**Problem Set5 Solution** **BE.462J/3.962J**
Issued: Day15 **Spring 2003**
Due: Day 17
5 points/problem (20 points total)

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1. Two hydrogels are prepared from two different candidate polymers for controlled release of a protein drug, interleukin-2 (IL-2). The gels exhibit the same swelling ratio at equilibrium, but gel A (formed by cross-linked polymer A, repeat unit molecular weight $M_0$ = 100 g/mole) has a molecular weight between cross-links of 3,000 g/mole, while gel B (formed from cross-linked polymer B, $M_0$ = 95 g/mole) has a molecular weight between crosslinks of 12,500 g/mole. Separate measurements showed polymer chains of polymer A and polymer B that have the same molecular weight have the same unperturbed end-to-end distance ($< r_0^2 >^{1/2}$). Finally, the hydrodynamic size *r* of the drug (IL-2) is one tenth the mesh size of gel B.

   a. Will diffusion of IL-2 through these two gels be the same or different? Calculate the ratio of the diffusion coefficient in gel A to that in gel B.

      *The diffusion of IL-2 through these gels will be slightly different, as assessed by the following calculation of the ratio of diffusion coefficients in gel A vs. gel B:*

      $Q_A = Q_B = Q$

      *The measurement showing equal unperturbed end-to-end distances for A and B chains of equal molecular weight gives us:*

$$\frac{< r_{0,A}^2 >^{1/2}}{< r_{0,B}^2 >^{1/2}} = \frac{\left(\frac{2M_{c,A}}{M_{0,A}}\right)^{1/2} C_{n,A}^{1/2} l_A}{\left(\frac{2M_{c,B}}{M_{0,B}}\right)^{1/2} C_{n,B}^{1/2} l_B} = \frac{\left(\frac{M_c}{M_{0,A}}\right)^{1/2} C_{n,A}^{1/2} l_A}{\left(\frac{M_c}{M_{0,B}}\right)^{1/2} C_{n,B}^{1/2} l_B} = \frac{\left(\frac{1}{M_{0,A}}\right)^{1/2} C_{n,A}^{1/2} l_A}{\left(\frac{1}{M_{0,B}}\right)^{1/2} C_{n,B}^{1/2} l_B} = 1$$

      *The above equation allows us to determine a relation between the mesh sizes of the two gels:*

$$\frac{\xi_A}{\xi_B} = \frac{\left(\frac{2M_{c,A}}{M_{0,A}}\right)^{1/2} Q_A^{1/3} C_{n,A}^{1/2} l_A}{\left(\frac{2M_{c,B}}{M_{0,B}}\right)^{1/2} Q_B^{1/3} C_{n,B}^{1/2} l_B} = \frac{M_{c,A}^{1/2}}{M_{c,B}^{1/2}} = 0.489$$

$$\frac{D_A}{D_B} = \frac{\left(1-\frac{r}{\xi_A}\right)e^{\left[\frac{-1}{Q_A-1}\right]}}{\left(1-\frac{r}{\xi_B}\right)e^{\left[\frac{-1}{Q_B-1}\right]}} = \frac{\left(1-\frac{r}{\xi_A}\right)}{\left(1-\frac{r}{\xi_B}\right)} = \frac{\left(1-\frac{0.1\xi_B}{0.489\xi_B}\right)}{\left(1-\frac{0.1\xi_B}{\xi_B}\right)} = 0.884$$

b. Explain in physical terms how two gels could have equal swelling ratios but different mesh sizes and diffusion rates for an entrapped drug. (It may be helpful to try to sketch the physical situation to explain the case described above where two gels have different molecular weights between cross-links but equivalent swelling ratios.).

*As we saw in our derivation of the equilibrium expression for the swelling of neutral hydrogels, Q depends on multiple factors- in addition to the molecular weight between crosslinks, it also notably depends on the polymer-solvent interaction parameter $\chi$. Thus two gels may have the same swelling ratio but different molecular weights between cross-links. The physical interpretation of this result is that a polymer which does not have highly favorable interactions with water may have less swollen chains (chains more collapsed to provide more polymer-polymer contacts). This gel may have a significantly higher Mc than a gel that has the same degree of swelling but which has more expanded chains (more favorable water-polymer interaction, lower $\chi$). This situation is schematically illustrated below. Gel diffusion theory ( as illustrated by the calculation above) says that when the swelling ratio is equal between two gels, the gel with the higher molecular weight between cross-links (drawing on left below), and thus larger mesh size, will allow faster diffusion of a drug through its structure. Though the local concentration of polymer per unit volume within the gel in these two cases can be equal, the constraint to diffusion presented by cross-links slows diffusion in the gel with lower Mc.*