12.215 Homework #4 **Due Monday November 18, 2002**

Statistics and Estimation

Question 1: Using the data from Homework 2, estimate using your own estimator (not polyfit) the coefficients of the quadratic polynomial that best fit the data in a least squares sense.
(a) Write the equation for the polynomial. What are the observations and unknown parameters in the polynomial (5-points).
(b) Write the above set of equations in matrix form (5-points).
(c) Form the least squares estimator and solve for the coefficients. You may use the Matlab matrix inversion routine inv and/or a calculator matrix inversion to solve the system of equations (10 points).

Question 2: Estimate the standards deviation of the errors in the measurements using the differences between the observed values and the polynomial fit. (10-points)

Question 3: What is the probability that the 8^{th} measurement (time 11 hr 01 m 57 sec) Measurement 86^{o} 40.0') differs from the polynomial fit due to random error assuming that the noise in the measurements is Gaussianly distributed and the data standard deviation computed in question 2 (10-points).

Question 4: Estimate the standard deviation of the peak in the polynomial (i.e., the covariance matrix of the estimate of the maximum value and the time at which the maximum occurs) based on the least squares estimate in Question 1**. Hint:** You need to write a linear relationship between the value of the polynomial at the peak and the time of the peak in terms of the coefficients of the polynomial. Since the least squares estimator gives you the covariance matrix of the coefficients of the polynomial, including correlations, you can use propagation of variance to obtain the result (20-points).

Question 5: In lecture 10, the equations of the forward model were given that related the latitude and longitude of MIT to the measured angles to the Sun. Using this forward model, compute by liberalized least squares the latitude and longitude of MIT. Using the data noise determined in Question 2, obtain the variance covariance matrix for the latitude and longitude estimates and compare with the results from question 4 (40-points).