8.333 Problem Set 7

1. Calculate the specific heat of an ideal gas of massless relativistic particles in d spatial dimensions (d arbitrary positive integer).

   Note: For such particles, the energy $E = c|\vec{p}|$.

2. (From Reif, Chap. 7) A system consists of N very weakly interacting particles at a temperature T sufficiently high that classical statistical mechanics is applicable. Each particle has mass m and is free to perform one dimensional oscillations about it's equillibrium position. Calculate the heat capacity of this system of particles at this temperature in each of the following cases:

   (a) The force effective in restoring each particle to it's equillibrium position is proportional to it's displacement x from this position.

   (b) The restoring force is proportional to x^3.

3. Consider N non-interacting quantum spins in a magnetic field $\vec{B} = B\hat{z}$ at a temperature T. The Hamiltonian for each spin is

$$H - -\mu \vec{S}.\vec{B} \tag{1}$$

   Assume that each spin has $\vec{S}^2 = S(S+1)$.

   (a) Calculate the partition function as a function of the temperature T and the magnetic field B.

   (b) Now calculate the specific heat C as a function of T, B. Consider specifically the two limits $\mu B << k_B T$ and $\mu B >> k_B T$, and discuss the behaviour in these limits. Plot the specifc heat as a function of T at a fixed field B.

   (b) The quantity $M_z = \mu \sum_i < S^{iz} >$ is the total magnetization of the system. Calculate the magnetization as a function of the field.

   (c) Calculate the zero field susceptibility $\chi = \left[\frac{\partial M_z}{\partial B}\right]_{B=0}$. Show that it satisfies the Curie law $\chi = \frac{c}{T}$ and find the constant of proportionality.

   (d) Now consider treating the spin as a classical vector of length S. Calculate the zero field susceptibility in the approximation. When does the full quantum result in (c) reduce to this classical result?