8.333 Problem Set 12

1. Consider a relativistic treatment of a gas of electrons. Assume that the energy of each electron is related to it's momentum by

$$E = \sqrt{p^2c^2 + m^2c^4} \tag{1}$$

Here c is the velocity of light and m is the electron rest mass.

(i) If the gas is at a density ρ, when do you expect that a non-relativistic treatment where the energy of each electron is approximated as

$$E \approx \frac{p^2}{2m} + mc^2 \tag{2}$$

will suffice?

(i) Determine the Fermi energy, Fermi momentum, and total ground state energy without assuming the non-relativistic limit.

(ii) From (ii) determine the degeneracy pressure as a function of the density.

(iii) Derive explicitly a criterion for the reduction of the results of (ii) and (iii) to those appropriate for non-relativistic particles to confirm the expectation in (i).

(iv) Now consider the limit $m = 0$. What do the results of (i) and (ii) reduce to in this limit? When is this limit an appropriate description of the system?

2. Consider a gas of classical atoms where the interaction potential takes the form

$$U = \sum_{\mu \neq \nu} U_{\mu\nu} \tag{3}$$

with $U_{\mu\nu} = V(|\vec{q}_\mu - \vec{q}_\nu|)$, and the function $V(x)$ vanishes for $x > a$ and is infinity for $0 < x < a$. Such a gas is called a "hard-sphere gas" with a the radius of the hard sphere.

(i) Calculate the second virial coefficient $B(T)$ for such a hard sphere gas, and comment on it's high and low temperature behaviour.

(ii) Calculate the first correction to the isothermal compressibility

$$\kappa_T = -\frac{1}{V}\left[\frac{\partial V}{\partial P}\right]_{T,N} \tag{4}$$

(iii) In the high temperature limit, reorganize the equation of state into the van der Waals form and identify the van der Waals parameters.