8.333 Problem Set 10

1. Find the pressure exerted by radiation on the walls of a cavity with which it is in equillibrium at a temperature T.

2. Consider matter in equillibrium with radiation at a temperature T. Let the matter be made of atoms each of which has some spectrum of states. Consider two such states m, n of an individual atom that have energy E_m and E_n respectively. Assume the $E_m > E_n$. Define $\hbar\omega \equiv E_m - E_n$.

   (a) In equillibrium, what is the ratio of the number of molecules N_m in state m to that N_n in state n?

   (b) The rate R_{nm} at which atoms in state n will be converted to states m will be proportional to N_n, and to the number of photons $\mathcal{N}(\omega)$ at frequency ω. Write

$$R_{nm} = B_{nm} N_n \mathcal{N}(\omega) \tag{1}$$

   The proportionality constant B_{nm} cannot be obtained without detailed calculations of the interaction of the atom with radiation.

   Now assume that in equillibrium this rate of conversion from n to m is exactly balanced by the rate R_{mn} at which states m get converted into states n. (This is known as detailed balance). This rate will be proportional to N_m. Write

$$R_{mn} = N_m r_{mn} \tag{2}$$

   The r_{mn} represents the rate for a single atom to make the transition from m to n. It is expected that this will be given as the sum of the rates of two processes: First, even in the absence of any radiation the atom may spontaneously decay from state m to n at some rate A_{mn}. Second, the presence of the radiation may stimulate decay at a rate that is proportional to the number of photons present at the frequency ω. So write

$$r_{mn} = A_{mn} + B_{mn} \mathcal{N}(\omega) \tag{3}$$

   Obtain an expression for A_{mn} and B_{mn} in terms of the coefficient B_{nm}. From this infer the enhancement in the decay rate r_{mn} due to the presence of radiation. (Note: This argument allows you to infer the phenomenon of stimulated emission of radiation, signalled by a non-zero value of B_{mn}.)

3. (From Reif) For quantized lattice vibrations discussed in connection with the specific heat of solids, the frequency ω of a propagating wave is related to it's wavevector $\vec{k}$ by $\omega = c|\vec{k}|$ where c is the sound velocity. On the other hand, in a ferromagnetic solid at low temperature quantized waves of magnetization (spin waves) have their frequency related to their wavenumber according to $\omega = Ak^2$ where A is a constant. At low temperature, find the temperature dependence of the heat capacity due to such spin waves.

4. (From Reif) The surface temperature of the sun is $T_0 = (5500K)$. It's radius is $R = (7 \times 10^{10} cm)$ while the radius of the earth is $r = (6.37 \times 10^8 cm)$. The mean distance between the sun and the earth is $L = 1.5 \times 10^{13} cm)$. In the first approximation, one can assume that both the sun and the earth absorb all the radiation incident on them.

   Assume that the earth has reached steady state so that it's mean temperature does not change in time despite the fact that the earth constantly absorbs and emits radiation.

   (a) Find an approximate expression for the temperature T of the earth in terms of the astronomical parameters mentioned above.

   (b) Calculate this temperature T numerically.

   Suggested reading: Reif, Chap. 9. pages 381-388.