14.451: Macroeconomics
Spring 2003

ProblemSet1

1 Human Capital in the Solow-Swan Model (Based on Mankiw, Romer, Weil 1992) *from BSM*

Assume that the production function is

$$Y = K^{\alpha} H^{\lambda} (AL)^{1-\alpha-\lambda}$$

where Y is output, K is physical capital, H is human capital, A is the level of technology, and L is labor. The parameters α and A are positive, and $\alpha + \lambda < 1$. L and A grow at the constant rates n and x, respectively. Output can be used on a one-for-one basis for consumption or investment in either type of capital. Both types of capital depreciate at the rate δ. Assume that gross investment in physical capital is the fraction s_k of output and that gross investment in human capital is the fraction s_h of output.

(a) Obtain the laws of motion for physical and human capital per unit of effective labor.

(b) What are the steady-state values of physical capital, human capital, and output, all per unit of effective labor?

(c) This augmented Solow-Swan model can be tested empirically with cross-country data if we assume that all countries are in their steady states. Derive a log-linear regression equation for output per worker. What problems would arise in estimating this equation by ordinary least squares?

(d) Derive an equation for the growth rate of output per unit of effective labor. How does this equation look when expressed as a linear approximation in the neighborhood of thesteadystate? If $\alpha = 0.3$,$\lambda = 0.5$,$\delta = 0.05$,$n = 0.01$,and $x = 0.02$,then what is the rate of convergence near the steady state? Compare the convergence rate in this augmented Solow-Swan model with that in the standard Solow-Swan model.

(e) Use the result from part (d) to derive a regression equation for the average growth rate of output per worker, $(1/T)\log[y(t+T)/y(t)]$, where T is the length of the observation interval. What problems arise in the econometric estimation of the rate of convergence, for example, if the levels of technology differ across the countries?

2 Distortions in the Solow-Swan Model (Based on Easterly 1993) *from BSM*

Assume that output is produced by the CES production function,

$$Y = [(a_F K_F^\eta + a_I K_I^\eta)^{\psi/\eta} + a_G K_G^\psi]^{1/\psi}$$

where $Y$is output; K_F is formal capital, which is subject to taxation, K_I is informal capital, which evades taxation; K_G is public capital, provided by government and used freely by all producers; a_F, a_I, $a_G > 0$; $\eta < 1$ and $\psi < 1$. Installed formal and informal capital differ in their location and form of ownership and, therefore, in their productivity.

Output can be used on a one-for-one basis for consumption or gross investment in the three types of capital. All three types of capital depreciate at the rate δ. Population is constant, and technological progress is nil.

Formal capital is subject to tax at the rate τ at the moment of its installation. Thus, the price of formal capital (in units of output) is $1 + \tau$. The price of a unit of informal capital is one. Gross investment in public capital is the fixed fraction s_G of tax revenues. Any unused tax receipts are rebated to households in a lump-sum manner. The sum of investment in the two forms of private capital is the fraction s of income net of taxes and transfers. Existing private capital can be converted on a one-to-one basis in either direction between formal and informal capital.

(a) Derive the ratio of informal to formal capital used by profit-maximizing producers.

(b) in the steady state, the three forms of capital grow at the same rate. What is the ratio of output to formal capital in the stady state?

(c) What is the steady-state growth rate of the economy?

3 Matlab Tutorial

To prepare us for programming Dynamic General Equilibrium models, I have prepared a Tutorial (dowload it from the web). For this first problem set follow the tutorial and acquire confidence with the basics of Matlab.

4 DP analytically: Stokey-Lucas *from SLP*

Consider the deterministic model of optimal growth: In each period t there is a single good, y_t, that is produced using two inputs: capital, k, in place at the beginning of the period, and labor n . In each period the economy has to decide how to allocate output between consumtion and investment:

$$c_t + i_t \leq y_t = F(k_t, n_t)$$

Capital is assumed to depreciate at a constant rate $0 < \delta < 1$:

$$k_{t+1} = (1 - \delta)k_t + i_t$$

Preferences over consumption are taken to be of the additively separable form:

$$\sum_{t=0}^{\infty} \beta^t U(c_t)$$

where $0 < \beta < 1$ is a discount factor. Assume that the size of the population is constant over time and normalize the size of the available labor force to unity. Then actual labor supply must satisfy:

$$0 \leq n_t \leq 1, \; for\ every\ t$$

Define $f(k) = F(k, 1) + (1 - \delta)k$ to be the total supply og goods available per worker.

a) Exercise 2.1 p.10 from Stokey-Lucas.

b) Exercise 2.2 p.12 from Stokey-Lucas.

c) Exercise 2.3 p.12 from Stokey-Lucas.