12.215 Modern Navigation

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Review of last Class

- Review of linear Algebra. Class will be based on the book "Linear Algebra, Geodesy, and GPS", G. Strang and K. Borre, Wellesley-Cambridge Press, Wellesley, MA, pp. 624, 1997
- Topics to be covered will be those later in the course
- General areas are:
  - Vectors and matrices
  - Solving linear equations
  - Vector Spaces
  - Eigenvectors and values
  - Rotation matrices

Today's class

- Analysis of Sextant measurements
- Homework was broken into a number of small steps:
  - Determining the maximum observed angle to the sun and time this maximum occurred
  - Obtaining the mean index error
  - Computing maximum elevation to the sun
  - Computing the atmospheric bending correction
  - Computing the latitude
  - Computing the longitude

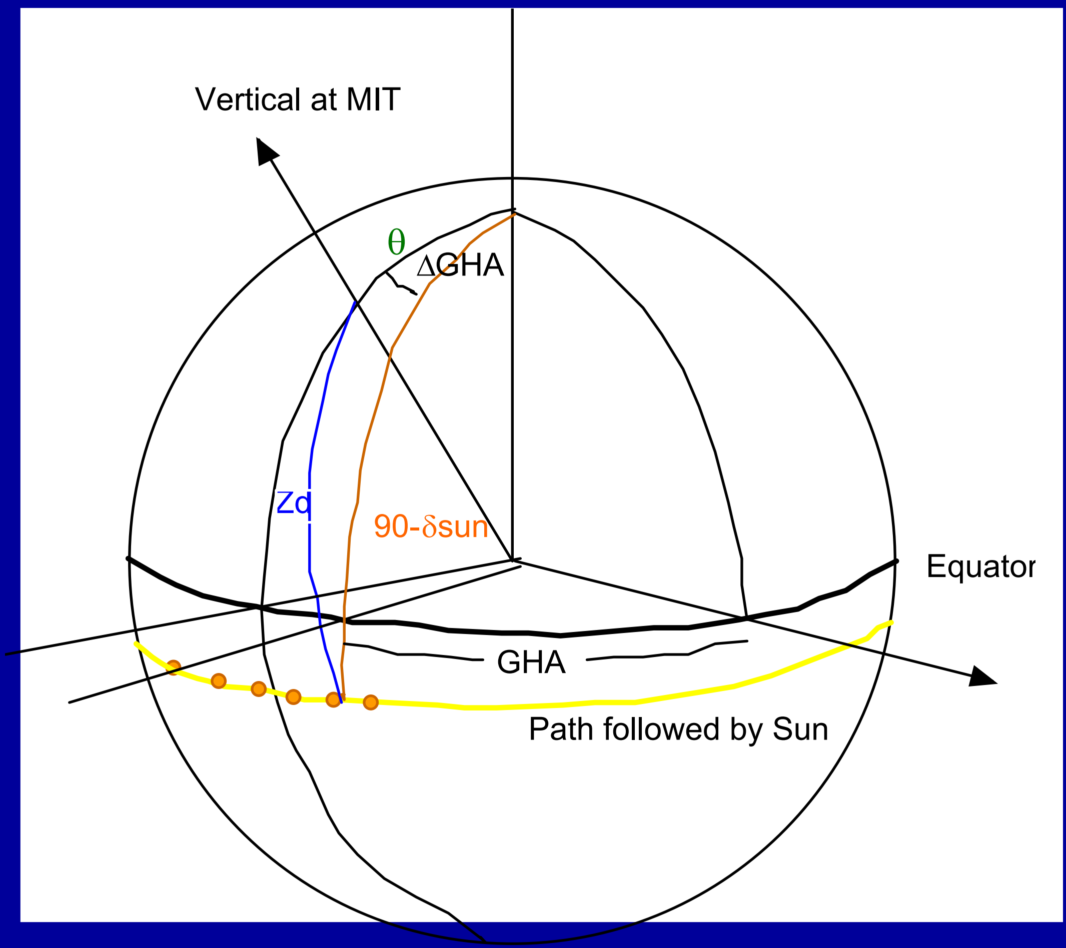
Simpler parts of calculation

- **Mean of index error:** Simply the sum of the values divided by the number of values
- Also we can compute a standard deviation about the mean (also called a root-mean-square (RMS) scatter). This gives is an indication of how well we can make measurements with the sextant. The standard deviation of our measurements was 1.5’
- We use this today and in later lectures we will show how to use this to allow us to estimate the uncertainty of our final latitude and longitude determination.

Atmospheric refraction

- We can use the simple formula given in class or we can look up the values in the Nautical Almanac.
- The formula result is slightly greater than 1’ since tan(ε) ~ 1
- Using the almanac we can explore how much this value will vary due to atmospheric conditions.
- (For latitude determination, atmospheric refraction becomes a bigger problem the closer we get to the pole where the meridian crossing elevation angle will be much smaller. It will also be a bigger problem in mid-winter than in mid-summer).

Geometry of measurement



- Spherical trigonometry that we can solve (we interpret on the meridian and so easy)

Spherical Trigonometry

- Based on the figure, we can write the solution for the zenith distance to the sun:

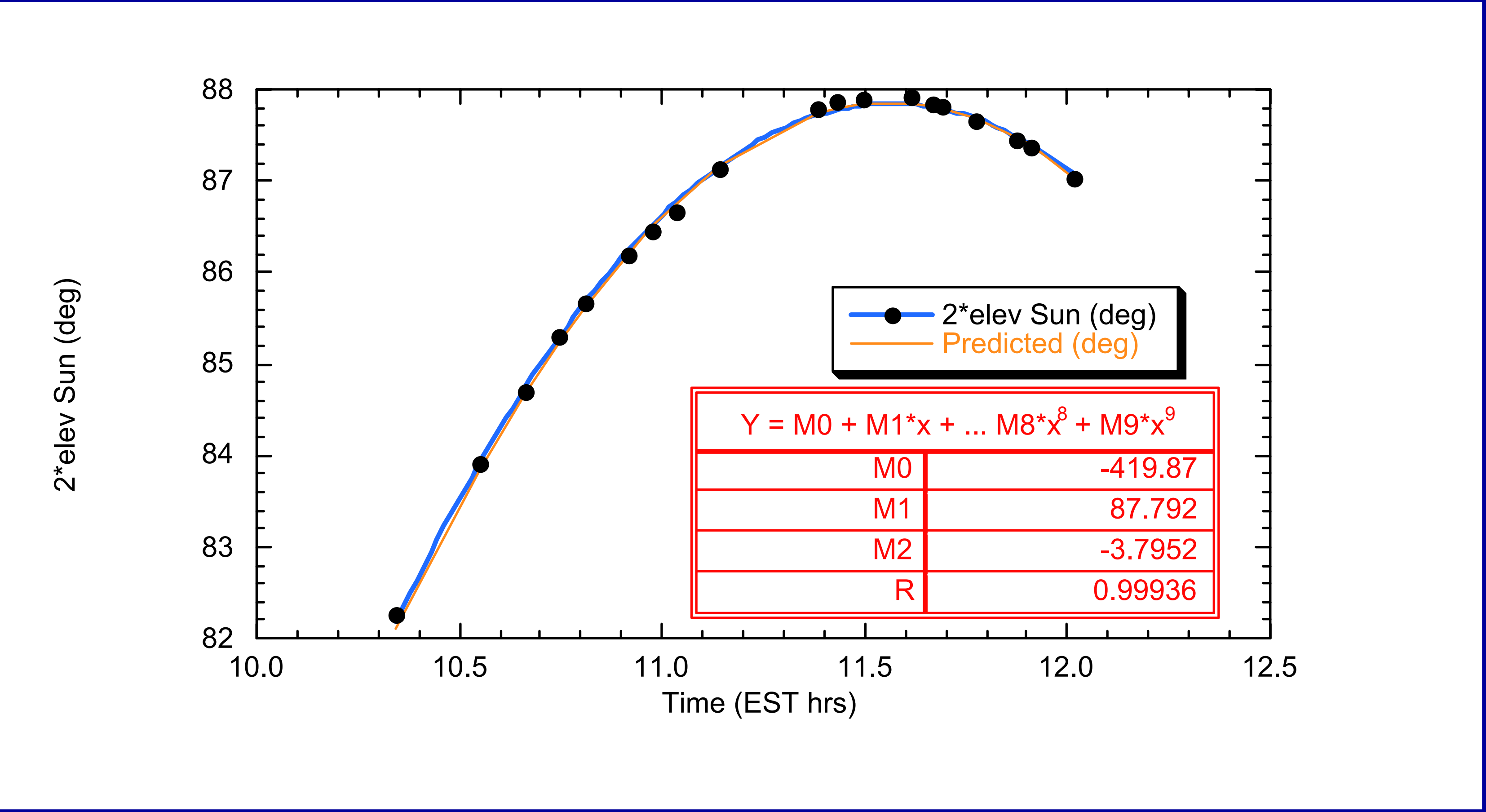
$$\cos Zd = \cos\theta\cos(90-\delta) + \sin\theta\sin(90-\delta)\cos(\Delta GHA)$$

- If we assume we know our latitude in longitude then we can compute the expected variations in the zenith distance to the Sun
- In addition, since we measured 2*(elevation to sun+refraction)+ index error , we can include this in what is called a “forward model”

Results of forward model

- GPS latitude 42.35990; longitude -71.0890
- Declination of Sun at MIT meridian crossing -3.68833
- Interpolating the Almanac GHA, UT meridian crossing 16.56125 hrs (-5 hrs to EST)
- $\cos\theta\cos(90-\delta)=-0.043344$; $\sin\theta\sin(90-\delta)=0.7373966$
- Mean index error -0.07166
- The forward model can be computed and compared to measurements.

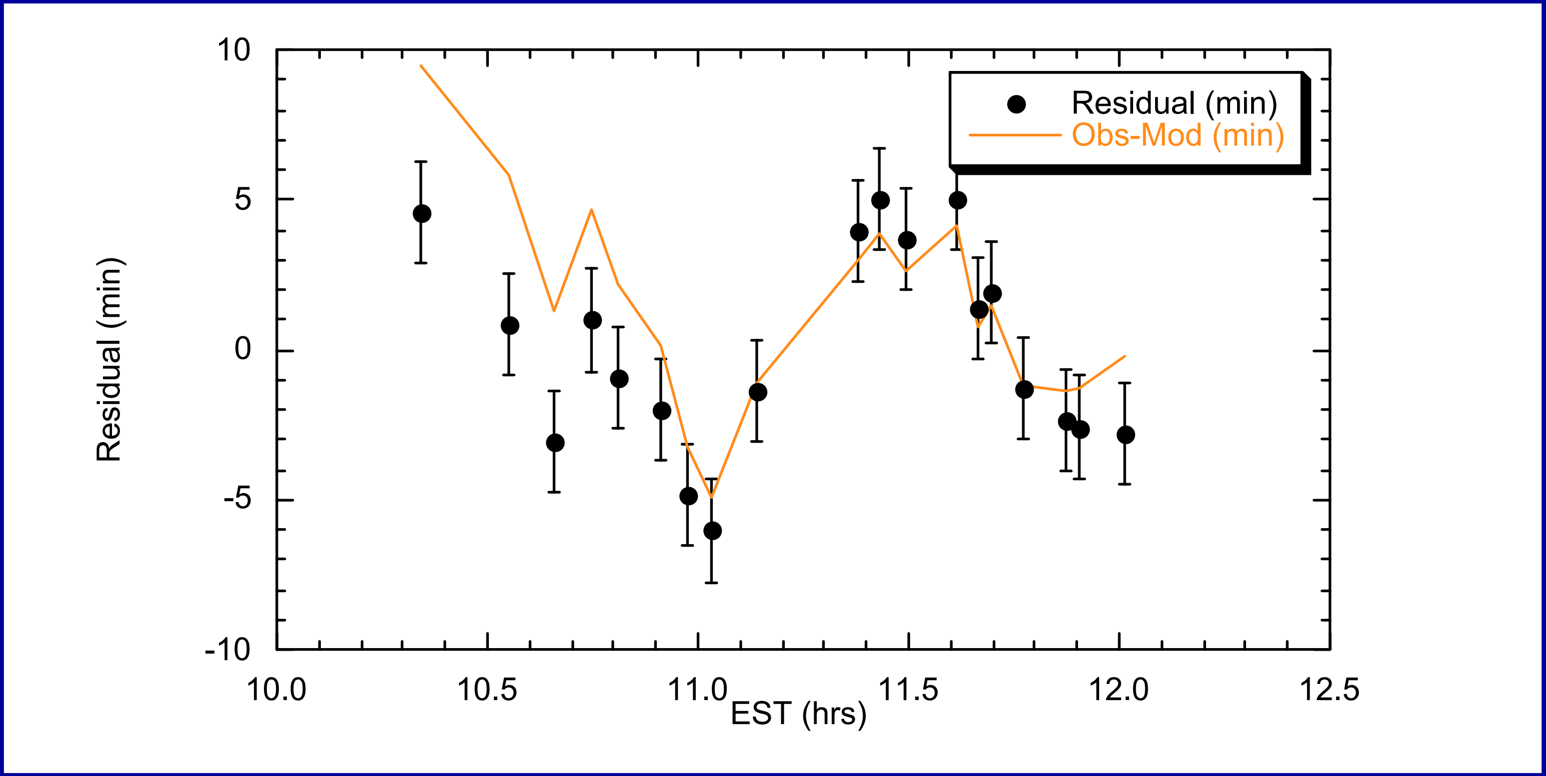
Forward Model Calculation



Comparison

- Agreement looks good but when totals are displaced the results can be be deceptive in that the details can not been seen.
- Normal to look at the difference between the observations and the model
- On the quadratic fit residuals we show “error bars” based on the index measurements. These are computed from sqrt(Sum(residuals^2)/(number-1)). Also called Root-mean-square (RMS) scatter
- In class we will vary the parameters of the model to see there effect on the fit to the data.

Residuals (Quadratric and Model)



The lines on the back circles are error bars based on index measurements

Some neglected effects

- The forward model does not account for the changing of the declination of the sun during the measurements. The declination changes by 1’ per hour. This could explain up to 2’ minutes of difference.
- The Sun also orbits during the measurements than therefore moves a little more than 15 deg in 1 hour (15.0033 degrees)
- We can refine the model to test these ideas.
- We can also change model parameters

Summary:

- Today we explored the latitude and longitude problem in more detail looking at the actual data collected with the sextant.
- Introduced the notion of a forward model for comparing with data and varying the parameters of the model to better match the observations.
- Differences between observations and models can be quantified with an estimated standard deviation or RMS scatter.
- These issues are returned to when we address statistics and estimation.