# 8.333 Problem Set 11

1. (from Huang) Give numerical estimates for the Fermi energy of

   (a) electrons in a typical metal; (b) nucleons in a heavy nucleus; (c) He-3 atoms in liquid He-3 (atomic volume = $46.2A^3$/atom). Treat all the mentioned particles as free particles.

2. *Pauli paramagnetism*

   Calculate the contribution of electron spin to it's magnetic susceptibiltiy as follows. Consider non-interacting electrons, each subject to a Hamiltonian

   $$\mathcal{H}_1 = \frac{\vec{p}^2}{2m} - \mu_0 \vec{\sigma}.\vec{B} \tag{1}$$

   where $\mu_0 = e\hbar/2mc$, and the eigenvalues of $\vec{\sigma}.\vec{B}$ are $\pm B$. (The orbital effect of the field has been ignored).

   (a) Calculate the grand potential $g$ at a chemical potential $\mu$.

   (b) Calculate the densities $n_\pm = N_\pm/V$ of electrons pointing parallel and antiparallel to the field.

   (c) Obtain the expression for the magnetization $M = \mu_0(N_+ - N_-)$, and expand the result for small $B$.

   (d) Sketch the zero field suscpetibility $\chi(T) = \left[\frac{\partial M}{\partial B}\right]_{B=0}$, and indicate it's behaviour at low and high temperatures.

   (e) Estimate the magnitude of $\chi/N$ for a typical metal at room temperature.

3. *Two dimensional fermi gas*:

   Consider a two dimensional gas of non-interacting fermions each of which is described by the Hamiltonian

   $$\mathcal{H}_1 = \frac{\vec{p}^2}{2m} \tag{2}$$

   (a) First consider zero temperature. Find the relationship between the Fermi energy $E_f$ and the particle density $n$.

   (b) Find the chemical potential in terms of the density and the temperature.

   (c) Calculate the low temperature specific heat of the gas.