Massachusetts Institute of Technology
Physics Department

Physics 8.20 — IAP 2003
Introduction to Special Relativity — January 6, 2003

Assignment 1 – **Corrected version**
Due January 13, 2003

Announcements

- Please remember to put your name at the top of your paper.
- Problem Sets can also be downloaded from http://web.mit.edu/8.20/
- Depending on our rate of progress, some of the problems at the end of the problem set may be delayed until Problem Set #2

Topics for this period

- Introduction and relativity pre-Einstein
- Einstein's principle of relativity and a new concept of spacetime
- The kinematic consequences of special relativity foundation for spacetime

Reading Assignment 1

- Resnick and Halliday, §1 and 2, and Supplement A
- French §1, 2, and 3.

  French §1 gives a preview of the remarkable consequences of relativity.
  French §2 gives a very good overview of the puzzling properties of light propagation discovered in the 19th century that led up to Einstein's formulation of relativity.
  Much of French §3 repeats R&H. You only need to read one source.
- Einstein §1–12 and Appendix 1.

Problem Set 1

1. **Speeds**

   What fraction of the speed of light does each of the following speeds represent? (If any calculation is required, use Newtonian mechanics; ignore any relativistic effects. In cases where calculation — as opposed to unit conversion — is required, comment on whether your Newtonian results are good approximations to the correct speed.)

   (a) An ant, travelling at 2 cm/sec.

   (b) The "T", travelling at 30 miles/hr.

   (c) A satellite orbiting the Earth in low-Earth orbit. The radius of the Earth is 6.4×10^6 m.

   (d) A proton (or a baseball, for that matter) dropped onto the surface of a neutron star from rest at a great distance. (A neutron star is a compact star with a mass of about 1.4 times the mass of the sun and a radius of about 10 km. The mass of the sun is 2.0×10^{30} kg. Calculate the kinetic energy of the proton or baseball when it crashes into the neutron star surface, and then calculate its velocity.)

   (e) A spaceship, starting from rest, accelerated at 9.8 m/sec^2 for 100 years.

2. **The travels of elementary particles**

   Elementary particles have very short average lifetimes (at least measured on our perceptual scale). In experiments at accelerators the particles are produced and then detected at points away from the point of production. If the rules of Newtonian physics were correct for particles travelling at great speeds, the particles would have to travel much faster than the speed of light in order to go so far before they decay.

   Assuming that the particle in question lives for the average lifetime of that species, compute its average speed using Newtonian mechanics:

   (a) A "muon" has a lifetime of 2.2×10^{-6} sec. It is observed 5 kilometers away from its point of production.

   (b) A "Lambda" hyperon has a lifetime of 2.6×10^{-10} sec. It is observed 10.5 meters away from its point of production.

   (c) A "B^0 meson" has lifetime 1.5×10^{-12} sec. It is observed 3 millimeters away from its point of production.

3. **Galilean transformation on a ship – Case I**

   A ship is travelling due east at a speed of 25 m/sec.

   - A ball is rolled due north on the deck of the ship at a speed of 5 m/sec relative to the ship. What is its velocity relative to the Earth?

- If the ball is rolled 30°s east of north at a speed of 5 m/sec relative to the ship, what is its velocity relative to the Earth?

4. Galilean transformation on a ship – Case II

A stone is dropped from rest from the mast of a ship moving with a velocity of 15 m/sec relative to the Earth. Choose the origins of the Earth based ("laboratory") coordinate frame (Σ) and the coordinate frame in which the ship is at rest (Σ') such that they coincide with each other and with the stone at the instant $t = 0$ that the stone is dropped. Choose your coordinate system so that the ship moves along the positive x-axis relative to the Earth and the mast points along the positive y-direction. Find the position of the stone (a) relative to the ship, and (b) relative to the laboratory after 2 seconds. [Remember that the acceleration of gravity is 9.8 m/sec^2.] Show that the results can be related using the Galilean transformation.

5. Frame independence of momentum conservation

Resnick and Halliday , §1, p. 33, Problems 8 and 9

6. Binomial expansion

We will be using the "binomial expansion" often in 8.20. It reads:

$$\begin{aligned}(1+\epsilon)^a &= 1+\frac{a}{1}\epsilon+\frac{a(a-1)}{2\cdot 1}\epsilon^2+\frac{a(a-1)(a-2)}{3\cdot 2\cdot 1}\epsilon^3 \\ &+ \frac{a(a-1)(a-2)(a-3)}{4\cdot 3\cdot 2\cdot 1}\epsilon^4+\ldots \end{aligned} \tag{1}$$

This expansion converges when $|\epsilon| < 1$.

(a) For what values of a does the expansion terminate with a finite number of terms?

(b) Use eq. (1) to derive an expansion for $(a+b)^c$ when $|b| < |a|$.

(c) Consider $1/\sqrt{1-v^2/c^2}$. Write this in the form $(1+\epsilon)^a$ and expand it using the binomial expansion and give the first four terms.

(d) If $v/c = 0.4$ how large an error would you make by keeping only the first two terms in part (c)? the first three terms?

7. Numbers for Michelson-Morley

In the M-M experiment of 1887, the length, ℓ of each arm of the interferometer was 11 meters, and sodium light of wavelength 5.9×10^{-7} meters was used. Suppose that the experiment would have revealed any shift larger than 0.0025 fringe. What upper limit does this place on the speed of the Earth through the supposed aether? How does this compare with the speed of the Earth around the Sun?

8. Michelson-Morley for a real wind

Resnick and Halliday, §1, p. 35, Problem 19.

9. Ehrenfest's thought experiment

Resnick and Halliday , §1, p. 36, Problem 25.

Since the answer is given in the back of the book, your job on this problem is to *explain* the answer!

10. Aberration, before and after Einstein

Resnick and Halliday, §1, p. 36, Problems 27 and 28. [The numerical answer to Problem 28 in the book seems to be wrong.]

11. Aberration due to the Earth's rotation

In class we discussed the stellar aberration generated by the Earth's motion around the Sun. The rotation of the Earth about its axis also causes stellar aberration.

(a) Explain why the amount of stellar aberration generated by the Earth's rotation depends the latitude of the observer.

(b) For an observer at a given latitude explain why the amount of aberration depends on the *compass direction* of the star being observed. Compare, for example, the aberration of a star viewed on the eastern or western horizon with one on the northern horizon and with one directly overhead.

(c) What is the largest aberration angle (the tilt of the telescope) due to the Earth's rotation alone for an observer a) at the North Pole, b) at the equator, and c) at latitutde 45° north.

12. Weighing the sun

Resnick and Halliday, §1, p. 36, Problem 29

13. Invariance of the wave equation

Starting from Maxwell's equations it is possible to derive a *wave equation* whose solutions represent electromagnetic waves. The equation, for the electric field, E, is

$$\frac{\partial^2}{\partial x^2}E(x,t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}E(x,t) = 0 \tag{2}$$

where c is the speed of light.

(a) Show that $E(x,t) = f(x \pm ct)$ is a solution to this equation *for any function, f.*

(b) Explain why this solution represents a wave travelling to the right or left (toward increasing or decreasing x) with speed c. Which sign corresponds to which direction?

(c) Show that eq. (2) *is not invariant* under the Galilean transformation $x' = x - vt$, $t' = t$. [See Resnick and Halliday, Problem 16 on page 35 for a useful hint.]

(d) Show however that eq. (2) *is invariant* under the Lorentz transformation $x' = a(x - vt)$, $t' = a(t - vx/c^2)$, where $a = 1/\sqrt{1 - v^2/c^2}$.

14. Relativity of simultaneity

A plane flies overhead an observer on the Earth. Treat both the Earth and the plane as inertial frames for this problem. The speed of the plane is v. When the plane is overhead a light signal is emitted from the center of the plane. Subsequently it is detected by observer A in the front of the plane and observer B in the rear of the plane. Both observers measure their distance from the center of the plane to be d.

(a) Assume the speed of light is c as measured by the observers in the plane. Explain why observers A and B agree that the light signal reaches them simultaneously. How much time does the light take to reach them?

(b) Assuming that the speed of light is also c as measured by an observer on the Earth, explain why the Earth-bound observer would say that the arrival of the light signal at A and at B *were not simultaneous events.*

15. A feeling for the Lorentz factor

Resnick and Halliday, §2, p. 82, Problem 4.

16. Inverse Lorentz transformation

Suppose two inertial frames are related by a Lorentz transformation:

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - vx/c^2) \end{aligned}$$

Solve for x, y, z, t in terms of x', y', z', t' and show that the transformation is identical except for $v \to -v$.

17. Lorentz transformation in an arbitrary direction

Suppose two inertial frames, Σ and Σ' move such that their coordinate axes are parallel, their origins coincide at $t = t' = 0$, and the origin of Σ' is observed to move with velocity $\vec{v}$ in Σ. Starting from the form of the Lorentz transformation when the relative motion is along a coordinate axis, derive the Lorentz transformation relating x', y', z', t' to x, y, z, t. [Hint: it will be useful to decompose $\vec{x}$ into $\vec{x}_{\parallel} = \hat{v}(\hat{v} \cdot \vec{x})$ and $\vec{x}_{\perp} = \vec{x} - \vec{x}_{\parallel}$.]