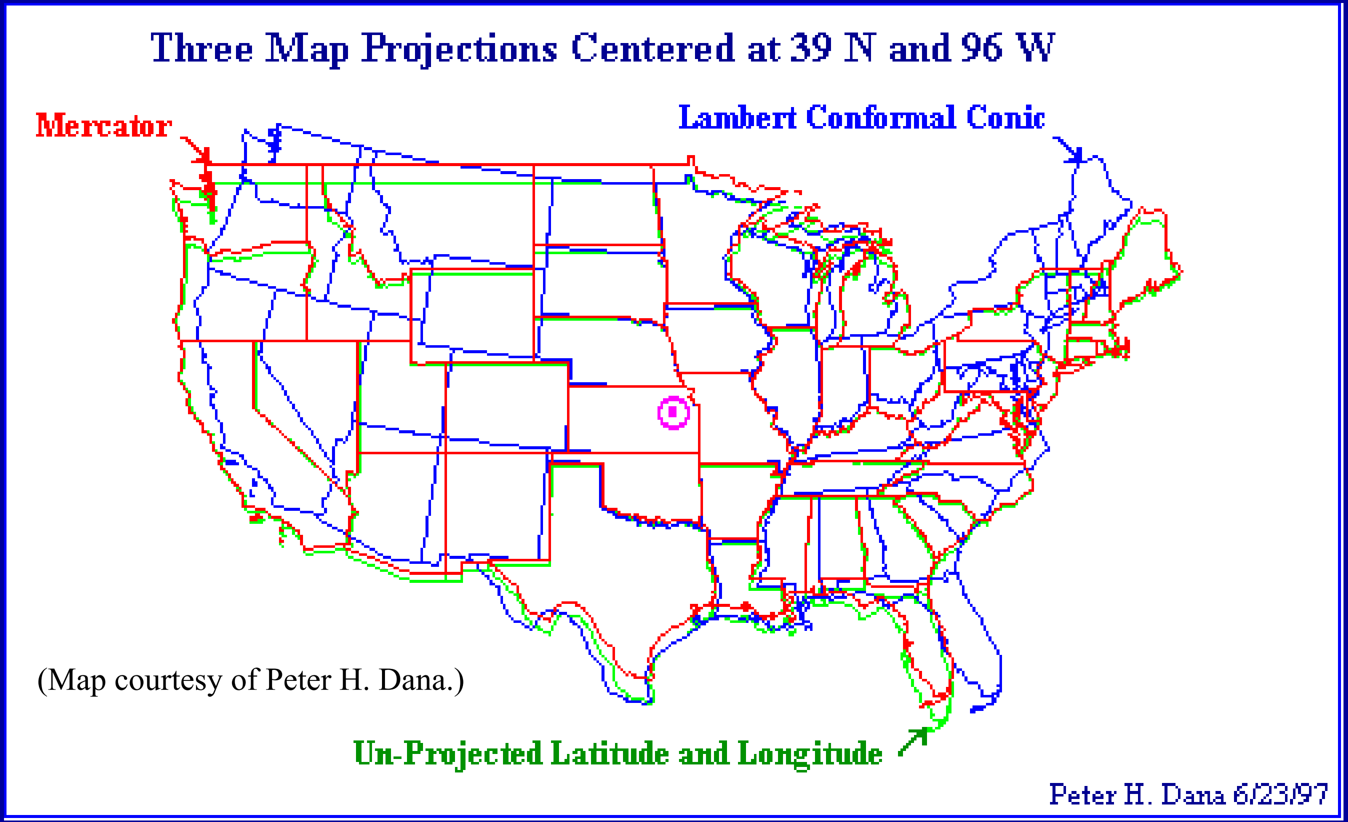
UTM coordinates

- Software for converting latitude and longitude to UTM coordinates is available at:
- ftp://ftp.ngs.noaa.gov/pub/pcsoft/utms/
- This software (available as PC executable and as Fortran source code) allows conversion to and from UTM and allows different ellipsoids to be used used
- NAD27 (most common on paper maps) use the Clarke 1866 ellipsoid while NAD83 (new North American Datum) using the WGS84 ellipsoid.

Map Projection resources

- Many web sites cover map projections. Some of the better ones are:
  www.colorado.edu/geography/gcraft/notes/mapproj/mapproj_f.html
  mac.usgs.gov/mac/isb/pubs/MapProjections/projections.html
  www.nationalgeographic.com/features/2000/exploration/projections/
- The mathematics involved in many projections can be found at
  mathworld.wolfram.com/MapProjection.html
- If time permits these sites can be examined for content in class.

North America under different projections



Summary

- Examined different classes of map projections and the mathematics behind dome of them
- When using a (paper) map, the important things to note are:
  - The ellipsoid used if an ellipsoidal projection
  - The nature of the projection itself
- Homework #3 due Monday Oct 28