Week 3 Review

What was covered:

- Review τ_{SS} vs. τ_{EQ}
- Osmotic flux
- Osmosis equations
- More measurements in cells

Review τ_{SS} vs. τ_{EQ}:

In the thin membrane approximation:

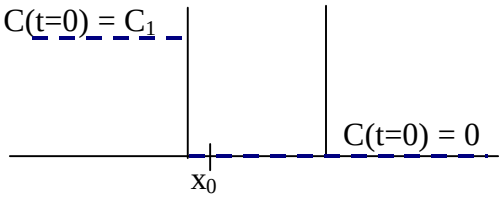
$$\tau_{SS} << \tau_{EQ}$$

What does this mean?

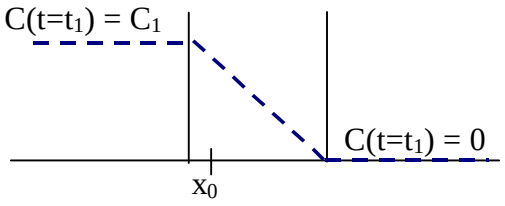
Example:

Given this initial solute concentration profile:

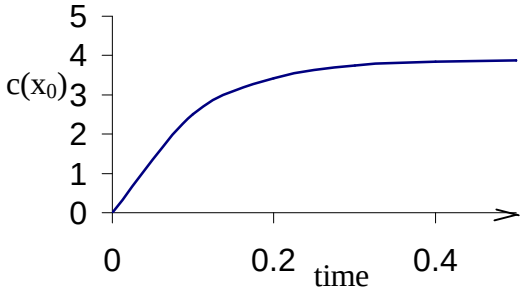
What doe the concentration at point x_0 look like as a function of time?



Well… We know that if you wait a little while (on the order of a couple of τ_{SS}) the system reaches steady state. That means that now the concentration profile in the membrane is linear but the bath concentrations haven't changed. Therefore, (assuming k =1) the profile now looks like this:

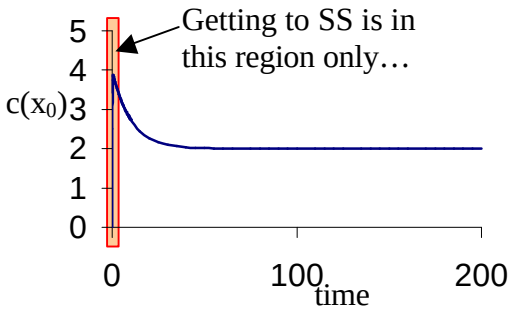


We know that the on this time scale (i.e. on times of the order of τ_{SS}), the concentration at any point in the membrane will behave as an exponential. So for this example, let's assume that τ_{ss}~0.1s then the c(x0,t) will look like:

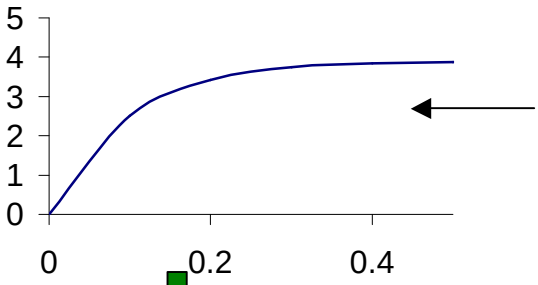


But now if we zoom out and look at the concentration profile at x0 on a much larger time scale (on a time scale on the order of τ_{EQ}), the concentration is going to tend to the equilibrium concentration. It will look like a different exponential (one with a much

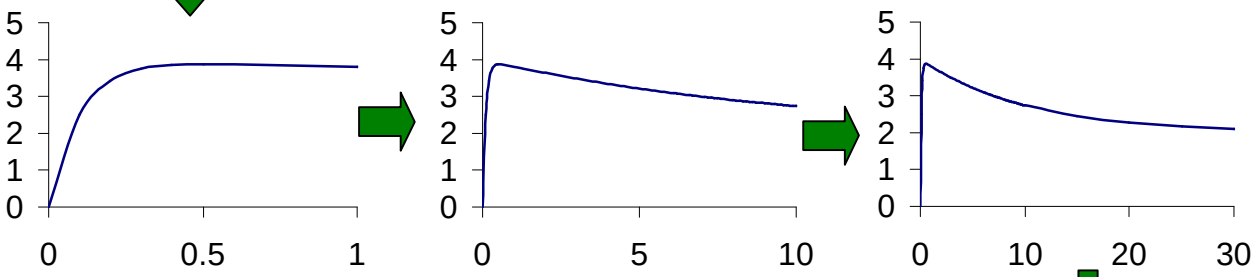
slower decay time constant). So if we assume that τ_{EQ}~10s, c(x0,t) on a big time scale will look like:

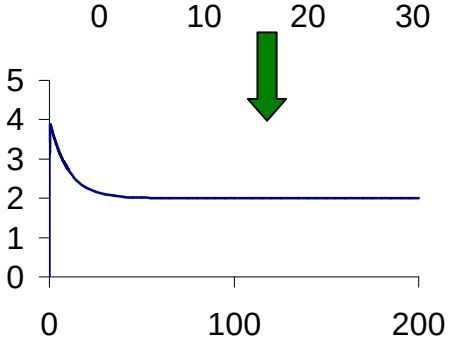


Steady state happens very fast compared to this change so it's taking place at the very beginning. But does this make sense? Well let's incrementally change the time scale of the plot so you can see how this effect happens:



So we have a graph of the concentration profile at a point in the membrane as a function of time. Let's slowly increase the time axis starting from small time scale (SS time scale) and going all the way to very large time scale (EQ time scale)





As you can see, there are some intermediary time scales (only somewhat bigger than τ_{SS} but still much smaller than τ_{EQ}), in which the concentration in the membrane does not look like any kind of exponential. The problem if you have a thick membrane is that you don't ever have a region that isn't intermediary (that is, on almost any time scale, the concentration does not look like an exponential…) But if $\tau_{EQ}>>>\tau_{SS}$, it's very hard to be in one of these intermediary time scales…

Anyway, hope this helped! For more help, look at problem 1 on problem set 3 and play with the diffusion simulation program on Athena. (or come ask a TA ☺)

What is osmotic flux?

Osmosis = Water Transport

f_s vs. Φ_v

	Units	What does it mean?
f_s= Diffusive flux	$\left[\frac{\text{mole}}{\text{s}\cdot\text{m}^2}\right]$	Represents the rate of change of the amount of solute normalized by surface area that the solute is going through
Φ_v= Osmotic flux	$\left[\frac{\text{m}^3}{\text{s}\cdot\text{m}^2}\right]=\left[\frac{\text{m}}{\text{s}}\right]$	Represents the rate of change of the volume of solvent normalized by the surface area that the solvent can go through

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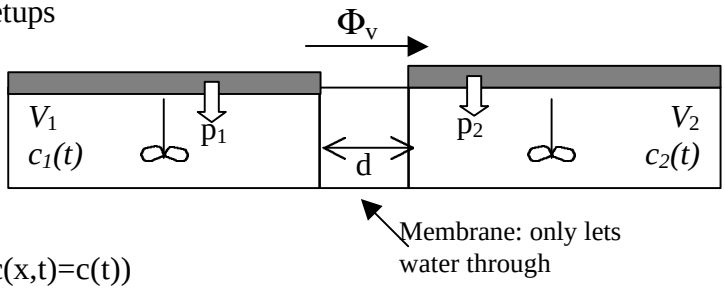
Osmosis Equations and Problem Setups

2 compartment model for osmosis:

A= cross-sect. area of membrane



Assume:

- well stirred baths (in baths c(x,t)=c(t))
- solute is can't go through membrane
- water is incompressible

If you assume water is incompressible then the system is <u>always</u> at steady state (see Continuity equation)

$\Phi_v(x,t) = -k\frac{\partial}{\partial x}(p-p)$ Darcy's Law

$c_\Sigma = \sum_i n_i c_i$ (where the i^{th} solute dissociates into n_i particles) Definition of Osmolarity

$p = RTc_\Sigma$ van't Hoff's Law

$-\frac{\partial}{\partial x}(r_m \cdot \Phi_v) = -\frac{\partial}{\partial t} r_m$ Continuity equation

Then once you assume SS (because water incompressible):

$\Phi_v = -\frac{1}{A}\cdot\frac{d}{dt}V_1$ Definition of volumetric flux

$L_v = \frac{k}{d}$ Hydraulic conductivity

$\Phi_v = L_v((p_1-p_1)-(p_2-p_2))$

+ Remember that you have conservation of solute in each compartment ($c_1(t)*V_1(t)$ is a constant) and conservation of solvent ($V_1(t)+V_2(t)$ is a constant)

Really the most useful equations (the ones you really want to understand). With these you can solve almost all the problems we will throw at you about osmosis. Things to remember though:

1. Be careful with the signs! Draw a diagram with which direction you are defining flux and be consistent. (the equations written here are only if you define the flux the way I've drawn it)
2. Check you unit! (always very helpful…)
3. Before starting the mathy part of the problem, stop and think about what will happen. Will the volume increase or decrease? At equilibrium, what will the concentrations of solutes have to be?
4. Once you finish all the math, check again to make sure that your answer makes sense…

Time behavior of volume in osmosis problem:

Plugging in from the equations above:

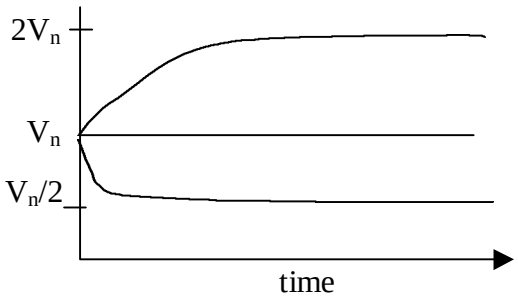
$$\Phi_v = -\frac{1}{A}\frac{d}{dt}(V_1(t)) = L_v((p_1 - p_1) - (p_2 - p_2))$$

but you also know that:

$$p = RTC_\Sigma = RT\left(\frac{N_\Sigma}{V(t)}\right)$$

this means that the differential equation for the time evolution of the volume of either compartment is NOT going to be linear! (since the right hand side will have 1/V terms…)

This means that V(t) is NOT an exponential!
However, we do know that it does asymptote to a final value and from numerical solvers we know it will look kind of like this:



Look in text or lecture notes for better version of the graph…

More measurements on cells!
How can we measure osmosis?

Measure cell volume at equilibrium when you stick the cell in different bath osmolarities. But wait! Isn't that going to be horribly non-linear? Yes, but if you plot it right it's pretty easy to interpret…

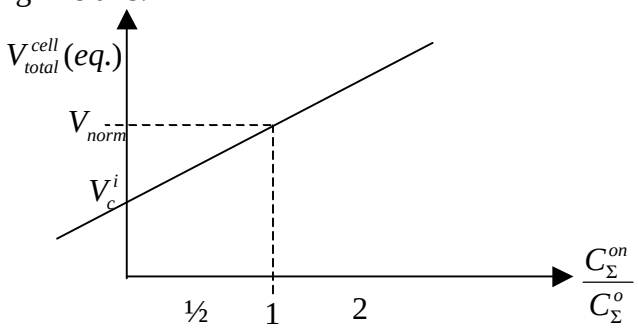
First off some definitions:

C_{Σ}^{on} = isotonic osmolarity.. This is basically the osmolarity equal to the normal osmolarity of the cell in it's normal environment.

Similarly, V_{norm} = isotonic cell volume is just the normal cell volume (before you do any experiment.)

Cell model: you assume that not all the cell is filled with water so you get: $V_{total}^{cell} = V_c^i + V_c\ (t)$ which says that the total cell volume is equal the volume of the water in the cell (which might change due to osmosis) plus the volume of everything else in the cell (won't change with osmosis)

So now onto the plot: We plot things on reciprocal axes because it makes things easy to interpret…
It will look something like this:



So basically what this graph says is:

- at infinite osmolarity, all the water will get sucked out of the cell and you will be left with the inactive (nonwater) volume of the cell (this is from the y-intercept on the graph)
- when you put a cell in a bath that has the same osmolarity as its normal (isotonic) environment the cell will neither shrink nor swell but stay the same size (Vnorm)
- if you put the cell in a higher osmolarity solution then its normal environment, the cell will shrink.
- similarly, if you decrease the osmolarity of the bath the cell will swell.

If you don't understand how to interpret this plots please find a TA and ask!