8.333 Problem Set 2

1. Consider a collection of N harmonic oscillators described by the Hamiltonian

$$H = \sum_{i=1}^{N} \left[\frac{p_i^2}{2m} + \frac{m\omega^2 q_i^2}{2} \right] \tag{1}$$

with q_i, p_i the coordinate amd momentum of the ith oscillator. In this problem, you will develop a description of their statistical properties through the microcanonical distribution.

First assume the oscillators may be described classically.

(a) Calculate the entropy S as a function of the total energy E.

(b) Calculate the energy E and the heat capacity C as a function of temperature.

(c) Find the joint probability density $P(q, p)$ for a single oscillator. Use this to calculate the mean kinetic energy and mean potential energy for each oscillator.

Now consider a quantum treatment of the oscillators. The Hamiltonian may be diagonalized to give the energy

$$E\left(n_i\right) = \sum_{i=1}^{N} \hbar\omega \left(n_i + 1/2\right) \tag{2}$$

where $n_i = 0, 1, 2, ...$ is the quantum occupation number of the ith oscillator.

(d) Repeat the calculations in parts (a) and (b) above.

(e) Find the probability $p(n)$ that a particular oscillator is in level n.

(f) Comment on the differences between the heat capacities for the classical and quantum treatments.

2. In this problem you will describe the statistical properties of the oscillators using the canonical distribution.

(a) Calculate the energy E and the heat capacity C as a function of temperature directly from the canonical distribution in both the clasical and quantum cases.

(b) Calculate explicitly the variance $< E^2 > - < E >^2$.

3. In this problem, you wil prove that there is no diamagnetism in clasical physics. Consider any system of charged particles described by the classical Hamiltonian $H(\vec{p}_i, \vec{q}_i)$ in the absence of any external magnetic fields. When a magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$ is turned on , the Hamiltonian becomes $H(\vec{p}_i - \frac{e}{c}\vec{A}_i, q_i)$. Here e is the charge of the particles and c is the velocity of light.

(a) Show that the (clasical) partition function is independent of the magnetic field $\vec{B}$

(b) Hence argue that the magnetization $\vec{M} = -\left\langle \frac{\partial H}{\partial \vec{B}} \right\rangle$ is zero.