8.333 Problem Set 4

1. The equation of state of an ideal elastic cylinder is

$$t = KT\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right) \tag{1}$$

where t is the tension, T the temperature and K is a constant. L_0, the length at zero tension is a function only of the temperature T.

(i) Write down the first law of thermodynamics for this system.

If the cylinder is stretched reversibly and isothermally from $L = L_0$ to $L = 2L_0$, show that:

(ii) the heat transferred is

$$Q = -KTL_0\left(1 - \frac{5}{2}\alpha_0 T\right) \tag{2}$$

where α_0, the linear expansivity at zero tension is expressed as

$$\alpha_0 = \frac{1}{L_0}\frac{dL_0}{dT} \tag{3}$$

(iii) The change of internal energy is

$$\Delta U = \frac{5}{2}KT^2 L_0 \alpha_0 \tag{4}$$

2. (i) Starting from $dU = TdS - pdV$, show that the equation of state $PV = Nk_BT$ implies that U can only depend on T.

(ii) What is the most general equation of state consistent with an internal energy that depends only on temperature?

(ii) A van der Waals gas is defined by the equation of state

$$\left[P - a\left(\frac{N}{V}\right)^2\right](V - Nb) = Nk_BT \tag{5}$$

Show that for this gas C_v is a function of temperature alone.

3. An exotic material is claimed to have the following properties.

(i) The laws of thermodynamics hold for this material.

(ii) The total entropy S of the material is determined in terms of it's internal energy E and volume V by

$$S = c\sqrt{UV} \tag{6}$$

where c is a constant.

(a) What is the relation between P, V and T for this material?

(b) What is it's isothermal compressibility $\kappa_T \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$?

(c) Calculate it's specific heat at constant volume.

(d) Consider the possibility of other related exotic materials obeying

$$S = cU^a V^b \tag{7}$$

with a, b real. For what values of a, b is this equation consistent with the laws of thermodynamics?

4. (From Reif) A solid contains N magnetic atoms having spin 1/2. At sufficiently high temperature, each spin is completely randomly oriented, *i.e* equally likely to be in either of it's two possible states. But at sufficiently low temperatures, the interactions between the magnetic atoms causes them to exhibit ferromagnetism, with the result that all their spins become oriented along the same direction as $T \to 0$. A very crude approximation suggests that the spin-dependent contribution $C(T)$ to the heat capacity of this solid has an appropriate temperature dependence given by

$$C(T) = C_1 \left(2\frac{T}{T_1} - 1\right) \quad if \quad 1/2T_1 < T < T_1 \tag{8}$$

$$= 0 \quad otherwise \tag{9}$$

The abrupt increase in specific heat as T is reduced below T_1 is due to the onset of ferromagnetism.

Use entropy considerations to find an explicit expression for the maximim value C_1 of the heat capacity.