Torsion Properties for Line Segments and Computational Scheme for Piecewise Straight Section Calculations Closed Thin walled Sections

the new material consists of the "corrections for ω and $Q\omega$

definition of

A = enclosed area

$$d\Omega_c = \left(h_c - \frac{J}{2\cdot A}\cdot\frac{1}{t}\right)\cdot ds = d\omega_c \qquad \omega_c = \int h_c\, ds - \frac{J}{2\cdot A}\cdot\int_0^s \frac{1}{t}\, ds = \frac{2\cdot A}{\int_0^b \frac{1}{t}\, ds}\cdot\int_0^s \frac{1}{t}\, ds$$

as

$$J = \frac{4\cdot A^2}{\int_0^b \frac{1}{t}\, ds}$$

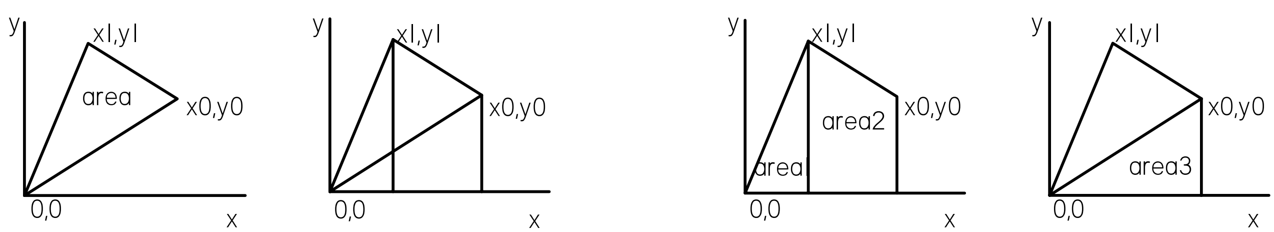
$$\text{circ\_integral} = \int_0^b \frac{1}{t}\, ds = \sum_i \frac{\sqrt{(\Delta X_i)^2 + (\Delta Y_i)^2}}{t_i} = \sum_i \frac{\Delta l_i}{t_i}$$

if we define segment length:

$$\Delta l_i = \sqrt{(\Delta X_i)^2 + (\Delta Y_i)^2}$$

we now need calculation of the enclosed area A in this expression
area of triangle determined by two points and the origin:

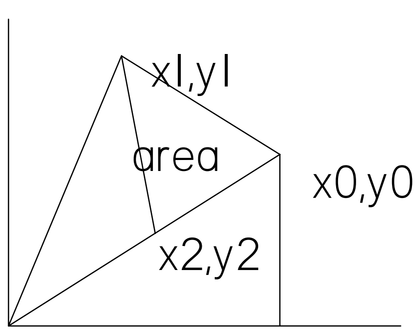
$$\text{area} = \text{area1} + \text{area2} - \text{area3}$$



$$\text{area1} := \frac{1}{2}\cdot y1\cdot x1 \qquad \text{area2} := \frac{1}{2}\cdot(y0 + y1)\cdot(x0 - x1) \qquad \text{area3} := \frac{1}{2}\cdot y0\cdot(x0)$$

$$\text{area} := \text{area1} + \text{area2} - \text{area3} \qquad \text{area simplify } \rightarrow \frac{-1}{2}\cdot y0\cdot x1 + \frac{1}{2}\cdot y1\cdot x0 \qquad \text{area\_2\_pts\_origin} := \frac{1}{2}\cdot(y1\cdot x0 - y0\cdot x1)$$

area between three points:



$$\text{area} := \frac{1}{2}\cdot(y1\cdot x0 - y0\cdot x1) - \frac{1}{2}\cdot(y1\cdot x2 - y2\cdot x1)$$

$$\text{area} := \frac{1}{2}\cdot(y1\cdot x0 - y0\cdot x1) + \frac{1}{2}\cdot(y2\cdot x1 - y1\cdot x2)$$

etc.........

$$\text{area\_enclosed} := \frac{1}{2}\cdot\sum_i \left(X_i\cdot Y_{i+1} - X_{i+1}\cdot Y_i\right)$$

for a straight line segment $\rho_c = \text{constant}$ $\Delta\omega_c = \rho_c \cdot L - \frac{2\text{area\_enclosed}}{\text{circ\_integral}} \cdot \frac{\Delta l_i}{t_i}$ and is linear along line

see torsion properties (open) for derivation of ρc part:

$$\Delta\omega_c = \frac{x_1 + x_0}{2} \cdot (y_1 - y_0) - \frac{y_1 + y_0}{2} \cdot (x_1 - x_0) - \frac{2\text{area\_enclosed}}{\text{circ\_integral}} \cdot \frac{\Delta l_i}{t_i}$$

$$\Delta\omega_c = xm \cdot (\Delta y) - ym \cdot \Delta x - \frac{2\text{area\_enclosed}}{\text{circ\_integral}} \cdot \frac{\Delta l_i}{t_i}$$

xm = mid-point
Δx = x1 - x0
Δy = y1 - y0

$$d\Omega_D = d\omega_D = h_D \cdot ds \quad \Rightarrow \quad \Omega_D(s) = \int_0^s h_D \, ds = \int_0^s h_C - x_D \cdot \sin(\alpha) + y_D \cdot \cos(\alpha) \, ds$$

calculation of ΩD and ωD aka ω identical to open $\Delta\Omega_D(s) = \Delta\omega c - x_D \cdot (y_1 - y_0) + y_D \cdot (x_1 - x_0)$

if we set $\Omega_{D0} = 0$ at the start of a line segment, then $\Omega_{D1} = \Omega_{D0} + \Delta\Omega_D$

calculate "centroid" of warping wrt shear center:

$$\Delta Q_{\Omega D_i} = \frac{a_i}{2} \cdot \left(\Omega_{D_i} + \Omega_{D_{i+1}}\right) \qquad \Omega_{Dcg} = \frac{\sum \Delta Q_{\Omega D}}{A} \qquad \omega_{D_j} = \Omega_{D_j} - \Omega_{Dcg}$$

$$I_{y\omega D} = \frac{t \cdot (s_1 - s_0)}{6} \cdot \left[2 \cdot (x_1 \cdot \omega D_1 + x_0 \cdot \omega D_0) + x_0 \cdot \omega D_1 + x_1 \cdot \omega D_0\right]$$

$$I_\omega = \frac{t \cdot (s_1 - s_0)}{3} \cdot \left[(\omega D_1)^2 + \omega D_0 \cdot \omega D_1 + (\omega D_0)^2\right]$$

$$I_{x\omega D} = \frac{t \cdot (s_1 - s_0)}{6} \cdot \left[2 \cdot (y_1 \cdot \omega_{D1} + y_0 \cdot \omega D_0) + y_0 \cdot \omega D_1 + y_1 \cdot \omega D_0\right]$$

first moment of ω needs "correction" also

$$q(s, x) = -\left[\frac{T_\omega}{I_{\omega\omega}} \cdot \left(Q_\omega - \frac{\int Q_\omega \, ds}{\int \frac{1}{t} \, ds}\right)\right]$$

thus a "correction" $\frac{\int Q_\omega \, ds}{\int \frac{1}{t} \, ds}$ is applied to Qω for the closed section. the ω is for the closed section (with it's correction applied)

first calculate ω as open which we did above, now calculate $\int Q_\omega \, ds$

we know Δω is piecewise linear over s (Δω = hD*Δs as hD constant over segment) thus over segment:

$$\omega(s) := \left[\frac{\omega 1 - \omega 0}{s1 - s0}\cdot s + \omega 0 - s0\cdot\left(\frac{\omega 1 - \omega 0}{s1 - s0}\right)\right] \qquad y = \frac{y_1 - y_0}{x_1 - x_0}\cdot x + y_0 - x_0\frac{(y_1 - y_0)}{(x_1 - x_0)}$$

$$Q_\omega(s) := \int_{s0}^{s} \omega(\sigma)\cdot t\, d\sigma + Q_{\omega 0}$$

$$\int_{s0}^{s} \omega(\sigma)\cdot t\, d\sigma \text{ collect}, s \rightarrow \frac{1}{2}\cdot\frac{\omega 1 - \omega 0}{s1 - s0}\cdot t\cdot s^2 + \frac{1}{2}\cdot\frac{2\cdot\omega 0\cdot s1 - 2\cdot s0\cdot\omega 1}{s1 - s0}\cdot t\cdot s - \frac{1}{2}\cdot s0\cdot\frac{-s0\cdot\omega 1 - \omega 0\cdot s0 + 2\cdot\omega 0\cdot s1}{s1 - s0}\cdot t$$

$$\Delta Q_\omega(s) := \int_{s0}^{s1} \frac{\int_{s0}^{s} \omega(\sigma)\cdot t\, d\sigma + Q_{\omega 0}}{t}\, ds$$

$$\int_{s0}^{s1} \frac{\frac{-1}{2}\cdot\frac{(\omega 1 - \omega 0)}{(-s1 + s0)}\cdot t\cdot s^2 - \frac{1}{2}\cdot\frac{(2\cdot\omega 0\cdot s1 - 2\cdot s0\cdot\omega 1)}{(-s1 + s0)}\cdot t\cdot s - \frac{1}{2}\cdot s0\cdot\frac{(s0\cdot\omega 1 + \omega 0\cdot s0 - 2\cdot\omega 0\cdot s1)}{(-s1 + s0)}\cdot t + Q_{\omega 0}}{t}\, ds \;\Bigg|\; \begin{matrix}\text{simplify}\\ \text{collect}, t, Q_{\omega 0}, s1, s0, t\end{matrix} \rightarrow \left(\frac{1}{6}\cdot\omega\right.$$

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$$\Rightarrow \quad \left(\frac{1}{6}\cdot\omega 1 + \frac{1}{3}\cdot\omega 0\right)\cdot s1^2 + \left(\frac{-1}{3}\cdot\omega 1 - \frac{2}{3}\cdot\omega 0\right)\cdot s0\cdot s1 + \left(\frac{1}{6}\cdot\omega 1 + \frac{1}{3}\cdot\omega 0\right)\cdot s0^2 + (s1 - s0)\cdot\frac{Q_{\omega 0}}{t}$$

$$= \quad (s1 - s0)\cdot\frac{Q_{\omega 0}}{t} + \left(\frac{1}{6}\cdot\omega 1 + \frac{1}{3}\cdot\omega 0\right)(s1 - s0)^2$$

a reference Heins calculates this increment as $\Delta Q_\omega(s) := \frac{1}{2}\left(Q_{\omega 1} + Q_{\omega 0}\right)\cdot\left(\frac{s1 - s0}{t}\right) + \frac{1}{12}\cdot(\omega 0 - \omega 1)\cdot(s1 - s0)^2$

it can be shown that these are equivalent, but it is not obvious what motivated the second form

from open development:

$Q_{\omega 1} := Q_{\omega 0} + \frac{1}{2}\cdot(\omega 1 + \omega 0)\cdot t\cdot(s1 - s0)$ linear => area is half end-point *t* distance, t constant

$$\left[\frac{1}{2}\left(Q_{\omega 1} + Q_{\omega 0}\right)\cdot\frac{(s1 - s0)}{t} + \frac{1}{12}\cdot(\omega 0 - \omega 1)\cdot(s1 - s0)^2\right] \;\Bigg|\; \begin{matrix}\text{simplify}\\ \text{collect}, t, Q_{\omega 0}, s1, s0\end{matrix} \rightarrow \left(\frac{1}{6}\cdot\omega 1 + \frac{1}{3}\cdot\omega 0\right)\cdot s1^2 + \left(\frac{-2}{3}\cdot\omega 0 - \frac{1}{3}\cdot\omega 1\right)\cdot s0\cdot s1 + \left(\frac{1}{6}\right.$$

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$$\left(\frac{1}{6}\cdot\omega 1 + \frac{1}{3}\cdot\omega 0\right)\cdot s1^2 + \left(\frac{-1}{3}\cdot\omega 1 - \frac{2}{3}\cdot\omega 0\right)\cdot s0\cdot s1 + \left(\frac{1}{6}\cdot\omega 1 + \frac{1}{3}\cdot\omega 0\right)\cdot s0^2 + (s1 - s0)\cdot\frac{Q_{\omega 0}}{t}$$

$$\frac{\int Q_\omega \, ds}{\int \frac{1}{t} \, ds}$$

correction is then:

$$Q_{\omega\_corr} = \frac{\sum_i \left[\left(\frac{s_{i+1} - s_i}{t_i} \right) \cdot Q_{\omega_i} + \left(\frac{1}{6} \cdot \omega_{i+1} + \frac{1}{3} \cdot \omega_i \right) (s_{i+1} - s_1)^2 \right]}{\sum_i \frac{s_{i+1} - s_i}{t_i}}$$

or ....

$$\frac{\sum_i \left[\frac{1}{2} \left(Q_{\omega_{1+1}} + Q_{\omega_i} \right) \cdot \left(\frac{s_{i+1} - s_1}{t_i} \right) + (\omega_i - \omega_{i+1})(s_{i+1} - s_1)^2 \right]}{\sum_i \frac{s_{i+1} - s_i}{t_i}}$$

$$\Delta l_i = s_{i+1} - s_i$$

and .

$$Q_{\omega_i} := Q_{\omega_i} - Q_{\omega\_corr}$$

$$Q_{\omega\_corr} = \frac{\sum_i \left[Q_{\omega_i} \cdot \frac{\Delta l_i}{t_i} + \left(\frac{1}{6} \cdot \omega_{i+1} + \frac{1}{3} \cdot \omega_i \right) (\Delta l_i)^2 \right]}{\sum_i \frac{\Delta l_i}{t_i}}$$

or ....

$$\frac{\sum_i \left[\frac{1}{2} \left(Q_{\omega_{i+1}} + Q_{\omega_i} \right) \cdot \frac{\Delta l_i}{t_i} + \frac{1}{12} (\omega_i - \omega_{i+1}) (\Delta l_i)^2 \right]}{\sum_i \frac{\Delta l_i}{t_i}}$$