8.333 Problem set 9

1. Please solve problems 3 and 4 from prob set 8 as part of this homework instead!

2. A gas of N atoms, each with a mass m, is confined by a spherically symmetric potential

$$V(r) = V_0 \left(\frac{r}{a}\right)^n \tag{1}$$

where r is the radial coordinate and $n > 0$.

First treat the motion of the gas atoms classically, and assume that the atoms are *distinguishable* obeying Boltzmann statistics.

(i) Calculate the free energy as a function of temperature.

(ii) Use this to calculate the entropy as a function of temperature and the specific heat.

Now assume that the atoms are *indistinguishable bosons*, but continue to treat the motion of any individual particle in the potential classically. To make the calculations simpler, specialize to $n = 2$.

(iii) Describe qualitatively the behaviour as the gas is cooled. Find the temperature at which a transition to the low temperature phase happens.

Now consider a full quantum treatment (assuming $n = 2$) where the motion of the atoms in the potential is treated quantum mechanically and they are assumed to be bosons.

(iv) Plot the shape of the distribution of the density of particles as a function of r at $T = 0$.

(v) Consider a non-zero temperature T which is still much smaller than $\frac{\hbar\omega}{k_B}$ where ω is the frequency of the trap. What is the change in the occupation of the ground state?