Chapter 7

Endogenous Growth II: R&D and Technological Change

7.1 Expanding Product Variety (Romer)

- There are three production sectors in the economy: A final-good sector, an intermediate good sector, and an R&D sector.

- The final good sector is perfectly competitive and thus makes zero profits. Its output is used either for consumption or as input in each of the other two sector.

- The intermediate good sector is monopolistic. There is product differentiation. Each intermediate producer is a quasi-monopolist with respect to his own product and thus enjoys positive profits. To become an intermediate producer, however, you must first acquire a "blueprint" from the R&D sector. A "blueprint" is simply the technology or know-how for transforming final goods to differentiated intermediate inputs.

- The R&D sector is competitive. Researchers produce "blueprints", that is, the technology for producing an new variety of differentiated intermediate goods. Blueprints are protected by perpetual patents. Innovators auction their blueprints to a large number of potential buyers, thus absorbing all the profits of the intermediate good sector. But there is free entry in the R&D sector, which drive net profits in that sector to zero as well.

7.1.1 Technology

- The technology for final goods is given by a neoclassical production function of labor L and a composite factor Z:

$$Y_t = F(\mathcal{X}_t, L_t) = A(L_t)^{1-\alpha}(\mathcal{X}_t)^{\alpha}.$$

The composite factor is given by a CES aggregator of intermediate inputs:

$$\mathcal{X}_t = \left[\int_0^{N_t} (X_{t,j})^{\varepsilon} dj\right]^{1/\varepsilon},$$

where N_t denotes the number of different intermediate goods available in period t and $X_{t,j}$ denotes the quantity of intermediate input j employed in period t.

- In what follows, we will assume $\varepsilon = \alpha$, which implies

$$Y_t = A(L_t)^{1-\alpha} \int_0^{N_t} (X_{t,j})^{\alpha} dj$$

Note that $\varepsilon = \alpha$ means that the elasticity of substitution between intermediate inputs is 1 and therefore the marginal product of each intermediate input is independent of the quantity of other intermediate inputs:

$$\frac{\partial Y_t}{\partial X_{t,j}} = \alpha A \left(\frac{L_t}{X_{t,j}}\right)^{1-\alpha}.$$

More generally, intermediate inputs could be either complements or substitutes, in the sense that the marginal product of input j could depend either positively or negatively on X_t.

- We will interpret intermediate inputs as capital goods and therefore let aggregate "capital" be given by the aggregate quantity of intermediate inputs:

$$K_t = \int_0^{N_t} X_{t,j} dj.$$

- Finally, note that if $X_{t,j} = X$ for all j and t, then

$$Y_t = AL_t^{1-\alpha} N_t X^\alpha$$

and

$$K_t = N_t X,$$

implying

$$Y_t = A(N_t L_t)^{1-\alpha} (K_t)^\alpha$$

or, in intensive form, $y_t = AN_t^{1-\alpha} k_t^\alpha$. Therefore, to the extent that all intermediate inputs are used in the same quantity, the technology is linear in knowledge N and capital K. Therefore, if both N and K grow at a constant rate, as we will show to be the case in equilibrium, the economy will exhibit long run growth like in an Ak model.

7.1.2 Final Good Sector

- The final good sector is perfectly competitive. Firms are price takers.

- Final good firms solve

$$\max \; Y_t - w_t L_t - \int_0^{N_t} (p_{t,j} X_{t,j}) dj$$

where w_t is the wage rate and $p_{t,j}$ is the price of intermediate good j.

- Profits in the final good sector are zero, due to CRS, and the demands for each input are given by the FOCs

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)\frac{Y_t}{L_t}$$

and

$$p_{t,j} = \frac{\partial Y_t}{\partial X_{t,j}} = \alpha A \left(\frac{L_t}{X_{t,j}}\right)^{1-\alpha}$$

for all $j \in [0, N_t]$.

7.1.3 Intermediate Good Sector

- The intermediate good sector is monopolistic. Firms understand that they face a downward sloping demand for their output.

- The producer of intermediate good j solves

$$\max \ \Pi_{t,j} = p_{t,j}X_{t,j} - \kappa(X_{t,j})$$

subject to the demand curve

$$X_{t,j} = L_t \left(\frac{\alpha A}{p_{t,j}}\right)^{\frac{1}{1-\alpha}},$$

where $\kappa(X)$ represents the cost of producing X in terms of final-good units.

- We will let the cost function be linear:

$$\kappa(X) = X.$$

The implicit assumption behind this linear specification is that technology of producing intermediate goods is identical to the technology of producing final goods. Equivalently, you can think of intermediate good producers buying final goods and transforming them to intermediate inputs. What gives them the know-how for this transformation is precisely the blueprint they hold.

- The FOCs give

$$p_{t,j} = p \equiv \frac{1}{\alpha} > 1$$

for the optimal price, and

$$X_{t,j} = xL$$

for the optimal supply, where

$$x \equiv A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}.$$

- Note that the price is higher than the marginal cost ($p = 1/\alpha > \kappa'(X) = 1$), the gap representing the mark-up that intermediate-good firms charge to their customers (the final good firms). Because there are no distortions in the economy other than monopolistic competition in the intermediate-good sector, the price that final-good firms are willing to pay represents the social product of that intermediate input and the cost that intermediate-good firms face represents the social cost of that intermediate input. Therefore, the mark-up $1/\alpha$ gives the gap between the social product and the social cost of intermediate inputs. (*Hint:* The social planner would like to correct for this distortion. How?)

- The resulting maximal profits are

$$\Pi_{t,j} = \pi L$$

where

$$\pi \equiv (p-1)x = \tfrac{1-\alpha}{\alpha} x = \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}.$$

7.1.4 The Innovation Sector

- The present value of profits of intermediate good j from period t and on is given by

$$V_{t,j} = \sum_{\tau=t} \frac{q_\tau}{q_t} \Pi_{\tau,j}$$

or recursively

$$V_{t,j} = \Pi_{t,j} + \frac{V_{t+1,j}}{1 + R_{t+1}}$$

- We know that profits are stationary and identical across all intermediate goods: $\Pi_{t,j} = \pi L$ for all t, j. As long as the economy follows a balanced growth path, we expect the interest rate to be stationary as well: $R_t = R$ for all t. It follows that the present value of profits is stationary and identical across all intermediate goods:

$$V_{t,j} = V = \frac{\pi L}{R}.$$

Equivalently, $RV = \pi L$, which has a simple interpretation: The opportunity cost of holding an asset which has value V and happens to be a "blueprint", instead of investing in bonds, is RV; the dividend that this asset pays in each period is πL; arbitrage then requires the dividend to equal the opportunity cost of the asset, namely $RV = \pi L$.

- New blueprints are also produced using the same technology as final goods. In effect, innovators buy final goods and transform them to blueprints at a rate $1/\eta$.

- Producing an amount ΔN of new blueprints costs $\eta \cdot \Delta N$, where $\eta > 0$ measures the cost of R&D in units of output. On the other hand, the value of these new blueprints is $V \cdot \Delta N$, where $V = \pi L/R$. Net profits for a research firm are thus given by

$$(V - \eta) \cdot \Delta N$$

Free entry in the sector of producing blueprints then implies

$$V = \eta.$$

7.1.5 Households

- Households solve

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t. \quad c_t + a_{t+1} \leq w_t + (1 + R_t)a_t$$

- As usual, the Euler condition gives

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta(1 + R_{t+1}).$$

And assuming CEIS, this reduces to

$$\frac{c_{t+1}}{c_t} = \left[\beta(1 + R_{t+1})\right]^{\theta}.$$

7.1.6 Resource Constraint

- Final goods are used either for consumption by households, or for production of intermediate goods in the intermediate sector, or for production of new blueprints in the innovation sector. The resource constraint of the economy is given by

$$C_t + K_t + \eta \cdot \Delta N_t = Y_t,$$

where $C_t = c_t L$, $\Delta N_t = N_{t+1} - N_t$, and $K_t = \int_0^{N_t} X_{t,j} dj$.

7.1.7 General Equilibrium

- Combining the formula for the value of innovation with the free-entry condition, we infer $\pi L/R = V = \eta$. It follows that the equilibrium interest rate is

$$R = \frac{\pi L}{\eta} = \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} L/\eta,$$

  which verifies our earlier claim that the interest rate is stationary.

- The resource constraint reduces to

$$\frac{C_t}{N_t} + \eta \cdot \left[\frac{N_{t+1}}{N_t} - 1\right] + X = \frac{Y_t}{N_t} = AL^{1-\alpha}X^{\alpha},$$

  where $X = xL = K_t/N_t$. It follows that C_t/N_t is constant along the balanced growth path, and therefore C_t, N_t, K_t, and Y_t all grow at the same rate, γ.

- Combining the Euler condition with the equilibrium condition for the real interest rate, we conclude that the equilibrium growth rate is given by

$$1 + \gamma = \beta^{\theta}\left[1 + R\right]^{\theta} = \beta^{\theta}\left[1 + \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} L/\eta\right]^{\theta}$$

- Note that the equilibrium growth rate of the economy decreases with η, the cost of expanding product variety or producing new "knowledge".

- The growth rate is also increasing in L or any other factor that increases the "scale" of the economy and thereby raises the profits of intermediate inputs and the demand for innovation. This is the (in)famous "scale effect" that is present in many models of endogenous technological change. Discuss....

7.1.8 Pareto Allocations and Policy Implications

- Consider now the problem of the social planner. Obviously, due to symmetry in production, the social planner will choose the same quantity of intermediate goods for all varieties: $X_{t,j} = X_t = x_t L$ for all j. Using this, we can write the problem of the social planner simply as maximizing utility,

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t),$$

subject to the resource constraint

$$C_t + N_t \cdot X_t + \eta \cdot (N_{t+1} - N_t) = Y_t = AL^{1-\alpha} N_t X_t^{\alpha},$$

where $C_t = c_t L$.

- The FOC with respect to X_t gives

$$X_t = x^* L,$$

where

$$x^* = A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}}$$

represents the optimal level of production of intermediate inputs.

- The Euler condition, on the other hand, gives the optimal growth rate as

$$1 + \gamma^* = \beta^{\theta} \left[1 + R^*\right]^{\theta} = \beta^{\theta} \left[1 + \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} L/\eta\right]^{\theta},$$

where

$$R^* = \tfrac{1-\alpha}{\alpha} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} L/\eta$$

represents that social return on savings.

- Note that

$$x^* = x \cdot \alpha^{-\frac{1}{1-\alpha}} > x$$

That is, the optimal level of production of intermediate goods is higher in the Pareto optimum than in the market equilibrium. This reflects simply the fact that, due to the monopolistic distortion, production of intermediate goods is inefficiently low in the market equilibrium. Naturally, the gap x^*/x is an increasing function of the mark-up $1/\alpha$.

- Similarly,

$$R^* = R \cdot \alpha^{-\frac{1}{1-\alpha}} > R.$$

That is, the market return on savings (R) falls short of the social return on savings (R^*), the gap again arising because of the monopolistic distortion in the intermediate good sector. It follows that

$$1 + \gamma^* > 1 + \gamma,$$

so that the equilibrium growth rate is too low as compared to the Pareto optimal growth rate.

- *Policy exercise:* Consider three different types of government intervention: A subsidy on the production of intermediate inputs; an subsidy on the production of final goods (or the demand for intermediate inputs); and a subsidy on R&D. Which of these policies could achieve an increase in the market return and the equilibrium growth rate? Which of these policies could achieve an increases in the output of the intermediate good sector? Which one, or which combination of these policies can implement the first best allocations as a market equilibrium?

7.1.9 Introducing Skilled Labor and Human Capital

notes to be completed

7.1.10 International Trade, Technology Diffusion, and other implications

notes to be completed

7.2 Increasing Product Quality (Aghion-Howitt)

topic covered in recitation

notes to be completed