BEST TYPE OF AIRPLANE FOR MINIMUM LANDING AND GETTING AWAY DISTANCES

by

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Best Type of Aeroplane of Minimum Landing Distance

Best Type of Aeroplane of Minimum Getting Away Distance.

Introduction.

During the past several years mechanics and performance of airplanes have been studied exhaustively. But the subject of minimum landing and also minimum getting away distance has not been treated theoretically except, "The Landing of Aeroplanes" (British Advisory Committee Report for Aeronautics No. 666) by Glauert and "Landing Run and Get Away for Standard Airplanes", by Klemin, as far as I have known.

Both Mr. Glauert and Mr. Klemin treated the Landing distance mathematically, but did not touch the best type of airplane which has a minimum landing distance and minimum getting away distance. As Mr. Glauert said in his report in the development of civil aviation, increasing importance will be attached to the ease and safety of landing. This is one of the most important reasons why this subject is discussed in this Thesis.

It has been a pretty long time since variable camber, variable area of aeroplane wing and reversible pitch propeller were suggested for improving the landing condition of aeroplanes. But these schemes for improving the landing distance have not been touched by any one. In this Thesis the best type of airplane of minimum landing distance and best type of minimum getting away distance will be treated taking into consideration the variable camber, variable area of aerofoil and reversible pitch airscrew.

It is a well known fact that any aerofoil which gives the greater lift gives the more improved landing.

Therefore the Handley Page's Slotted Wing will be discussed because this form of wing has greater lift coefficient than those of any other which are existing.

1. Landing distance of aeroplane when the engine is shut off.

1. Equation for landing distance when Engine is shut off.

For the practical method of landing an aeroplane it may be represented closely in the following manner.

The aeroplane crosses the edge of the aerodrome at Height ho, sufficient to give the necessary clearance over trees or buildings and glides down to Height ho at a constant speed V.

The elevator control is then used to flatten out the flight path until the speed has fallen to the stalling speed Vo when the aeroplane begins to settle on the ground and eventually is brought to rest by the drag of the airforces and of the friction of the under carriages and tail skid. The distance from the edge of the aerodrome to the place at which the aeroplane is stopped, may be more useful for practical landing than the distance from the ground at which the airplane is touched to the position at which the plane is stopped.But for simplicity the usual meaning of the landing distance, that is, the distance after the aeroplane touches the ground will be discussed as the landing distance. Consider the case of a landing with the engine shut off in which the pilot flattens out after a dive, places his machine in a stalling attitude and gradually loses speed, until the wheels and skid touch the ground simultaneously.

The equation of motion after touching the ground then becomes, assuming the attitude is constant and to be that of stalling angle of incidence.

$$\frac{W}{g}\frac{dv}{dt} = K_D \rho S v^2 - \mu(W - K_L \rho S v^2) \quad \cdots\cdots (1)$$

W	Weight of the machine
g	Gravity
v	speed of the machine
S	surface area of the wings
μ	frictional coefficient of the ground
K_L	lift coefficient of the wings
K_D	total drag coefficient
	i.e. drag coefficient of the wings
	Plus coefficient of plate x $\frac{\text{equiv.area of body}}{S}$

Let Vo be the stalling velocity of the machine and put $\lambda_0 = \frac{K_D}{K_L}$ then the equation (1) becomes

$$\frac{dv}{dl} v = -\lambda_0 \frac{v^2}{V_0^2} - \mu \left(g - \frac{v^2}{V_0^2}\right)$$

Integrate the limit $l = 0 \quad v = V_0 \quad ; \quad l = L \quad v = 0$

$$L = \frac{V_0^2}{2g(\lambda_0 - \mu)} \log_e \frac{\lambda_0}{\mu} \quad \text{.........} \quad (2)$$

In this case of the engine being shut off the landing distance is the function of stalling velocity, ratio of $\frac{K_D}{K_L}$ and i.e. λ_0 and frictional coefficient of the ground.

The above equation easily shows that this length will be made shortest when stalling speed i.e. landing speed is least and λ_0 is largest.

Friction of the ground could be made great if we made a special scheme by which a great friction on the ground would be given. But this scheme is not practical because the ground must be seriously injured.

Therefore we have two methods of improving the landing distance.

(1) One is to increase the wing area when about to land.

(2) The other is to increase the maximum lift coefficient when landing.

The former means the variable wing area the latter means variable camber wing and Handley Page's Slotted Wing. All above methods have not yet been put into practice owing to the mechanical difficulties in their application.

2. The Landing Distance of Variable Wing Area

Three possible ways of enlarging the wing area when landing are:

(a) The wing of the machine which flights as a monoplane consists of two parts which can be separated gradually. The wing area of a biplane increases about twice when landing.

(b) A part of a wing is folded into the fuselage while flying in the sky and it may be pulled out from the fuselage when landing.

(c) The tip of the wings are telescopic in direction of the span and may be pulled out to the greater area when landing.

By (b) and (c) methods the wing area can not be increased so much that the landing speed is decreased effectively. If (a) can be realized the landing speed must be decreased a great deal. Therefore (a) is the only worthy one to be considered.

For convenience sake the monoplane which can be separated into a biplane when landing will be called (a) type monoplane.

Before the computation of the landing distance when (a) type monoplane is realized, the landing distance of a biplane must be calculated. As Mr. Glauert, R.A.F. 15 biplane of zero stagger will be adopted for calculating. At stalling speed the drag coefficient being increased 0.015, mean friction coefficient of the ground has been estimated as 0.05 for wheels and 0.50 for tail skid. Assuming that one sixth of the total weight rests on the tail skid, the value adopted for

$$\mu = \frac{0.50 \times 1}{6} + \frac{0.05 \times 5}{6} = 0.12$$

$$K_D = 0.069 + \tfrac{1}{4}(0.069) = 0.081$$

assuming the body resistance is the one forth of wing resistance at stalling attitude.

$$K_L = 0.538 \times 0.94 = 0.505$$

where 0.94 is efficiency of biplane effect.

$$\lambda_0 = \frac{0.081}{0.505} = 0.160$$

$$\mu_0 = 0.12$$

Take 40 m.p.h. $= 58.6$ ft/sec as the stalling speed

$$L = \frac{V_0^2}{2g(\lambda_0 - \mu)} \log_e \frac{\lambda_0}{\mu} = \frac{58.6^2 \times 2.3}{2 \times 32(0.160 - .12)} \log_{10} \frac{.160}{.12}$$

$$= 382 \text{ feet}$$

382 feet is the landing distance of the biplane.

If we take the monoplane with the same weight as the biplane the monoplane usually has a little greater velocity (say 5%) than the biplane therefore as stalling speed 42 m.p.h. will be used for a monoplane. It will be assumed that (a) type monoplane has the following property.

$\overline{W}$ = Weight of total usual biplane

W = Weight of wing (15% of total weight of the machine)

W_1 = Extra weight for mechanisms to separate the Monoplane to a biplane
= 50 percent of wing weight

The gap cannot be so great as usual biplane if (a) type monoplane would be applaced therefore gap is assumed 75% of the chord of the wing which has 80% of efficiency of biplane effect

Vo = stalling speed of the machine as a biplane

Total weight of (a) type monoplane

$$= \overline{W} + \frac{15}{100}\overline{W} \times \frac{50}{100}$$

$$= 1.08\,\overline{W}$$

$\therefore 1.08\,\overline{W} = \rho K_L \times 0.8\, V_0'^2 \times 2A$ Where A is the area of the monoplane wing.

Therefore $\frac{1.08}{1.6} V_0 = V_0'^2$

or $V_0' = .82\, V_0 = 32.8$ m.p.h $= 48$ feet/sec

The stalling speed of the (a) type monoplane is 48 feet per second.

The body resistance of the monoplane may be approximately the same as the usual biplane with the same area of wings when the machine becomes the biplane. So we get the drag coefficient and lift coefficient for the (a) type monoplane taking 0.80 as a biplane effect efficiency for lift when landing.

$$K_D' = 0.069 \times 2 + 1/4 \times 0.069 = 0.156$$

$$K_L' = 0.437 \times 2 \times 0.8 = 0.699$$

$$\lambda_0' = \frac{0.156}{0.699} = .223$$

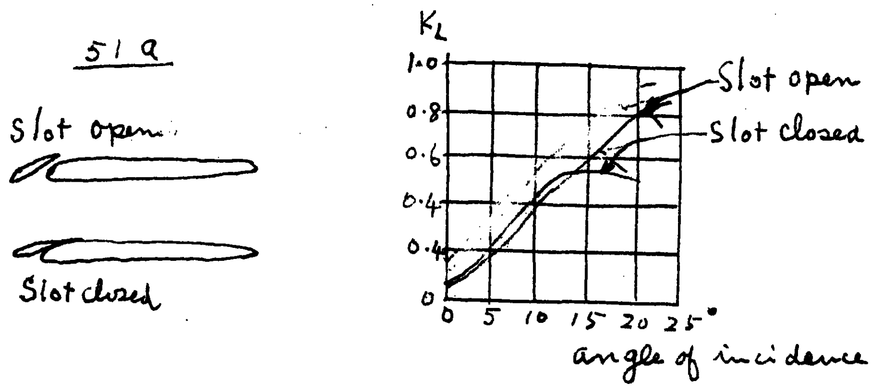
The landing distance

$$L = \frac{48^2}{2g(.223 - .12)} \log_e \frac{.223}{.12} = 207 \text{ feet}$$

34.6% of the landing distance is cut down by adopting an (A) type monoplane.

3. The landing distance of the Monoplane and the Biplane equ ipsd by the Handley Page's Slotted Wings and the Variable Camber of Wings. Mr. Handley Page designed very peculiar aerfoils with slots which can be opened as shown on figure 1.

Figure 1.



These forms of the wings modoles were tested at the National Physical Laboratory in England showing the astonishing results with the slots opened the total lifts on the slotted wings were greatly improved on some aerofoils and with the slots closed practically all the advantages of the ordinary sections.

<u>(a) (51A) Biplane</u>

Section (51A) has the section with the slot closed and under side gap filled up being with a R.A.F. 15.

(51A) has the leading edge of the aft main aerofoil with a slight Phillips entry as shown in Figure 1 and the maximum lift coefficient is increased from 0.52 to 0.80 at 20 degrees in increase of 54 percent, and lift drag ratio is higher with the slot open at all angles above 12 degrees of angle of incidence.

Extra weight of equipment adopting (51A) for an aeroplane wing may be estimated under 25% of the total wing weight.

If (51A) aerofoil can be used as the wing of a biplane, its landing distance will be computed as follows, assuming the landing is done at 20 degrees of angle of incidence.

Handley Page's many biplane combination tests have shown us that with the necessary biplane correction the slotted monoplane can be applied to the biplane.

W_1 = Weight of (51A) = 1.25 w
W_2 = 1.04 W
V_2 = landing speed of the plane

$$1.04\ W = \gamma K_{L_2} \rho V_2^2 A$$
$$= 1.04 \times \gamma K_L \rho V_0^2 A$$

where γ is correction for lift of biplane.

$$\therefore V_2^2 = 0.656\, V_0^2$$
$$V_2 = 0.81 V_0 = 32.4^{\ m.p.h} = 47.5^{\ feet/sec}$$

K_{D_2} = drag coefficient of wings $+ 0.069 \times \frac{1}{4}$
$= 0.1239$

$\lambda_2 = 0.0737$
$\mu = 0.12$

The landing distance $= \dfrac{58.6^2}{2g(0.197 - .12)} \log_e \dfrac{.197}{.12} = 284$ feet

284 feet is the required landing distance of (51A) type of bi-

plane.

(b) R. A. F. Biplane with flap along all length.

Biplane of R. A. F. 9 upper wing with flap 0.385 of the chord and lower wing has no flap. The British Advisory Committee Report for Aeronautics 1913-1914 shows us that the increase of lift coefficient is very much less in the case of the biplane.

It might be expected that increase of lift coefficient in the biplane would be half that found in the case of the monoplane, actually the increase is only about one third and for larger angles of flap than 15 degrees to the chord no further increase of lift coefficient is obtained.

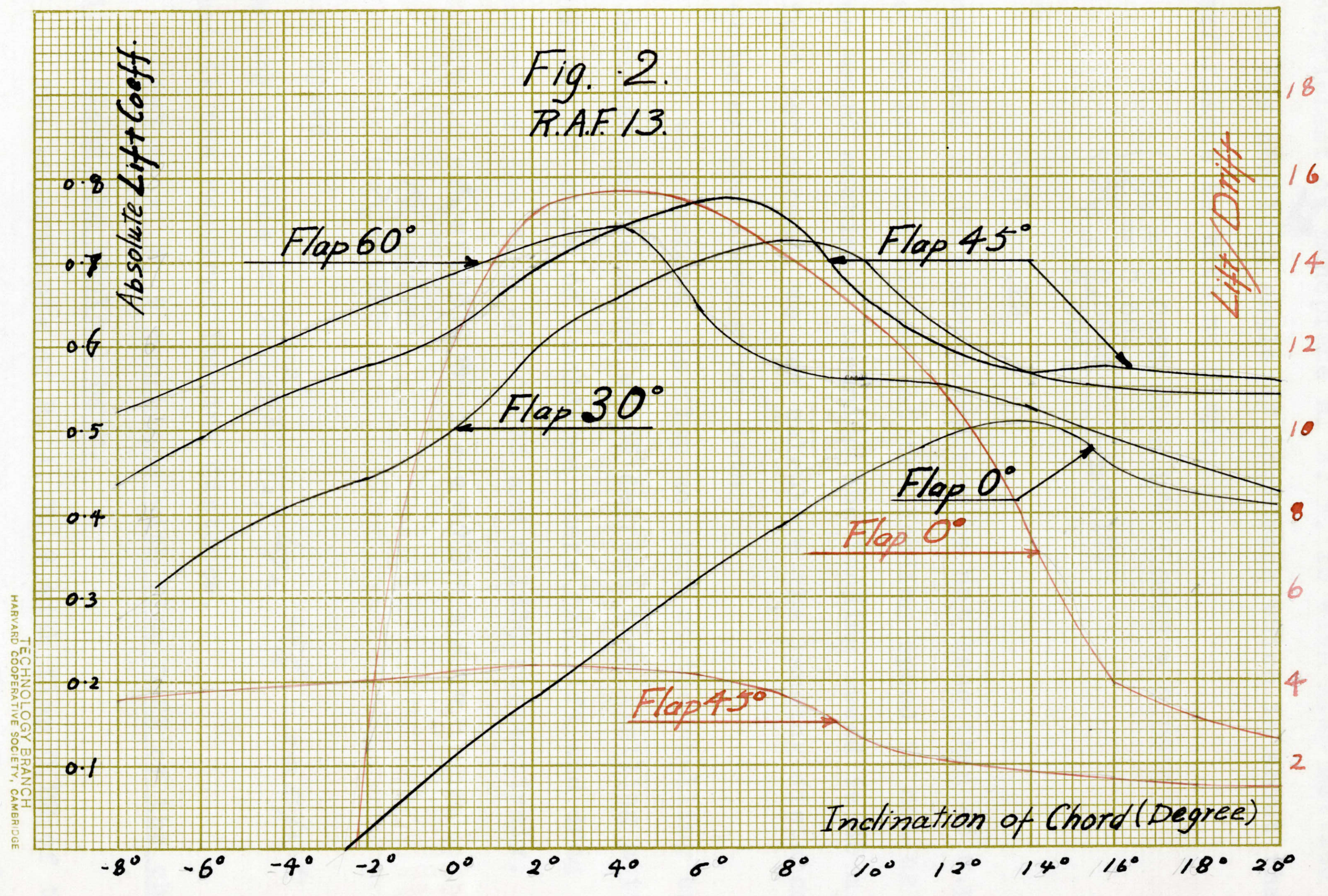
As flap 15 degrees to the chord lift is maximum 0.625 in its value at 15 degrees angle of incidence where
In no flap, maximum lift is 0.57 at 15 degree angle of incidence where $L/D = 6$

$$V_3' = \sqrt{\frac{.57}{.625}}\, V_3 = 0.955\, V_3$$

The landing distance may be shortened roughly 10% which is not so effective.

R.A.F. 15 biplane with flaps extending along all length both upper and lower wings was tested in National Physical Laboratory in England but the report is not in our hands yet. However we can presume fromthe above results that biplanes with flaps extending along all length, both upper and lower wings, cannot be expected to improve so much as a monoplane with flap extending along the whole length.

(c) R. A. F. 13 Monoplane with flap 0.385 of the chord extending along all length. From Fig. 2 we get the drag and



Absolute Lift Coeff.
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
Lift/Drift
18
16
14
12
10
8
6
4
2
Flap 60°
Flap 45°
Flap 30°
Flap 0°
Flap 0°
Flap 45°
Inclination of Chord (Degree)
-8°
-6°
-4°
-2°
0°
2°
4°
6°
8°
10°
12°
14°
16°
18°
20°
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Fig. 2.
R.A.F. 13.

lift coefficient of monoplane R. A. F. 13 with flap settled zero degree at the stalling angle of incidence which is about the same performance as with no flap.

$$\begin{cases} K_{D_4} = 0.0693 \text{ plus } 1/4\ 0.0693 = 0.0866 \\ K_{L_4} = 0.505 \\ \lambda_4 = \dfrac{0.0866}{0.505} \doteq 0.171 \\ \mu = .12 \end{cases}$$

Stalling speed of this monoplane may be not less than 45 m.p.h. because the lift coefficient at the landing speed is not as great as usual machine. Assuming the landing speed is 45 m.p.h. $\doteq$ 66 feet per second, we can calculate the landing distance as 290 feet using the above given data.

If we regulate the flap so that at the stalling speed it is inclined to the chord at 45 degrees then stalling attitude is 7 degrees of angle of incidence where the lift and drag coefficient of the Wing are 0.775 and 0.1830 respectively.

$$\begin{cases} K'_{D_4} = 0.1830 + \frac{1}{4}\,0.0693 = 0.200 \\ K'_{L_4} = 0.775 \\ \lambda'_4 = .258 \end{cases}$$

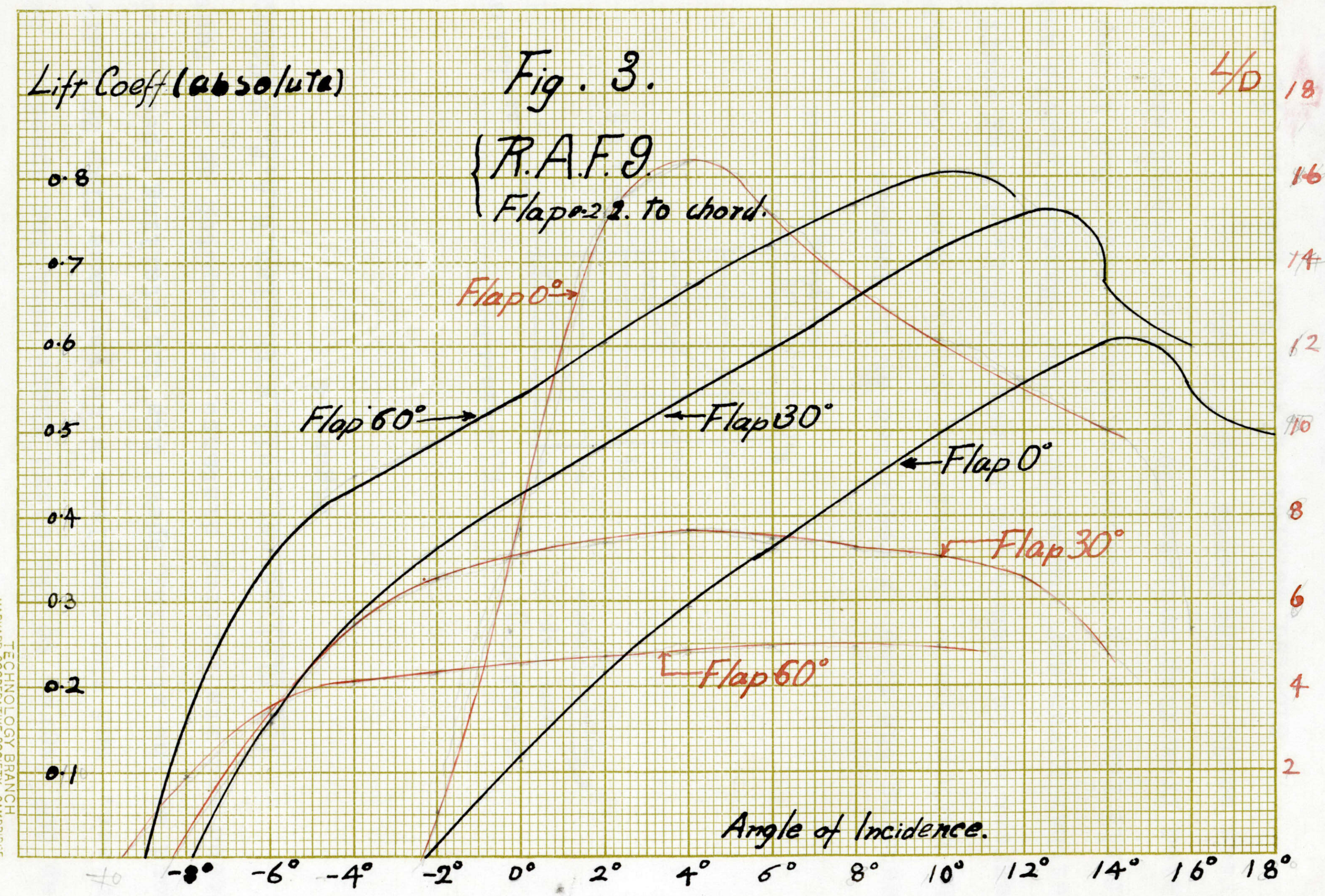
Stalling speed

The landing distance $= \dfrac{53.2^2}{64(.258-.12)} \log_e \dfrac{.258}{.12} = 245$ feet.

245 feet is the required landing distance.

(d) R. A. F. 9 monoplane with flap 0.22 of the chord extending along the whole length.

The experiments of R. A. F. 9 with flap 0.22 of the chord were made in N.P.L/ which are shown on Fig. 3. Applying the same treatment to the R. A. F. 9 monoplane with flaps 0.22


Lift Coeff (absolute)
Fig. 3.
R.A.F. 9.
Flap 0·2 to chord.
L/D
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
18
16
14
12
10
8
6
4
2
Flap 0°
Flap 60°
Flap 30°
Flap 0°
Flap 30°
Flap 60°
Angle of Incidence.
-8° -6° -4° -2° 0° 2° 4° 6° 8° 10° 12° 14° 16° 18°
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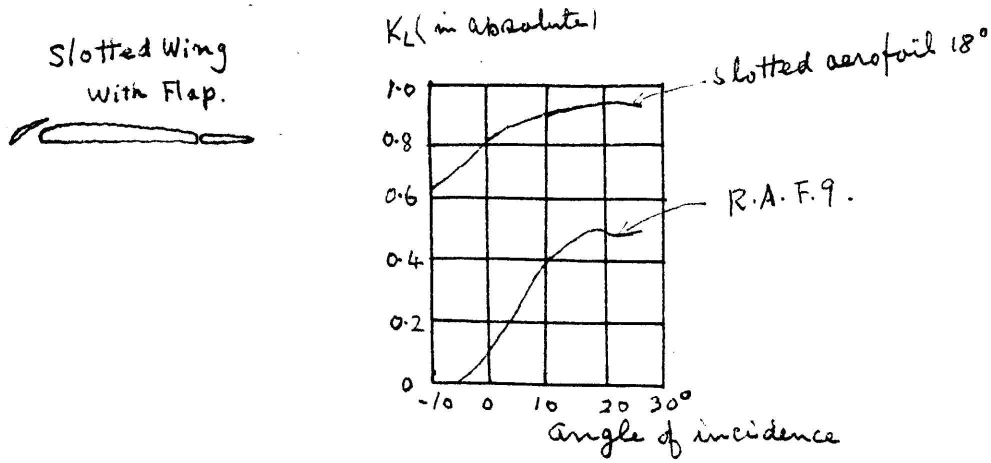
of the chord extending along the whole length. The landing distance 206 feet can be obtained as 10 degrees of angle of incidence and flap 60° degree to the chord where stalling speed is 52.6 feet per sec. and the monoplane with the same flap 30 degrees to the chord at the stalling speed has the stalling speed 53.5 feet per second and its landing distance is computed as 253 feet.

(e) Slotted aerofoil with flap R.A.F. 9 Monoplane

Figure 4



Assuming the flap is regulated at 18 degrees to the chord when the machine lands at stalling speed. Then $K_L = 0.915$ at 18 degrees of incidence can be seen on Fig. 4 Drag coefficient is neither given nor suggested in the Handley Page's announcement of his results on Aviation or Aerial Age. But it may be estimated from flap inclined to the chord without slot at 14 degrees angle of incidence $L/D = 3.72$.

Slotted aerofoil always has greater L/D values above 12 degrees of incidence. Therefore slotted aerofoil with flaps (18 degrees to the chord) has about the same L/D value but a little greater than the flap aerofoil.

Estimate $L/D = 4$

$K_{L_5} = .915$ and drag coefficient of the wing is 0.229 therefore the drag coefficient of the machine

$K_{D_5} = .256$

$$\lambda_5 = \frac{.256}{.915} = .280$$

$$V_5'^2 \times .915 = 0.604\, V_5^2$$

Where 0.604 is the R. A. F. 9 lift coefficient at stalling speed

$\therefore V_5' = 34.2$ m.p.h. $= 50.2$ feet/sec

The landing distance computed in the same manner is about 230 feet. If 3 slotted wing with flap inclined to the chord 20 degree at stalling speed is used as an aeroplane wing then stalling speed is 46.3 feet per second and the landing distance can be reduced to 210 feet.

II Reversible Pitch Propeller

The variable pitch propeller has been suggested as a brake to reduce the run of an aeroplane when landing and thus to enable it to be made in a small landing ground. In this case the blade would be given limits of variation sufficient to allow them to be reversed and exert a backward thrust. Thus when the aeroplane touched the ground the blades would be reversed and the engine opened out to obtain the maximum reversed thrust.

Let us consider whether any appreciable braking effect can be obtained by running the engine on the ground while the propeller can be reversed so that it may give the maximum

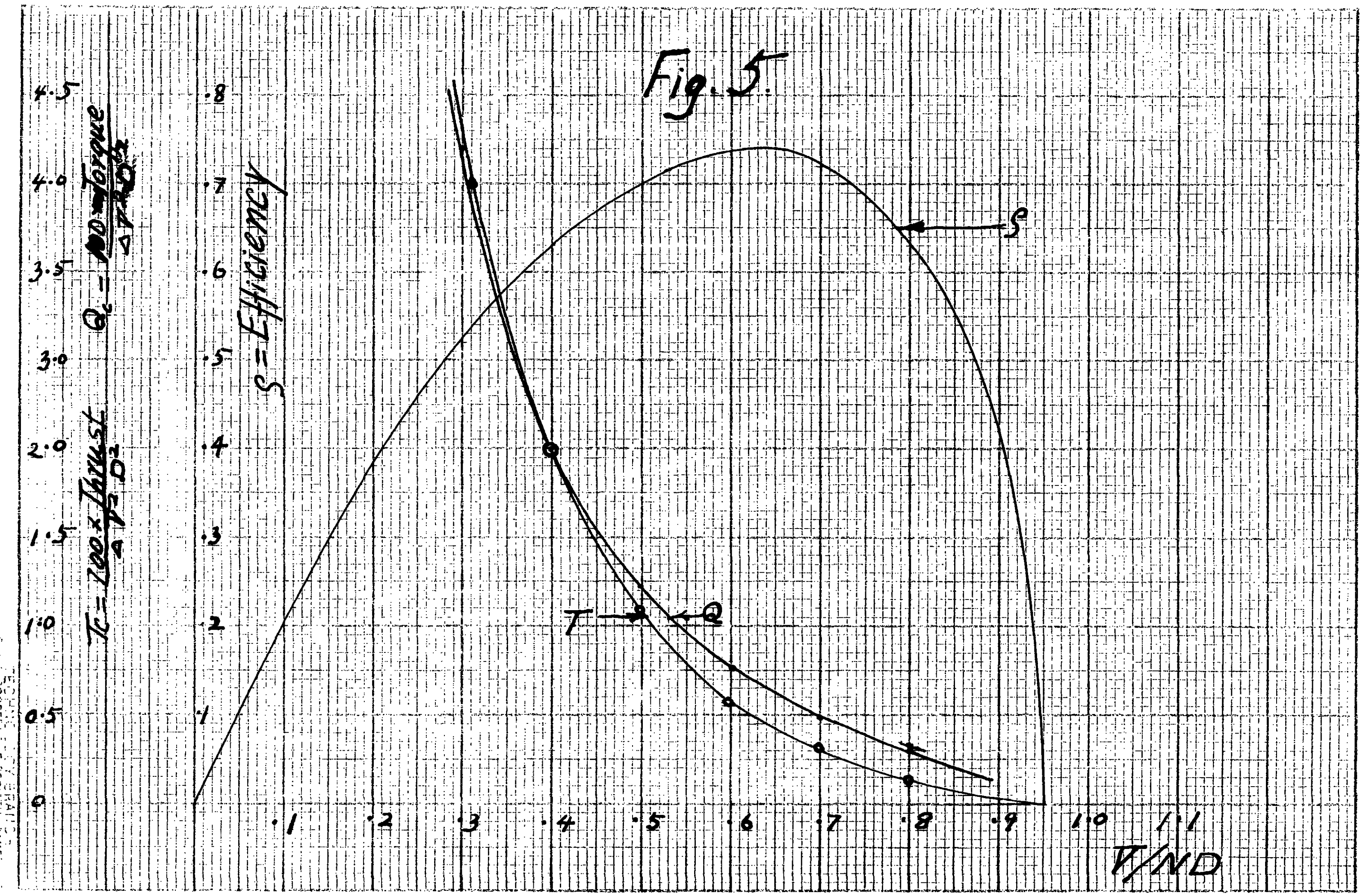


Fig. 5
$T_c = \frac{100 \times Thrust}{\Delta V^2 D^2}$
$Q_c = \frac{100 \times Torque}{\Delta V^2 D^3}$
S = Efficiency
S
T
Q
V/ND
0
0.5
1.0
1.5
2.0
3.0
3.5
4.0
4.5
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
1·1

negative that is backward thrust.

As an example take the propeller No. 17 of Dr. Durand's report No. 14 for National Advisory Committees for Aeronautics of U. S. A.

Assume the propeller to have a diameter of 9.5 feet and that it is used with a Liberty Engine having a maximum speed of 1,700 r.p.m. in the sea level.

The V/nd ratio (where V is speed of the machine; n is number of revolution per minute; D diameter) at a speed of 110 m.p.h. is $\frac{110 \times 1.46}{27.4 \times 9.5} = 0.62$

The thrust coefficient from Plate X in Durand's report is 0.525 and the thrust is given by the formula

$$T = \frac{T_c D^2 V^2 \Delta}{100}$$

Where Δ is the density of the air on the ground

$$\Delta = 0.0761 \text{ lbs. per cub. foot}$$

Using the formula

$$T = \frac{0.525 \times (9.5)^2 \times (160)^2 \times 0.0761}{100}$$

$$= 920 \text{ lbs.}$$

where its efficiency is 73.8%. The torque is given by the formula

$$Q = \frac{Q_c D^3 V^2 \Delta}{1000}$$

Where $Q_c = 0.70$

Applying the formula

$$Q = \frac{0.70 \times 9.5^3 \times 160^2 \times 0.0761}{1000}$$

$$= 1140 \text{ lbs. foot}$$

The power under the condition is $\frac{1140 \times 2\pi \times 27.4}{550} = 358$ h.p.

This is the limit which Liberty can work out. Negative thrust of propeller is investigated experimentally and is reported in the Advisory Committee Report No. 30 and Eifel's "Influence de la Valeur et de la Variation du pas Obtenue par le Decalage des Pales", in his "Etudes L'Helice Aerienne". But these are not applicable to the reversible pitch propeller. Experimental results of models of negative pitch propellers in wind tunnel or those of full sized negative pitch propeller have not been given hence it is very difficult to calculate the backward thrust of negative pitch propeller and it may not be expected to compute precisely these thrust without knowing of lift and drift coefficient of the sections of the blades at the negative angle of incidence.

4. Negative Thrust of Reversible Pitch Propeller

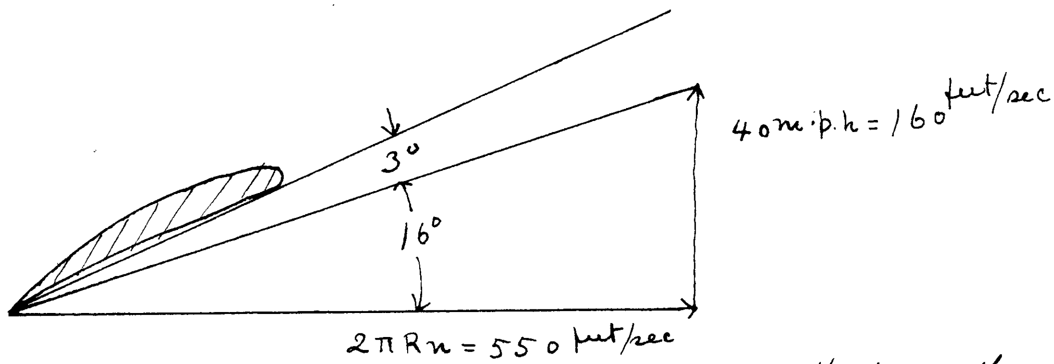
The propeller adopted above has the following properties:

Efficiency --	73.8 percent	at
Thrust --	920 lbs	110 m.p.h.
Torque --	1140 lbs. foot	1700 r.p.m.

If we assume roughly that all area of the propeller assembled on 2/3 radius then we have as mean drift to lift ratio of the blade 1/15 in its value so that the propeller efficiency has 0.738 percent in its efficiency at the given conditions. Because

$$\left\{\begin{array}{l} \phi = \tan^{-1}\left\{\dfrac{\text{speed of the machine}}{2\pi r n}\right\} = 16^\circ \\ \gamma = \tan^{-1}\dfrac{1}{15} = 4^\circ \\ \text{efficiency} = \dfrac{\tan 16^\circ}{\tan(16^\circ + 4^\circ)} = 0.738 \\ \text{angle of incidence is assumed as } 3^\circ \end{array}\right.$$

Fig. 6



Wher R = 2/3 Radius of the propeller

Thrust T = 920 lbs.

$$= (L_c \cos 16° - \frac{L_c}{15} \sin 16°)(550 \sec 16°) A_1$$

Where A_1 is developed area of the propeller
L_c mean lift coefficient of the propeller

$$\therefore A_1 L_c = \frac{920}{306000} = 0.00300$$

A is about 9 square foot

mean lift coefficient

L_c = 0.000330 in lb. foot sec. unit

= 0.14 in absolute unit

Check

Check for above assumption, Torque $= (D_c \cos 16° + L_c \sin 16°)(550 \sec 16°)^2 A_1 R$

$$= (\frac{.00300}{15} \cos 16° + .003 \sin 16°)(550 \sec 16°)^2 A_1 R$$

= 1050 lb. foot

1050 lb. foot is a little less than 1140 lbs. foot but near enough for above rough assumption.

Before proceeding with the discussion of experimental result of aerofoil R. A. F. 6 at negative angle of incidence should be considered. R.A.F. 6 is the only one of the aerofoils of which characteristics for negative angle of incidence are known .

The value of the lift coefficient and lift to drag ratio are shown in Fig. 7. for R. A. F. 6 wing section.

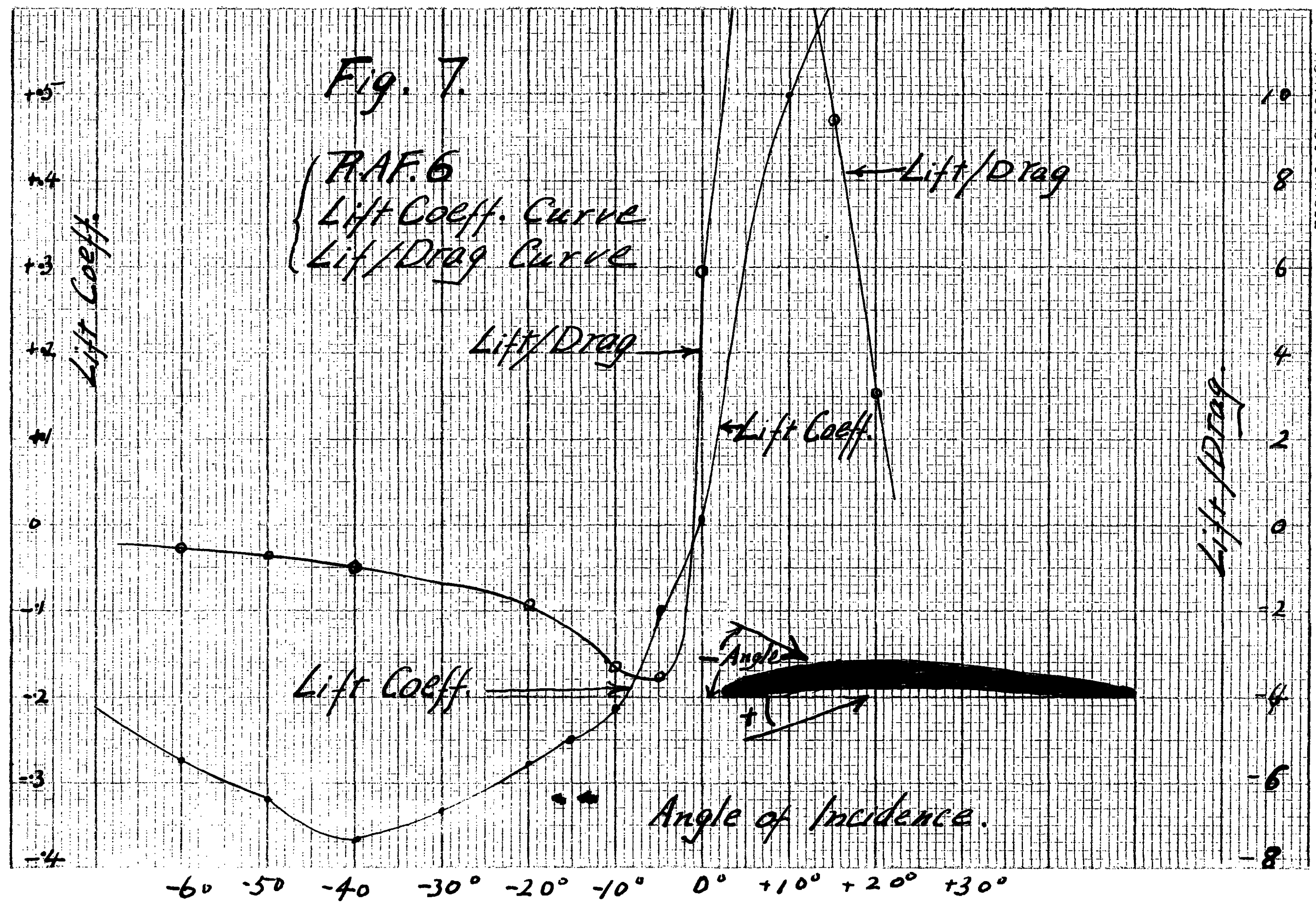


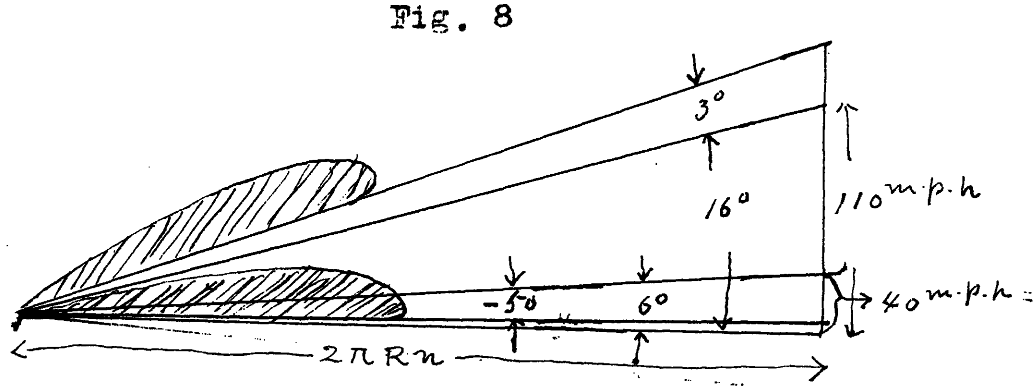
Fig. 7.
R.A.F. 6
Lift Coeff. Curve
Lift/Drag Curve
Lift/Drag
Lift/Drag
Lift Coeff.
Lift Coeff.
Angle
Angle of Incidence.
Lift Coeff.
Lift/Drag
+.5
+.4
+.3
+.2
+.1
0
-.1
-.2
-.3
-.4
1.0
8
6
4
2
0
-2
-4
-6
-8
-60
-50
-40
-30°
-20°
-10°
0°
+10°
+20°
+30°

In Bairstow's Applied Aerodynamics p.p. 128 the author says that small camber of the under surface is of little importance for the shape of the curve although a modification known as R. A. F. 6a has been used on many occasions and differs from R. A. F. 6 only in the fact that in the form the under surface is flat. Fig. 7 shows us that lift to drag ratio is the maximum in its absolute value (-3.6) at - 5 degree of incidence. It is a very clear matter that we can get the greater maximum lift to drag ratic in absolute value than -3.6 using the flat under surface and the considerably greater value at least than - 7 which is the maximum lift to drag ratio of flat plate that can be expected.

A good designed reversible pitch propeller will have a maximum lift to drag ratio over the -10 in absolute value of negative angle of incidence.

Estimating rather low value -7 (in absolute) will be used as the mean maximum lift to drag ratio of the negative pitch, propeller which may be at a negative few degrees of incidence say -5°.

Fig. 8



for negative pitch propeller. For the propeller which was applied before that is No. 17

$\phi = 16^\circ$ and angle of incidence

is 3° at 110 m.p.h. and 1700 r.p.m.. Therefore if the propeller blade is twisted 18° about the axis perpendicular to the propeller shaft then the angle of incidence becomes -5° when the speed of the machine is 40 m.p.h. that is, stalling speed of the machine and at the full speed of the engine.

Torque for the negative pitch propeller it will be

$$Q' = A_1 L_c' \left(\tfrac{1}{7}\cos 6^\circ - \sin 6^\circ\right) V_R^2 R$$
$$= 0.039\, A_1 L_c' V_R'^2 R$$

On the other hand the propeller has the following torque at 110 m.p.h. and full speed of the engine.

$$Q = \left(\tfrac{1}{15}\cos 16^\circ + \sin 16^\circ\right) L_c A_1 R V_R^2$$
$$= 0.293 \times A_1 L_c R V_R^2$$
$$\therefore \frac{Q'}{Q} = \frac{.039 L_c' V_R'^2}{.293 L_c V_R^2}$$

but $\frac{L_c'}{L_c}$ is estimated about 1/3 from Fig. 7

Therefore $\frac{Q'}{Q} < \frac{1}{27}$ is the torque ratio at full speed of the engine. Torque corresponding to the maximum lift to drag ratio of negative pitch which is very much less than that of the usual type of propeller.

In this case of reversible pitch propeller, it is so far from full utilization of engine power to use angle of negative incidence corresponding to the maximum lift to drag ratio.

5. Backward Thrust of Negative Pitch Propeller at the Maximum Lift Coefficient of The Blades.

Greater negative lift coefficient of R.A.F. 6 is -3.59 at -40° and at that point lift to drag ratio is -1.08. Negative lift coefficient may be expected at about -40°on any usual aerofoil but greater lift coefficient and less drag coefficient can be clearly expected by good shaped aerofoil for negative pitch propeller. At first we will try to estimate the negative thrust using the same lift and drag coefficient of R.A.F. 6. The torque can be roughly estimated at the following formula at the condition of a revolution per minute is 1700 and speed of the machine is 40 m.p.h.

$$Q'' = A_1 R \left(\frac{1}{1.08}\cos 6^{\circ} - \sin 6^{\circ}\right) 0.369\, V_R''^2$$

where 0.369 is the lift coeff.

$$= 9 \times 3.16\ (.870 - .105)\ 0.369 \times 0.00237\ (550)^2 \times (\text{Sec. } 6^{\circ})^2$$

$$= 5710 \text{ lb. foot}$$

Thrust $T'' = A_1 (L_c \cos\phi + D_c \sin\phi)(550 \sec 6^{\circ})^2 \times 0.00237$

$= 9 \times (.369 \cos 6^{\circ} + .343 \sin 6^{\circ}) \times 303000 \times 0.00237$

$= 2620$ lbs.

Brake Power $= \rho N^2 D^5 Q_c \times 2\pi N$

$= \rho N'^2 D^5 Q_c' \times 2\pi N'$

Where { ρ is density of air
N & N' are revolution of the propeller
are torque coeff. s.
D is the diameter of the propeller }

$$\therefore N'^3 = N^3 \times \frac{Q_c}{Q_c'} = 1700^3 \times \frac{1140}{5710}$$

$$= 994^3$$

994 r.p.m. is the speed of the negative pitch propeller when 358 horse power of the engine is worked out and when its angle of incidence is corresponding to the maximum lift coefficient of the blades.

Therefore the negative thrust which can be put out from 358 horse power is given by

$$T'' = 2680\left(\frac{994}{1700}\right)^2$$

$$= 895 \text{ lbs.}$$

At full power of the Liberty engine revolution of the negative pitch propeller becomes $994 \times \sqrt[3]{\frac{400}{358}} = 994\sqrt{1.037}$

$=$ 1030 r.p.m. and then the thrust becomes 960 lbs.

If the machine is at rest, then $\phi = 0$ and torque and thrust are calculated as the following values.

Torque = 7040 lb. foot at 1700 r.p.m.
Thrust = 2390 lbs. at 1700 r.p.m.

Negative thrust which can be put out when the machine is at rest and when the engine power is utilized fully is as follows.

Thrust = 762 lbs. where propeller revolution 962 per minute

We have arrived at such conclusion about the backward thrust of the negative pitch propeller when the angle of incidence of maximum lift coefficient of the blade is used.

The backward thrust = 960 lbs. at 40 m.p.h. of the machine speed.

the backward thrust- 760 lbs. at the rest of the machine.

6. The Landing Distance of Biplane with Reversible Pitch Propeller.

In the front page we know that 400 horse power Liberty engine can work out about 960 lbs. of the negative thrust at stalling speed of the aeroplane while the propeller speed is about 1030 revolutions per minute.

On the other hand when the machine is at the rest, negative thrust reduces to about 760 lbs. while the propeller revolution is about 960 per minute.

A 400 horse power machine may usually be about 4500 lbs. in its total weight.

Then

4500 lbs = total weight of the machine
860 lbs = Negative mean thrust while landing.

The landing distance can be computed by applying the following formula.

$$\frac{W}{g}\frac{dv}{dt} = -K_D \rho S v^2 - \mu(W - K_L \rho S v^2) - \text{Thrust}$$

~~Thrust~~

Where μ = friction coef. of the ground

Integrate the ~~about~~ differential equation and we can get

$$\text{Landing distance} = \int_{V_0}^{0} \frac{dv^2}{\frac{2g(\lambda_0 - \mu)v^2}{V_0^2} + 2g\mu\left(1 + \frac{T}{\mu W}\right)}$$

where $\lambda_0 = K_D/K_L$ and V_0 is stalling speed

$$\text{or landing distance} = \frac{V_0^2}{2g(\lambda_0 - \mu)} \log_e \frac{\lambda_0 + \frac{T}{W}}{\mu + \frac{T}{W}}$$

By substitution of the following number in the above equation

$$V_0 = 58.6 \text{ feet pec.}$$

$$g = 32 \text{ feet/sec}^2$$

$$\mu = 0.12$$

$$\lambda_0 = 0.197$$

$$\frac{T}{W} = \frac{860}{4500} = 0.192$$

The landing distance

$$= \frac{58.6^2}{64(0.197-0.12)} \log_e \frac{0.197+.192}{0.12+.192}$$

$$= 1450 \times \log_e 1.23$$

$$= 1450 \times 0.0969$$

$$= 140 \text{ feet}$$

140 feet is the landing distance.

7 F. Conclusion for the Landing Distance.

The landing distance of usual type of biplane as well as monoplane with variable area of wing; with all kind of variable camber of wings and finally the landing distance of biplane with reversible pitch propeller have been computed. For easy comparing of these landing distances, following table is made.

Type of planes	Landing Distance
No. 1 Usual R. A. F. 15 biplane	310 feet
No. 2 (a) type monoplane	210 feet
No. 3 (5/a) slotted wing monoplane	284 feet
No. 4 R.A.F. biplane with flaps	280 feet
No. 5 R.A.F. 13 monoplane with flap	210 feet
No. 6 Slotted Aerofoil with flap R.A.F. 9 monoplane	210 "
No. 7 R.A.F. biplane with reversible pitch propeller	140 "

No. 2 monoplane in the sky and biplane when landing has a very good landing distance which is only 210 feet.

No. 2 is only one possible method by which the landing distance, can be improved effectively when we only consider the limit of variable wing area. However, it is almost impossible and will be always difficult that a monoplane in the sky having good performance can be separated into biplane by using some mechanisms. No. 2 is a pretty good scheme for improving the landing distance only theoretically but is not at all good for practical application on account of the great difficulties of mechanisms.

No. 3 and No. 4 do not so much improve the landing distance as shown on the above table.

No. 5 is pretty good improving of the landing distance and this scheme can be accomplished with no difficulty and almost without extra weight for mechanism. Therefore, monoplane with flap extending along whole length of the wing is a good and preferable scheme by which the landing distance is improved.

As British Advisory Committee Report shows us, 0.22 of the chord flap gives better results than 0.385 of the chord flap for any points of view.

No. 6 is also one of the good schemes but some difficulty and also some extra weights for slat-mechanism must be accompanied. Therefore, No. 5 which is monoplane with flap along all length of the wing is better than No. 6 which is slotted wing with flap.

No. 7 the biplane equiped by reversible pitch propeller gives us a wonderful result which is only 140 feet in landing distance.

It is supposed that much practical and theoretical work remains to be done before full possibilites of the reversible pitch propeller can be realized. However, the realization of the reversible pitch propeller is not unexpected in the near future with or without some sacrifice for propeller efficiency.

Therefore, we get the final conclusion that the reversible pitch propeller can improve the landing distance more than any other scheme.

The aeroplane with reversible pitch propeller is the best type for minimum landing distance and the monoplane with a flap along all the length of the wing is the next to the former.

III The Best Type of Aeroplane of Mininmum Getting Away Distance.

It is much more difficult to get the exact determination of the getting away length because the attitude of the machine may change during the run and also thrust of the propeller varies during the run.

For every machine therefore is clearly a series of best attitude for getting away during the run.

At any rate, when the machine starts off, the equaton of motion becomes

$$\frac{W}{g}\frac{dv}{dt} = T - K_D v^2 S - \mu(W - K_L S v^2)$$

The motion is the same as that previously employed and T is thrust of the propeller. The frictional coefficient of the ground is very small in this case because as soon as the machine starts off the skid does not touch the ground unless the center of gravity of the machine is very backward.

Therefore we can put as a constant 0.05 which was used as the frictional coefficient of the wheels.

To solve the equation thrust must be written as some functions

of velocity of the machine where this substitution must hold good for low speeds.
But unfortunately any wind tunnel tests of propeller do not show us their characristecs at the part of which abscisa (Speed divided by nD) is less than 0.2.
Therefore wecan not discuss their thrusts durin g the getting away run using the wind tunnel tests of propellers.
The thrust of propeller at rest can be obtained by very easy experiment, and the thrust at getting away speed can be calculated from the wind tunnel model experiments.
For the usual propeller the thrust at getting away speed is graeter than this of at the rest of the machine and the thrusts during the getting away run are cleary between them.
The thrusts of propeller during the run can not be able be written by some functions of speed of the machine without knowing the charactcristics of the propeller at very low speeds.
So it is assumed that T is a const which exists between the the places of starting off and getting away, and gives the the correct value for getting away distance when we solve the equation.

Then the equation becomes

$$\frac{W v\, dv}{g\, dl} = T - K_D v^2 \rho - \mu (W - K_L \rho v^2)$$

or

$$\frac{W v\, dv}{g\, dl} = T - \mu W - (\lambda - \mu) K_L \rho v^2 \text{ ------------(3)}$$

Integrate the limits l is equal to zero at v is equal to zero; l is equal to L ie the getting away distance at v is equal to V_0 ie the getting away speed of the machine.

$$L = \frac{V_0^2}{2g(\lambda - \mu)} \log_e \frac{T - \mu W}{T - \lambda W} \text{ --------------(4)}$$

The equation (4) shows us that getting away distance is proportional to the square of getting away speed and is about ~~reciprocal~~ Proportional to λ.

Relations between the getting away distance and λ and also T/W can be shown easily by applying approximations,

$$L = \frac{V_0^2}{2g(\lambda - \mu)} \log_e \frac{1 - \mu W/T}{1 - \lambda W/T}$$

$$= \quad '' \quad \left\{ \log(1 - \mu W/T) - \log(1 - \lambda W/T) \right\}$$

where both $\mu W/T$ and $\lambda W/T$ are small values.

Or

$$L = \frac{V_0^2}{2g} \left\{ W/T + \frac{W^2}{2T^2} (\lambda + \mu) \right\}$$

hence the getting away distance varies as square of the getting away speed , it is very efficient to reduce that speed but in this case of getting away the reduction of that speed will introduce the dangerous accident because any aeroplane becomes unstable under certain limit of the speed while this limit is not so conciderably low.

8 Rough Calculation of Getting away Distance of Usual type of Aeroplane.

The aeroplane of 400 brake horse power Liberty engine ..; its total weight is 4500lbs and stalling speed is 40 miles per hour; the propeller of 17 of Doctor Durand is 9.5 feet in its diameter ; will be used for calculation of the getting away distance.

It will be assumed that as soon as the machine starts of the propeller thrust line becomes in the level, and total drag to lift ratio 1/7 ie 1/7 = 0.14.

At the conditions of

{ speed = 40 m.ph. = 58.6 feet per second.
revolution of the propeller = 1570 r.pm
= 26.2 r.p.s.

The propeller No 17 type of Doc . Durand has the following properties which were shown on the Fig.5 previously.

$$\begin{cases} v/nD = 0.235 \\ Q = 5.85 \\ T = 6.9 \end{cases}$$

the torque $= \dfrac{5.85 \times 9.5 \times 0.0761}{1000} = 1320$ lbs ft.

the thrust $= \dfrac{6.9 \times 9.5 \times 5.85 \times 0.0761}{100} = 1650$ lbs

the brake H.P. $= \dfrac{1320 \times 2 \times 26.2}{559 \quad 550} = 395$ horse power.

thrust T will be used as 1500 lbs. during the run , because the thrust is a little less at the begging part of the getting awy run. The getting away distance of the machine is calculated in the above rough assumptions.

The req. distance is equal to

$$\frac{58.5^2 \times 2.3}{64(0.14-0.05)} \log_{10} \frac{1500-0.05 \times 4500}{1500-0.14 \times 4500}$$

or $1370 \log 1.47$

or 230 feet

230feet is the getting away distance of the machine using the some assumptions.

On the above computation 1500 lbs. may be a little too great as the mean value of the thrust during the getting away run, but this is little important for the purpose of compering the types and finding out the best type of the machine of minimumgeting away distance.

9 The Getting Away Distance of The Other Types of Aeroplanes.

(a) The Getting Away distance of (a) Type of Monoplane which can be Separated into Biplane.

(a) type of monoplane which can be separated into biplane reduces the getting away speed about 20 % while total drag to lift ratio is little different compered with anusual type of the aeroplane .

Therefore the getting away distance were conciderably reduced if (a) type of monoplane could get away as a biplane . But it can not be realized because not only the mechanics for to separate monoplane into biplane is very difficult in itself, but such small speed tends the machine tounstable.

(a) type of monoplane is not worthy to be concidered in this problem.

(b) The Getting Away Distance of

Slotted Wing Aeroplane.

Slotted wing monoplane can reduce the g etting away speed about 20% in its value but drag to lift ratio becomes more than 1 5 which willbe seen from the previous figure.

$$\text{The req. distance} = \frac{48X2.3}{64(0.2-0.05)} \log \frac{1500-0.05X4500}{1500-0.2X4500}$$

$$\fallingdotseq 180 \text{ feet.}$$

The distance is conciderably short, but as(a) type of mono monoplane great cutting off the getting away speed introduce an unstable condition.

(We can expect that (a) type of monoplane's getting away distance is shorter than 180 feet.)

(c) R.A.F.15 Monoplane with Flap

Extending all The Length.

(Flap is 0.22 of the chord 60 degrees to the chord at getting awayspeed)

In this case 8percent decreasing of getting away speed cuts off the distance but hte increasing of e drag to lift ratio elongates the distance with graet deal, because drag tolift is increased to 0.25 which will be seen from the figure .

The req. distance is

$$\frac{53.2^2 \times 2\cdot3}{64(0.25-.05)} \log_{10} \frac{1500-225}{1500-0.25\times4500}$$

or 268feet.

R.A.F.9monplane of slotted wing with flap gives about the same length of getting away distance.

The si distance of getting away discussed on this paragh (c) is greater than that of usual type of aeroplane.

(d) The Getting Away Distance of The Usual Typeo of Aeroplane Equiped by Vari able Pitch Propeller.

By variable pitch propeller , we can get the greater thrust at the low speed of the machine , because best angle of attack can be used for the blades at which lift to drift ratiois vry great and revolution mustbe increased with a concidérable amount, but thrust is propo-

rtional to square of number of revolution.
Rough estimation of increasing of the thrust with will be tried which a variable pitch propeller can makes.
The thrust of a propeller is given by

integration of $(dL.\cos\phi - dD.\sin\phi)$

where dL and dD are lift force and drag of the element of the blades; ϕ is the angle of

$$\tan^{-1}\frac{\text{speed of the machine}}{2\pi.R.\text{revolution p.s.}}$$

But for the range of very slow speed of the machine as durong the getting away run the angle is very small and we can put $(dL.\cos\phi - dD.\sin\phi)$ the element thrust as only dL.

By the same reason the elementtorgue can be put as $R(dD + dL\phi)$ approximately during the run. {$\phi = 0$ at rest $\phi = 6° \times \frac{1}{57.3}$ at the end of the run $\phi = 3° \times \frac{1}{57.3}$ will be taken as the mean value}
For easy and quick calculation of the thrust , the same assumptions as the previously applied ,ie all the area of thepropeller blades were assembled at two third of the radius
We knew already that this assumption hold good because

it checked pretty well.

Now this assembled area is inclined to the plane of rotation at about 20 degrees in u-sual propell er , taking the safe side 18 degrees will be used as the inclination of the area of the blades.

The lift coefficient of propeller-blade-aerofoil has 0.56 in absolute units when the mean of the several aerofoils in British Advissary Commitee Report are taken. at 18 degrees of angle attack while lift to drag ratio is less than 5 in the average.

On the other hand at 3 degrees of angle of attack, mean lift coefficient is 0.28 while lift to r drag ratio is graeter than 17 in evry aerofoil examined in National Phyical Labolatory, taking the safe side 16 will be used as the ratio.

If a variable pitch propeller is used when getting away and the best angle of incidence 3 degrees is utilized for the propeller blades.

Then at the same revolution the thrusts and the torques of the two kind of propellers can be compered as follows

$$\frac{\text{Thrust of varia. type}}{\text{Thrust of usual type}} = \frac{0.28}{0.56} = 0.5$$

at same rev.

$$\frac{\text{Torque of varia. type}}{\text{Torque of usual type}} = \frac{(0.28/16)}{(0.56/5)} = 6.4$$

$$= \frac{0.28/16 + .28\times\phi}{0.56/5 + .56\times\phi} = \frac{0.28/16 + .28\times .052}{0.56/5 + .56\times .052} = \frac{1}{5.2}$$

at $\left\{\begin{array}{l}\text{same rev.}\\ \phi = 3^\circ \times \frac{1}{57.3} = 0.052\end{array}\right.$

Therefore revolutons of the two kind of propeller are given in their ratio when the same power of engine is used

$$\frac{\text{Rev. of varia type}}{\text{Rev. of usual type}} = (\text{cube root of } 5.2) = 1.73 \text{ at Same power.}$$

Finally thrust ratio of the two is given by

$$\frac{\text{Thrust of varia. type}}{\text{Thrust of usual type}} = 0.5 \times \text{square of } 1.73 = 1.50$$

where the same power of engine and the same propellers but variable pitch and fixed pitch type are compered.

When a variable pitch propeller is used , the thrust can be increased 50% during the getting away run.

In real variable pitch propeller , the increasing of the

thrust at low speed may not be so much as 50 % , becuase 73 % of increasing in its revolutions will break the propeller . But it is a clear matter that the increasing percentage of the thrust when variable pitch propeller is used is greater than 20% because bending moment is decreased while centrifugal force is increased.
when the increasing ampunt is assumed only 15 % of an usual type of propeller , the getting away distance is calculated as 187 feet which is short enough .

10 Conclusion for Best Type of
Minimum Getting Away Distance.

The quetion of the length of getting away offeres less difficulties in commertial flying , since for getting away there can be abetter selection of ground . It is clear that the distance of landing is more important in commertial flying than that of getting away distance.

1 Decreasing of getting away speed is one of the most effective methode to get short getting away distance , but it is not good sheme for the practical application.

(a) type of monoplane gives very short distance but this type may never be used for reducing the getting away distance.

Slotted wing type of aeroplane is only one good type i in many types ofwhich can reduce the getting away speed.

2 Variable pitch propeller reduces the length of gettig away run very well without cutting off the speed. Therefore the aeroplane equiped by a variable pitch propeller is the best type of aeroplane for minimum distance of getting away run.

11 CONCLUSION FOR BEST TYPE OF MINIMUM LANDING AND MINIMUM GETTING AWAY DISTANCE.

1 Reversible pitch propeller (including the variable pitch) gives minimum distance of the landing and also minimum getting away while all practical condition s are satisfactory.

2 Variable wing area reduces the landing distance , but it is not good for practical reasons ; this acheme gives very short getting away distance , but not good for the same reasons.

3 Variable camber of wing:

(a) Slotted wing gives a little better landing length than an usual type of aeroplane ; and the getting away distance is pretty short , but variable pitch propeller is better than on account ~~of the~~ of the practical reasons.

(b) Flap extending along all the length of monoplane wing reduces the distance very well ; but has the very bad getting away distance.

THE AEROPLANE EQUIPED BY AREVERSIBLE PITCH PROPELLER (INCLUDING A VARIABLE PITCH) IS THE BEST TYPE OF THE airplane WHICH HAS MINIMUM LANDING AND ALSO MINIMUM GETTING AWAY DISTANCE WHILE ALL THE PRACTICAL CONDITIONS ARE SATISFACTORY IF SUCH KIND OF PROPELLER WERE REALIZED.

Aditional Works:

Contents | Pages

Landing Run Distance of 6 Slotted
Handley Pages Wing Biplane

With 6 slotted R.A.F 15 wingthe lift coefficient reaches the abnormal value of 1.96 in absolute unit at 45 degree of incidence while the coefficient of R.A.F.15 has the value 0.538 at stalling angle of incidence14 degree.

As already mensioned , the Handley Pages slotted monoplane can be applied to biplane calculation as the usual type of wing.

Extra weight of equipment for 6 slotted Handley Pages wing may be estimated under the total wing weight. Take the extra weight as the total weight of R.A.F. biplane win_.

$$W_6 = 2W$$

$$W_6 = 1.15W$$

$V_6 =$ the landing speed of the plane

$$1.15W = \zeta K_{L6} \rho V_6^2 A$$

$$= 1.15 \zeta K_L \rho V_0^2 A$$

where ζ is correction of the biplane effect

$$V_6^2 = 1.1 \frac{.538}{1.96} V_0^2$$

$$= 0.316 V_0^2$$

Therefore $V_6 = 0.562 V_0 = 22.5$ m. p.h. $= 33$ feet per se

The value of lift to drag ratio of 6 slotted wing is not given but we can estimated lift to drag ratio is 5 from the figure which shows L to D ratio of the 5 slotted and 7 slotted.

$$\text{Therefore } K_D = \frac{1.96}{5} + \frac{0.069}{4}$$

$$= 0.41$$

$$\lambda_6 = 0.41/1.96 = 0.209$$

The landing rundistance

$$= \frac{33^2 \times 2.3}{64(0.209-0.12)} \log_e \frac{0.209}{0.12}$$

$$= 106 \text{ feet}$$

It seems that the above computation is not correct in the practical case because 45 degree of incidence is too much inclined to be kept during the landing run.

When the machine lands with 45 degree of incidence the body must be inclined to the ground over than 40 degree and as soon as the skid touches the ground the machine rotates rather rapidly about the skid until the under carriadges touch the ground while the machine is running.

Therefore the both the lift and drag coefficients becomes less than those of at 45 deree of incidence.

The experiments of Handley Pages wings show us that when the number of slotts is increased the value of L to D ratio is flattened at greater angle of incideces than 10 degree.

And their maximum difference is only "one" at the range of large angle of incidence. While the body resistance is pretty small quantity in this case, and the body resistance has little influence upon the landing distance.

The difference 1 of Lto D ratio makes only 10 percent difference of landing distance.(If we take L to D ratio as 6 in its value then we get 117 feet as the landing distance.)

At any rate the landing distance of this type can be estimated as a little over than 100 feet.

The landing of this type may be pretty difficult but not impossibly difficult because the controability is very bad at such small.speed.

The Distance of Diving and Flattening out.

When an aeroplane lands , the aeroplane acrosses the edge of the aerodrome at height H which is sufficient height to give clearance over the trees and buildings . And then the plane glides with constant speed or dives with variable speeds to the height H_0.

150 feet may be high enough for H in usual; and H_0 is rather arbitrary, because H_0 is depending upon the performance of the machine and also the pilot.

The heavy and high speed machine is needed to be flattened out at higher position, and light and slow speed machine is contrary to the former one.

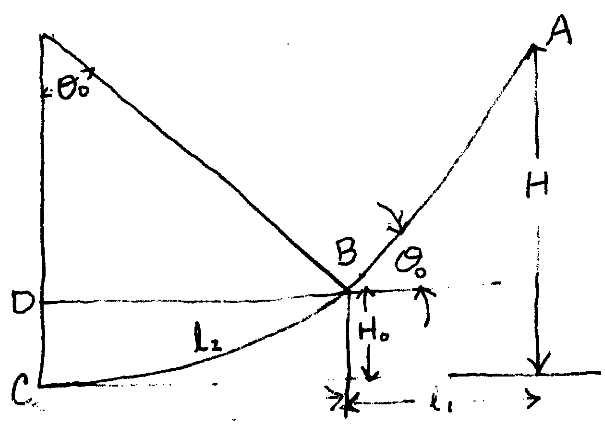
If . flattening out the machine is done gradually which is an usual and better than suddenly flattening out , then the path of the aeroplane is curved from the straight lined gliding down or diving path.

The curved path may be some what different from a circle but it can be assumed as a circle while the centirpedal force is created by increasing the lift coefficient of the aeroplane due to increasing the angle of incidence of the wings.

The machine glides or dives with the path inclined to the horizontal from A to B while the engine is str-ottled down near to shut off.

At B the pilot begines to flatten out his machine until the machine touches the ground at which the machine has a little vertical speed , especially a skilful pilot let his machine almost no vertical speed when his machine touches the ground.

Vertical speed is negligible small when compared with the horizontal speed, consequently the path is tange-ntial at C to the ground.



Therefore $OC = OB = R =$ radius

angle BOC $\equiv \theta$ the angle between the path and horizontal.

$R - R\cos\theta_o \equiv H_o$

or $R = H_o / 2\sin^2\frac{\theta_o}{2}$

horizontal distance of BC $=$ BD $=$ $R\sin\theta_o$

$= H_o \cot\frac{\theta_o}{2}$

The horizontal distance is only function of θ_o when H_o is given.

Deeper diving angle gives the shorter horizontal distance ofdiving , but the deeper diving creates the greater speed at B and hence the speed of the machine at C may not be decreased to the stalling speed of the machine which is most important factor of the landing runing distance. The large gliding angle cutts distances short horizontal distances of gliding and flattening out, but increases the runing distance when the height of H_o is given.

If $\theta_o = 30^o$

$H = 150$ feet

$H = 80$ "

Then

$$R = \frac{H}{2\sin\frac{\theta_0}{2}} = \frac{H}{.134} = 600 \text{ feet}$$

$$l_2 = \text{distance BC} = \frac{R\,30}{57.3} = 314 \text{ feet}$$

$$\text{horiz. distance BC} = 80\cot 15^{\circ} = 300 \text{ feet}$$

Assuming the angle of incidence is not so much different from the normal one and taking the drag coefficient is about the same and drag is changed by only speed; the speed at B can be obtained as follows.

$$\frac{W\,du}{g\,dt} = W\sin 30^{\circ} - K_D A u^2$$

put $\begin{cases} \lambda = \dfrac{\text{total resistance}}{W} \\ V_0 = \text{normal speed} \end{cases}$

or

$$\frac{Wu\,du}{g\,dl} = W\sin 30^{\circ} - K_D A u^2$$

$$= W\sin 30 - \lambda W\, u^2 / V_0^2$$

Integrate the above equation using the limites

$$u = u \text{ at } l = l_2 \text{ and } u = V_0 \text{ at } l = 0$$

$$\frac{V_o^2}{2g\lambda}\log_e \frac{\sin 30^\circ - \lambda}{\sin 30^\circ - \lambda u^2/V_o^2} = l_2$$

or
$$\frac{\sin 30^\circ - \lambda}{\sin 30^\circ - \lambda u^2/V_o^2} = e^{2g\lambda l_2/V_o^2}$$

or
$$\left(\frac{u}{V}\right)^2 = \sin 30^\circ - \frac{\sin 30^\circ - \lambda}{e^{2g\lambda l_2/V_o^2}}$$

By substituting $\lambda = 1/6$ $l = 314$ feet & $V = 90$ m.p.h $= 132$ feet per sec

$u/V_o = 1.14$

If we take 3° as the normal angle of incidence, then the angle of incidence of R.A.F.15 biplane is 2° at B which is calculated from the diffence of lift coefficient by using the following formula

$$W\cos = K_L' \gamma A \, V_o^2 \cos\theta_o$$
$$= K_L' \gamma A \, (V_o \times 1.14)^2$$

where K_L is the lift coefficient at 3 angle of incidence, K_L' is the angle of incidence at B .

$$K_L'/K_L = \frac{0.866}{1.32} = .66$$

From the above equation we get 2° as a the angle of incidence at B.

Centripetal force for the circular motion is

$$\frac{W\ V_0^2}{g\ R} = \frac{W \times 2300}{32 \times 600}$$

if $\begin{cases} V = \sout{90}\ 90 \times 1.15 \text{ m.p.h.} = 152 \text{ feet per sec} \\ R = 600 \text{ feet} \end{cases}$

$= 1.2W$

Therrefore the load due to lift during the flattening out at the given conditions is a little over than 2W.

Any aeroplane is strong enough to be loaded by 2W.

From the above calculation we get following results

$$\begin{cases} u = 1.15\ V_0 \quad \text{at} B \\ \text{load due to lift} = 2.2W \\ \text{angle of incidence at} B = 2 \end{cases}$$

Now we have to calculate hou much speed of the machine can be reduced at C by flattening out. R.A.F.15 usual type will be applied at first for computation of the speed atC.

(a) The Speed of R.A.F Biplane after Flattening out.

The equation of motion along the path during flattening out becomes

$$\frac{W\,du}{g\,dt} = W\sin\theta - (C_D A + bW)u^2$$

where
- θ is the angle between the path and horizontal
- C_D is drag coefficient of wings
- bWu^2 is body resistance

or

$$\frac{W\,du}{g\,dt} = W\left\{\sin\theta - \left(\frac{C_D}{K_L V_0^2} + b\right)u^2\right\}$$

where
- K_L is the lift coefficient of the wings at the normal angle of incidence
- V_0 is the normal speed

or

$$\frac{u\,du}{dl} = \sin\theta - \left(b + \frac{C_D}{K_L V_0^2}\right)u^2$$

$$\begin{cases} \theta = 30 & \text{at B i.e. } l = 0 \\ \theta = 0 & \text{at C } l = l_2 = 314 \text{ feet.} \end{cases}$$

If we assume the inclination of the path is linear function of the length of the path, then

$$\theta = 30^\circ\left(1 - \frac{l}{314}\right)$$

where 314 feet is the total length of the path when $H_o = 80$ feet and diving angle is 30° at the begging of flattening out.

In the above equation K_L and V_o are known and constants and also b can be assumed as a constant approximatery. But C_D the drag coefficient of the wings is changed by angle of incidence, therefore it is necessary that the dragcoefficient C_D is written byfuction of angle of incidence or the length of the path.

We can see easily that the drag coefficient curve of R.A.F. 15 wing between 2 and 14 degrees of incidence which is the useful range for this calculation, is represented by a kind of exponential function.

$$C_D = \alpha e^{\beta x}$$

Where { α and β are consttants; x is angle of incidence between 2 and 14 degrees.

angle of incidence	C_D (lbs ft sec units)	α	β
0°	0.0000194	0.0000194	—
2°	0.0000246	"	0.120
4°	0.0000351	"	0.140
8°	0.0000486	"	0.148

angle of incidence	C_D	α	β
8°	0.0000656	0.0000194	0.150
10°	0.0000860	"	0.149
12°	0.000109	"	0.144
14°	0.000149	"	0.145

The drag coefficient can be represented by

$$C_D = 0.0000194e^{0.146x}$$

The constants 0.0000194 and 0.146 have not been obtained by least square methode , but they hold pretty good between the angles 2° to 14°.

We have to change x the angle of incidence to the length of the path.

$$C_D = 0.0000194\ e^{0.146x}$$

$$= 0.0000194\ e^{0.292 + 0.00362\ell}$$

$$= c_1\ e^{c_2 + c_3\ell}$$

Now the equation of motion during flattening out becomes

$$\frac{u\ du}{g\ dl} = \sin\left\{30^\circ (1 - \ell/314)\right\} - \left(b + \frac{c_1 e^{c_2 + c_3\ell}}{K\ V}\right) u^2$$

and $\sin 30(1 - \ell/314)$ beocmes . $0.50 - 0.0012\ell$ approximately.

put $\begin{cases} 0.50 = a_1 \\ 0.0012 = a_2 \\ u^2 = U \end{cases}$

$$\frac{dU}{2g} \left(b + \frac{c_1 e^{c_2 + c_3 l}}{K V} \right) U \, dl = (a - a l) \, dl$$

This is a Bernoulis equation which can be integrated using the integrating factor of

$$e^{\int 2g \left\{ b + \frac{c_1 e^{c_2 + c_3 l}}{K_2 V_0^2} \right\} dl}$$

or

$$e^{2g \left(bl + \frac{c_1 e^{c_2 + c_3 l}}{c_3 K_2 V_0^2} \right)}$$

Then the equation becomes

$$e^{2g \left(bl + \frac{c_1 e^{c_2 + c_3 l}}{c_3 K_2 V_0^2} \right)} \left[dU + U 2g \left\{ b + \frac{c_1 e^{c_2 + c_3 l}}{K_2 V_0^2} \right\} dl \right]$$

$$= \left\{ e^{2g \left(bl + \frac{c_1 e^{c_2 + c_3 l}}{c_3 K_2 V_0^2} \right)} \right\} (a_1 - a_2 l) \, dl$$

or

$$d\left(U e^{2g \left(bl + \frac{c_1 e^{c_2 + c_3 l}}{c_3 K_2 V_0^2} \right)} \right) = e^{2g \left(bl + \frac{c_1 e^{c_2 + c_3 l}}{c_3 K_2 V_0^2} \right)} (a_1 - a_2 l) \, dl$$

The left hand side of the above equation is the exact differential which can be integrated directely but the right hand side can not be integrated directely. If we can expand $e^{c_2 + c_3 l}$ as a series of $(c_2 + c_3 l)$ then the right hand side can be integrated but unfortunatery

$e^{c_2+c_3 l}$ (at l = 314) = $e^{2.05}$ and the series is not converged when it is expanded

Unless we express C_D by some other functions of l the equation of motion during the flattening out can not be integrated.

C_D will be expanded by quadratic equation of angle of incidence ie by ($a + bx + cx^2$) where a, b and c are constants and x is a variable representing angle of incidece in degrees.

We can find constants a, b, c using the least square methode.

The result is shown as follows.

$C = 0.000011 \quad 0.0000065x \quad 0.00000056x$

The computation

at 0°	0.0000194	= a
" 2°	0.0000240	= a + 2b + 4c
" 4°	0.0000351	= a + 4b + 16c
" 6°	0.0000656	= a + 6b + 36c
" 10°	0.0000860	= a + 10b + 100c
" 12°	0.000109	= a + 12b + 144c
" 14°	0.000189	= a + 14b + 196c

From the above 8 equations we can get the following

3 normal equations

$$\begin{cases} 0.000521 = 8a + 56b + 600c \\ 0.00522 = 56a + 600b + 6770c \\ 0.0597 = 600a + 6770b + 274000c \end{cases}$$

Solving the above equations simaltaneusly, we obtaine

$a = 0.000011$

$b = 0.0000065$

$c = 0.00000056$

Therefore

$$C_D = 0.000011 + 0.0000056x + 0.00000056x^2$$

C_D is expressed by quadraticequation with pretty presice. Since the coefficient of x^2 is very small and has a little effect on C_D, the term can be neglected with about the same presice if we add a little correction on the coefficient of x.

$$C_D = 0.000011 + 0.0000068x$$

will be used as the equation which can be written as a function of lengthof the path

$$C = 0.000011 + 0.0000068\left\{2 + \frac{(14-2)\ell}{314}\right\}$$

$$= 0.0000246 + 0.00000026\ell$$

put $\begin{cases} 0.0000246 = f_1 \\ 0.00000026 = f_2 \end{cases}$

The equation of motion then becomes

$$\frac{u\,du}{g\,dl} = (a_1 - a_2 l) - \left(b + \frac{f_1 + f_2 l}{K_2 V_0^2}\right) u^2$$

put $U = u^2$

$$dU + 2g\left(b + \frac{f_1 + f_2 l}{K_2 V_0^2}\right) U\,dl = 2g(a_1 - a_2 l)\,dl$$

This is a Belnoulis equation which can be integrated using the integrating factor.

$$\text{The integrating factor} = e^{\int 2g\left(b + \frac{f_1 + f_2 l}{K_2 V_0^2}\right) dl}$$

$$= e^{2g\left(bl + \frac{f_1 l + f_2 l^2/2}{KV}\right)}$$

$$d\left(U e^{2g\left(bl + \frac{f_1 l + f_2 l^2/2}{K_2 V_0^2}\right)}\right)$$

$$= 2g(a_1 - a_2 l)\, e^{2g\left(bl + \frac{f_1 l + f_2 l^2/2}{KV}\right)}\, dl$$

On the above equation the left hand side is exact difr- ential and exponential of the right hand side can be expanded in the series which is integrated easily because $e^{2g\left(bl + \frac{f_1 l + f_2 l^2/2}{K_2 V_0^2}\right)} = e^{0.24}$ at $l = 314$ feet assuming the body resistance is $(W/18)$ at normal speed of 90 m p h.

By integrating after applying the expantion of exponential the equation of motion becomes as follows.

$$U\,e^{2g\left(bl + \frac{f_1 l + f_2 l^2/2}{KV}\right)}$$

$$= 2g(a_1 l - a_2 l^2/2) + 4g^2 a_1\left(bl^2/2 + \frac{f_1 l^2/2 + f_2 l^3/6}{\frac{K}{2}V_0^2}\right)$$

$$- 4g^2 a_2\left(bl^3/3 + \frac{f_1 l^3/3 + f_2 l^4/8}{\frac{K}{2}V_0^2}\right)$$

Substitute the limites

$$\begin{cases} l = 0 \\ U = V^2 = (90 \times 1.15)^2 = 23000 \end{cases} \quad (m.p.h)^2 \quad (feet/sec)^2$$

$$\begin{cases} l = 314 \\ U = U \end{cases}$$

$$U = \frac{V^2 - 7530}{\left(e^{0.244}\right)} = \frac{23000 - 7530}{1.27}$$

$$= 23200$$

Therefore u = 152 feet per second after flattening out.

(b) Gliding and Flattening Distance of R.A.F.15 Biplane.

The above calculation suggests us that when an aeroplane is flattening out from height of about 100 feet to the ground after diving with a great inclined angle to the horizontal direction, its speed after flattening out is still very higher than its stalling speed. Therefore it is clear that total landing distance (including gliding , flattening out, and landing run distance) is made shorter when an aeroplane glides at great angle of incidence ——— but at less angle of incidence corresponding to the stalling speed. cosequentry with a little larger speed than the stalling speed.

If an aeroplane glides at a large angle of incidence but a little less than the stalling angle of incidence then total drag coefficient is not so much changed and we can use an average value of coefficients.

If we can use a costant value as the drag coefficient , then the equation of motion can be integrated very easily and result is more simple.

Equations of motion during flattening out are

$$\begin{cases} \dfrac{W}{g}\dfrac{u\,du}{dl} = \theta W - K_D A u^2 & \text{along the path} \\[2ex] \dfrac{W}{g}\dfrac{u^2\,d\theta}{dl} = -\left(W - K_L A u^2\right) & \text{perpendicular to the path} \end{cases}$$

where u is the speed ; θ is the angle of flight path to the horizontal at any instant.

It is covenient to use a subsidery speed V defined by

$$W = K_L A V^2$$

Finally we assume as shown on the front page that a mean value of K_L & and K_D may be used , and cosequetry a mean value of λ may be used for the drag to lift ratio

Then the two equations are writen

$$\begin{cases} \dfrac{u\,du}{dl} = g\left(\theta - \lambda\dfrac{u^2}{V^2}\right) \\ \dfrac{u^2 d\theta}{dl} = +g\left(1 - \dfrac{u^2}{V^2}\right) \end{cases}$$

Eliminate V from these equations

$$u\frac{du}{dl} - \lambda u^2\frac{d\theta}{dl} = g(\theta - \lambda)$$

or

$$u\,du - \lambda u^2 d\theta = g(\theta - \lambda)\,dl$$

As before the angle of the inclination of the path to the horizontal is assumed that it is defined by

$$\theta = \theta_0\left(1 - \frac{l}{l_2}\right)$$

Therefore $d\theta = -\theta_0\,dl/l_2$

$$u\,du + \frac{\lambda\theta_0 u^2}{l_2}dl = g\,(\theta - \lambda)dl$$

or $$du^2 + \frac{2\lambda\theta_0 u^2}{l_2}dl = g\{\theta_0(1-\frac{l}{l_2}) - \lambda\}dl$$

Using the integrati ng factor

$$e^{\frac{2\lambda\theta_0 l}{l_2}}\{du^2 + \frac{2\lambda\theta_0 u^2}{l_2}dl\} = e^{\frac{2\lambda\theta_0 l}{l_2}}\,2g\{\theta_0(1-\frac{l}{l_2}) - \lambda\}dl$$

Interating

$$e^{\frac{2\lambda\theta_0 l}{l_2}}u^2 = 2g\frac{e^{\frac{2\lambda\theta_0 l}{l_2}}(\theta_0 - \lambda)}{\frac{2\lambda\theta_0}{l_2}} - 2g\int e^{\frac{2\lambda\theta_0 l}{l_2}}\frac{l}{l_2}dl$$

$$= \quad '' \quad - 2g\frac{\theta_0 l\,e^{\frac{2\lambda\theta_0 l}{l_2}}}{l_2(\frac{2\lambda\theta_0}{l_2})} + 2g\frac{\theta_0 e^{\frac{2\lambda\theta_0 l}{l_2}}}{l_2(\frac{2\lambda\theta_0}{l_2})^2}$$

Applying the limits

($u = u$ at $l = l_2$) and ($u = u_0$ at $l = 0$)

$$u\,e^{2\lambda\theta_0} - u_0 = 2g\left\{\frac{(\theta_0 - \lambda)(e^{2\lambda\theta_0} - 1)l_2}{2\lambda\theta_0} - \frac{l_2 l}{2\lambda} + \frac{\theta_0 l_2(e^{2\lambda\theta_0} - 1)}{(2\lambda\theta_0)^2}\right\}$$

$$= 2g\left\{\frac{(\theta_0 - \lambda)2\lambda\theta_0 l_2}{2\lambda\theta_0} - \frac{l_2(1 + 2\lambda\theta_0)}{2\lambda} + \frac{l_2}{2\lambda}\right\}$$

$$= 2g\{(\theta_0 - \lambda)l_2 - \theta_0 l_2\}$$

$$= -2g\lambda l_2$$

or

$$u(1 + 2\lambda\theta_0) - u_0 = -2g\lambda l_2$$

Therfore

$$\frac{l}{2} = \frac{u_0^2 - u^2(1 + 2\lambda\theta_0)}{2g\lambda} \text{-----------------} (A)$$

The height of flattening out is

$$h = \int dl \times \tan^{-1}\theta = \int \theta dl = \theta_0(l - l^2/2l_2)$$
$$= (1/2)\, l_2\theta_0 \text{---------------------------} (B)$$

From the equations(A) and (B) the length of the path and the height of flattening out can be required when an aeroplane glides at greate angle of incidence and cosequentry with a small inclination of the path to the ~~ground~~ horizontal.

The speed u is a little larger than the stalling speed and the speed u is the stalling speed with which the aeroplane lands on the ground.

R.A.F.15 biplane which was used for calculation of landing run distance has the properties at h the stalling angle of incidence as shown below.

$0.197 = 0.2$

stalling speed = 40 m.p.h.

If the aeroplane glides down at 10 degree of incidence at which the machine has 59.5 m.p.h. as the speed and 0.18 as the drag to lift ratio.

Then

$u_0 = 58.6$ feet per sec.

$u = 87.4$ feet per sec.

the inclination of the path at the begging of lattening out = tan0.18 = 10 degree = θ.18 in radiün

$\fallingdotseq 0.19$

The required distance is

$$\frac{1}{2} = \frac{(87.4)^2 - 58.6^2(1 + 2 \times 0.18 \times 0.19)}{2 \times 32 \times 0.19}$$

$$= \frac{7600 - 3980}{12.1}$$

$\fallingdotseq$ 299 feet

= 300 feet

The required height for flattening out h

= 1 2 0.18 300 = 27 feet

If the R.A.F15 biplane glides with 10 degre of incidence from the edge of the aerodrome at which the machine is 100 feet height from the ground , and then flattening out from the height of 27 feet to the groud the total landing distance is requird as follows.

The total landing distance

= gliding distance + flattening out distance +landing run distance

= 410 feet + 300 feet + 320 feet

= 10 30 feet

The Total Landing Distance of The Aeroplane with Flaps (022 of the chord of monoplane)

The monoplane with flaps which is ajusted 30 degree to the chord at the stalling speed 53.5 feet per sec. has the drag to lift ratio of ~~0.25~~ 0.25 at stalling speed.

If the aeroplane glides at 10 dgree of incidence, then its apeed is expected as 66 feet per sec. and drag to lift ratio a little larger than 0.25.

Applying the formulae (A) and (B), we get

$$\frac{l}{2} = \frac{4350 - 3100}{16}$$

$$= 78 \text{ feet}$$

$$h = 9.7 \text{ feet}$$

The total landing distance

= 360 feet + 78 feet + 250 feet

= 690 feet