Massachusetts Institute of Technology
Physics Department

Physics 8.20 **IAP 2003**
Introduction to Special Relativity **January 22, 2003**

Assignment 4
Due January 29, 2003

Announcements

- Please remember to put your name at the top of your paper.
- Problem Sets can also be downloaded from http://web.mit.edu/8.20/
- It is possible that some problems will be omitted if we do not get to cover the necessary material in class. Please watch for announcements!
- The problems are confined to special relativity especially relativistic momentum and energy, and the kinematics of reactions and decays. Although there are no problems on general relativity, you are responsible for the basic concepts as presented in lecture and the reading.

Topics for this period

- Relativistic collisions and decays
- Four vectors and transformation properties
- The Lorentz transformation of energy and momentum
- Invariants
- The equivalence principle
- Looking forward to general relativity
  ,

Reading Assignment 3

- Resnick and Halliday, §3, Supplements C (on General Relativity)
- French, §7 (and §6, if you haven't studied it yet)
- Einstein, §18 – §29. This is the important reading.

Problem Set 3

1. **"Hyperbolic" space travel** Assume that a rocket can produce an acceleration $g_0 = 10\text{m}/\text{sec}^2$. [As described in lecture, this acceleration is approximately the same as that due to gravity at the surface of the earth. Astronauts will be able to live comfortably in this spaceship, as if in earth's gravity.) Assume, in addition, that in travelling to any destination the rocket *will accelerate half the way and decelerate during the second half of the journey.*

   (a) Calculate the travel time *as measured by the space traveller* to the moon. (Assume the moon is at a distance of 382,000 km.) Compare with the Galilean answer.

   (b) Answer the same questions for travel to Neptune, assumed to be at a distance of 4.5×10^9 km.

   (c) Answer the same questions for travel to Alpha Centauri, assumed to be at a distance of 4.3 light years away. What is the velocity of the rocket, in the earth's reference frame, at the half way point of the journey?

   (d) What is the value of β that would enable a second astronaut, travelling at constant speed, to travel from the earth to Alpha Centauri in the same travel time as that taken by the rocket described above?

2. $\mathbf{E = mc^2}$ **I.**

   When methane burns competely to form CO_2 the reaction is:

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

   the energy released is 212.8 Kilocalories per gram molecular weight (KCal/mole) of methane. Suppose 1000 kg of methane gas is combined with just enough oxygen to burn completely. What is the mass equivalent of the energy released in the reaction? [You will first have to convert KCal/mole to MKS units, *ie.* to joules/kg.]

3. $\mathbf{E = mc^2}$ **II.**

   The net result of the fusion reaction that fuels the sun is to turn four protons and two electrons into one helium nucleus,

$$4p^+ + 2e^- \rightarrow \ {}^4He^{++}.$$

   Other particles are given off (neutrinos and photons), but you can assume they eventually show up as energy. The masses of the relevant nuclei are as follows:

   $m_p = 1.6726 \times 10^{-27}$ kg.

$m_e = 9.1094 \times 10^{-31}$ kg.
$m_{\,^4He} = 6.6419 \times 10^{-27}$ kg.

(a) How much energy is released when a kilogram of protons combines with just enough electrons to fuse completely to form helium?

(b) How many kilograms of methane would you have to burn to produce the same amount of energy?

4. $\mathbf{E = mc^2}$ III.

Assume that the heat capacity of water is constant and equal to 4.2 joules/°gm. How much does the mass of a kilogram of water increase when it is heated from freezing (0°C) to boiling (100°C)?

5. Enormous energies

Quasars are the nuclei of active galaxies in the early stages of their formation. A typical quasar radiates enery at the rate of 10^{41} watts. At what rate is the mass of this quasar being reduced to supply this energy? Express your answer in solar mass units per year, where one solar mass unit, $1\text{smu} = 2 \times 10^{30}$kg is the mass of our sun.

6. Classical physics and the speed of light

(a) How much energy would it take to accelerate an electron to the speed of light according to "classical" (before special relativity) physics?

(b) With this energy what would its actual velocity be?

7. A useful approximation

(a) Show that for an extremely relativistic particle, the particle speed, u differs from the speed of light, c, by

$$\Delta u = c - u = \frac{c}{2}\left(\frac{m_0 c^2}{E}\right)^2$$

where m_0 is the rest mass and E is the energy.

(b) Find Δu for electrons produced by

i. MIT's Bates Accelerator Center, where $E = 800$ MeV.

ii. The Jeffreson Lab (in Newport News, Virginia), where $E = 6$ GeV.

iii. The Stanford Linear Accelerator Center (in Palo Alto, California), where $E = 50$ Gev.

8. Pressure of light I.

The solar constant (energy per unit area) at the top of the earth's atmosphere is 100 watts per square meter. [Remember, a watt is a joule/sec.] What is the pressure exerted by sun's light on the Earth? You should assume that the light is absorbed, not reflected.

9. Pressure of light II.

French, §6, Problem 6-8, page 201.

10. Useful kinematic relationships

Resnick and Halliday, §3, Problem 22, page 119.

11. Sticking reaction

French §6, Problem 6-3, page 200. This problem is easiest if you use invariants.

12. Impossible processes

French §6, Problem 6-14, page 202. Again, this problem is easiest if you use invariants.

13. Compton scattering

Derive the relationship between scattering angle and wavelength change for Compton scattering.

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$$

Explain what the notation means. What is $\Delta\lambda$? What is $\cos\theta$? [You can derive this using energy and momentum conservation or you can use invariants to make this problem easier.]

14. Limits of production

French §7, Problem 7-3, page 225.

15. An ultimate energy

Magnetic fields in the universe accelerate charged particles in intergalactic space up to enormous energies. These energetic particles rain down on the Earth as "cosmic rays". Particle physicists conjecture that there is an ultimate limit to the energy of cosmic rays due to their collisions with the "cosmic background radiation" (CBR) that suffuses the Universe. Suppose the cosmic rays are protons (with rest mass $m_p = 1.67 \times 10^{-27}$ kg.). Suppose the CBR consists of photons of energy 2.1×10^{-4} eV. [This is the energy corresponding to the temperature ($E = kT$) of $2.3°K$ that characterizes the background radiation.] The reaction that degrades the energy of the cosmic rays is "pion production"

$$\gamma + p \to \pi^0 + p,$$

where γ is a CBR photon and the π^0 is a "meson" with rest mass, $m_{\pi^0} = .240 \times 10^{-27}$kg.

(a) What is the threshold energy of the proton, *ie.* what is the minimum proton energy necessary for this reaction to go in the frame where the photons have energy 2.1×10^{-4} eV?

Hint 1: As always, the use of invariants will make this problem much simpler.

Hint 2: Consider the threshold condition in the center of mass of the final proton and pion. What is their configuration at threshold?

(b) Suppose the proton has just enough energy to make the reaction possible. What is its energy after the collision?

16. A magic energy

The K^- meson and the Λ^0 hyperon are two commonly encountered unstable particles. For example, they are commonly produced in air showers by cosmic rays and several of each have whizzed through you during the time you have been working on this problem set. The reaction

$$K^- + p \to \Lambda^0 + \pi^0$$

can be used to make Λ^0's at rest in the laboratory by scattering K^- mesons off a stationary proton (hydrogen) target.

(a) Find the energy of the incident K^- beam required to just produce Λ^0 hyperons at rest in the lab.

(b) What is the π^0 energy for this "magic" K^- energy?

(c) Check momentum conservation.

(d) Could the process be run the other way? That is, could a π^0 beam (assuming one was available) be used to make a K^+ at rest by the reaction $\pi^0 + p \to \Lambda^0 + K^+$?

The rest energies of all the particles involed are: $m_p c^2 = 939\text{MeV}$, $m_{K^\pm} c^2 = 494\text{MeV}$, $m_{\pi^0} c^2 = 135\text{MeV}$, $m_{\Lambda^0} c^2 = 1116$ MeV.

17. s, t, and u

When particles scatter off one another, the fundamental physics effects are the same in all reference frames even though the energies of the particles and the angles at which they scatter differ from frame to frame. That means that the interesting information can depend only on Lorentz invariant combinations of the four momenta p_i^μ.

Consider the general "two-to-two" scattering process,

$$a + b \rightarrow c + d$$

where the four momentum and rest mass of particle i are p_i^μ and m_i, respectively, where $i = a, b, c, d$. Of course $p_i^2 = m_i^2 c^4$.

(a) Show that all the products of the form $p_i \cdot p_j$ can be expressed in terms of the three variables,

$$\begin{aligned} s &\equiv (p_a + p_b)^2 \\ t &\equiv (p_a - p_c)^2 \\ u &\equiv (p_a - p_d)^2 \end{aligned}$$

and the rest masses m_i.

(b) Show that s, t, and u are related by

$$s + t + u = \sum_{i=a,b,c,d} m_i^2 c^4$$

(c) Show that when the masses are all equal $s \geq 4m^2$ and $t, u \leq 0$.