8.333 Problem Set 1

1. Hamiltonian dynamics of a classical particle:

   A classical particle in three dimensions is described by the Hamiltonian

   $$H = \frac{\vec{p}^2}{2m} + V(\vec{q}) \tag{1}$$

   where $(\vec{q}, \vec{p})$ are the position and momentum respectively. The particle has mass m and $V(\vec{q})$ is it's potential energy.

   (i) Derive the equations of motion for $(\vec{q}, \vec{p})$, and show that they are equivalent to Newton's law for the evolution of q.

   (ii) Show explicitly that the Hamiltonian is conserved during the time evolution.

   (iii) Now consider a particle in one dimension described by the oscillator potential $V(q) = \frac{K}{2}q^2$. Sketch the trajectories in phase space.

   (iii) Show by explicit calculation for the oscillator in (iii) that the volume of an infinitesimal region in phase space does not change under time evolution.

2. Density of states of a system of relativistic particles:

   Consider N independent quantum particles each with energy-momentum relation

   $$\epsilon = c|p| \tag{2}$$

   in a large one dimensional box of length L. Assume periodic boundary conditions.

   (i) What are the allowed values of p for each particle?

   (ii) Calculate the number of allowed states with total energy less than some specified amount E.

   (iii) Using (ii) calculate the density of allowed states as a function of E, N, and L. How does it depend on E for large N?

3. Density of states of free spins in a magnetic field:

   Consider a system of N spin-1/2 quantum objects ($N >> 1$) in a Zeeman magnetic field B. The energy of each spin is given by

   $$\epsilon = -\mu B s_z \tag{3}$$

   where $s_z = \pm 1/2$ and μ is a constant. Calculate the total number of states in an energy interval between E and $E + \Delta E$. (You may assume that$|E| >> \Delta E >> \mu B$).