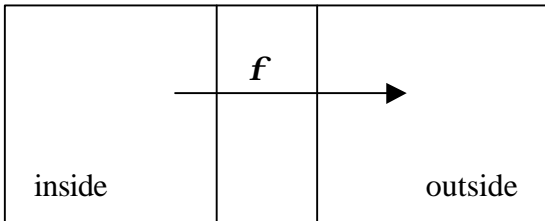# Week 6 Review

What was covered:
- Ion transport
- Active transport
- Steady state, rest, quasi-equilibrium, and equilibrium
- Indirect and direct effect of active transport
- Circuit review

Ion transport:

The membrane is only permeable to ion n that has a valence, z_n. (Valence is the amount of charge on one ion.)

So if there is a difference in concentration between the inside and the outside, the ions are going to move down the concentration gradient because of **diffusion**. But this will begin to make charge build up on the membrane, that effect, **drift**, will cause the ions to flow in the opposite direction. Therefore, the flux will be due to both diffusion and drift.

Flux is due to 2 phenomena:

| Diffusion | Drift |

$$f = -D_n \frac{\partial c(x,t)}{\partial x} - \boldsymbol{m}_n z_n F c_n(x,t) \frac{\partial \boldsymbol{y}(x,t)}{\partial x}$$

Because the movement of charged transport is easier to think about in terms of current, we convert flux to current density.

$J \qquad z \; F \qquad x$

$$_n(\ ,\ ) \qquad _n \quad _n \frac{_n(\ ,\ )}{} \qquad _n \; _n \quad ^2 \quad (\ ,\ ) \frac{(\ ,}{}$$

Of course there is still the Continuity Equation:

$$\frac{\partial J_n(x,t)}{\partial x} = -z_n F \frac{\partial c_n(x,t)}{\partial t}$$

If the membrane is permeable to more than one ionic species, you will end up with a Nernst-Planck Equation and a Continuity Equation for each ion. So, if there are k permeable ionic species that makes 2k equations. (In this example, there is only 1 permeable ionic species and therefore there are 2 equations)

However, we have 2k+1 unknowns (in this example, there are 3: J_n, c_n, and ψ). So we need one more equation. Remember, 8.02? Well you can use it here! We can use Gauss's Law and the definition of potential and get: $\dfrac{\partial^2 y}{\partial x^2} = -\dfrac{1}{e}(\text{total charge})$

Poisson's Equation:

$$\frac{\partial^2 y}{\partial x^2} = -\frac{1}{e}\sum_n z_n F c_n(x,t)$$

Electroneutrality approximation

To simplify the equations:
On the length scale of cells and on the time scale of biological processes, we can make the approximation that the total charge is 0. That is:

$$\sum_n z_n F c_n(x,t) = 0$$

That's because all the charge build up on the membrane occurs very fast (charge relaxation time is on the order of nanoseconds), is on the order of nanometers ("Debye length" is ~1nm for physiological conditions, and the amount is very small compared to the amount of ions in the bath.

Steady State Electrodiffusion through membranes

Steady State still has the same definition ($\dfrac{\partial}{\partial t} \to 0$). This means that:

$\dfrac{\partial J_n(x,t)}{\partial x} = -z_n F \dfrac{\partial c_n(x,t)}{\partial t} = 0$ from the continuity equation and so it implies that J_n is constant.

Then you can solve for J_n through the membrane from the Nernst-Planck equation (see the lecture notes):

$$J_n = G_n(V_m - V_n)$$

where Vm is the total membrane voltage ($V_m=y(0)- y(d)$). G_n is the conductivity (units of S=1/Ω) of the ion species n through the membrane. It's a function of the ion mobility, valence and concentration in the membrane.
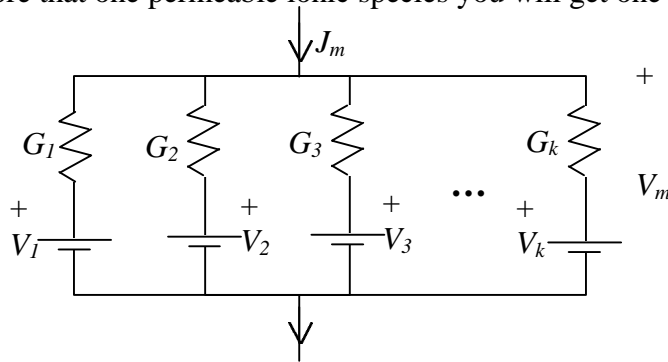Finally, V_n is defined as the Nernst Equilibrium Potential:

$$V_n = \frac{RT}{z_n F}\ln(\frac{c_n(outside)}{c_n(inside)})$$

From this we define a circuit model for the conductance of one ionic species through the membrane:
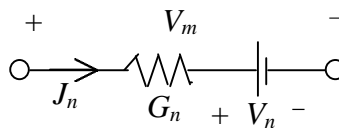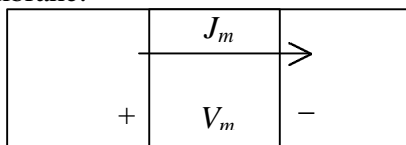
So, if we have more that one permeable ionic species you will get one branch in the circuit model:

What is the Nernst equilibrium potential?

$$V_n = \frac{RT}{z_n F} \ln(\frac{c_n(outside)}{c_n(inside)}) \approx \left(\frac{60}{z_n} \log(\frac{c_n^o}{c_n^i}) \right) mV \text{ at room temperature (T = 25°C)}$$

This means that if $V_m = V_n$ then ionic species n is **not** moving across the membrane.

What's the problem with this model?

Well, since all the transport is passive, you will eventually make the cellular concentrations change but this doesn't happen…
The cell uses active transport to maintain its internal concentration of ions constant. So to account for this we add active transport as current sources in the model:
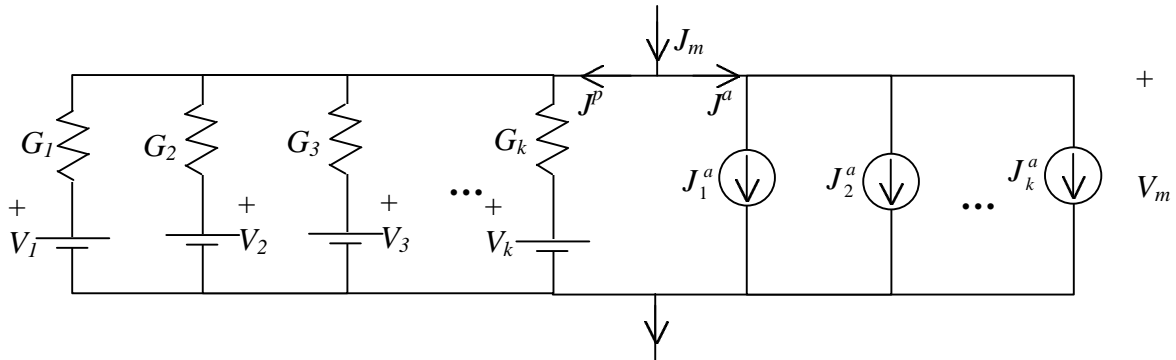
Steady State, Rest, Equilibrium, and Quasi-Equilibrium: What's the difference?

(Steady State and Equilibrium still have the same definitions)

Steady State: nothing changes with time (i.e. $\frac{\partial}{\partial t} \to 0$)

Rest: the net flux of charge particles is zero (i.e. $J_m = \sum_n J_n^p + J_n^a = 0$. This is only one

equation…) This implies that: $V_m = \frac{1}{\sum_n G_n}\left(\sum_n G_n V_n - J_n^a \right)$

Quasi-Equilibrium: The net flux of each species is 0 (i.e. $J_n^p + J_n^a = 0$ for all n. This is a set of k equations: one equation for each permeable ionic species…)

Equilibrium: There is no flux of anything (i.e. all the $J = 0$). This can only happen if the active transport is blocked.

Indirect vs. Direct Effect of Active Transport

Indirect Effect:
If you turn off the active transport, then if you wait long enough the concentration in the cell will begin to change. (over a long time at least several hours)

Direct Effect:
If the active transport is electrogenic, then turning it off will cause an immediate change in V_m (see equation for V_m at rest).

Electrogenic active transport means that $\sum_n J_n^a \neq 0$.

Example: The Na/K ATPase covered in lecture pumps more Na+ than K+ every iteration:
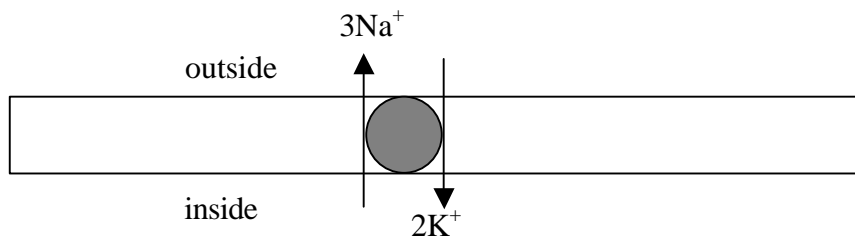
Review Circuits

Remember our friends KVL and KCL. If you don't, come talk to a TA for a quick refresher course. (Also, G_n is conductance so now the constitutive relationship is $J_n = G_n V_n$)