Bargaining Theory

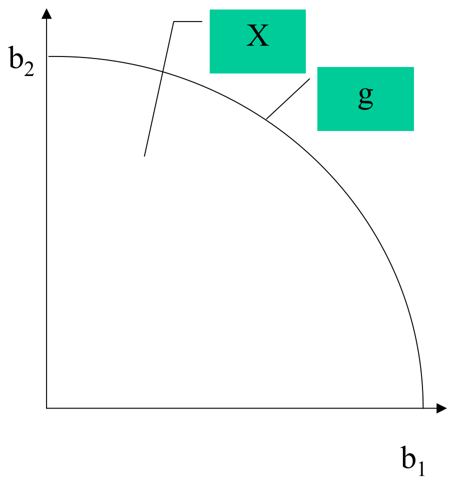
MIT 14.126 Game Theory

Sergei Izmalkov
Muhamet Yildiz

Bargaining Theory

- Cooperative (Axiomatic)
  - Edgeworth
  - Nash Bargaining
  - Variations of Nash
  - Shapley Value
- Non-cooperative
  - **Rubinstein-Stahl (*)** (complete info)
  - Asymmetric info
    - Rubinstein, Admati-Perry, Crampton, **Gul, Sonenchein, Wilson; Abreu and Gul**
  - Non-common priors
    - Posner, Bazerman, **Yildiz (*)**

Rubinstein-Stahl Model



- $N = \{1,2\}$
- X = feasible expected-utility pairs $(x,y \in X)$
- $U_i(x,t) = \delta_i^t x_i$
- $D = (0,0) \in X$ disagreement payoffs
- g is concave, continuous, and strictly decreasing

Timeline

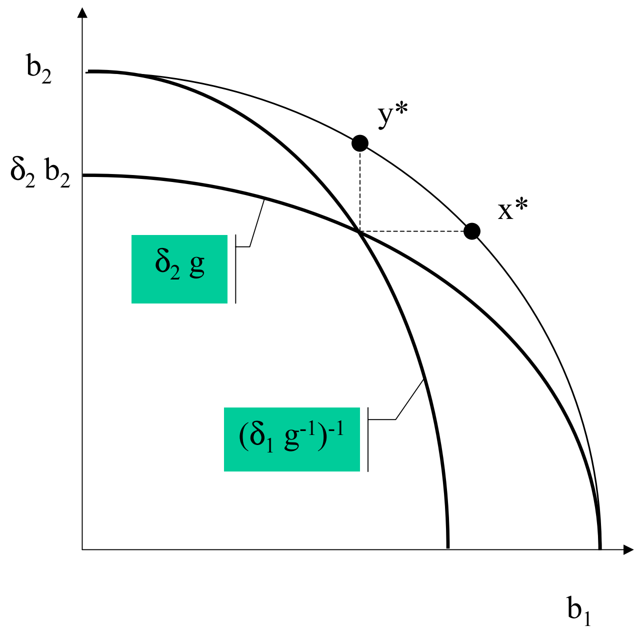
$T = \{0,1,\ldots, t,\ldots\}$

At each t, if t is even,

- Player 1 offers some x
- Player 2 Accepts or Rejects the offer
- If the offer is Accepted, the game ends yielding x
- Otherwise, we proceed to t + 1

if t is odd,

- Player 2 offers some y
- Player 2 Accepts or Rejects the offer
- If the offer is Accepted, the game ends yielding y
- Otherwise, we proceed to t + 1

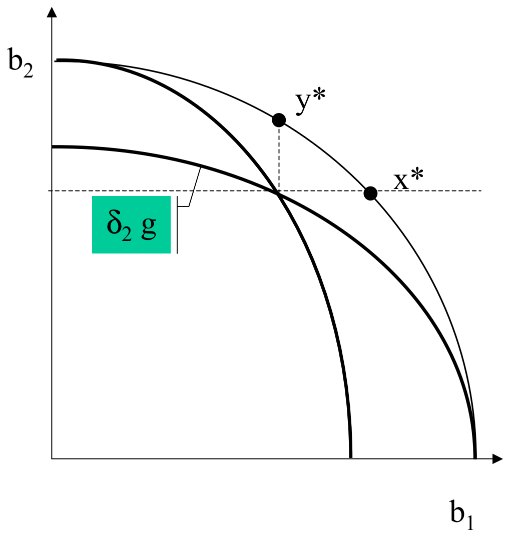


Theorem [OR 122.1]

The following is the unique subgame-perfect equilibrium:

- player 1 always offers x*;
- player 2 accepts an offer x iff $x_2 \geq x^*_2$;
- player 2 always offers y*;
- player 1 accepts an offer y iff $y_1 \geq y^*_1$;

Proof (it is a SPE)



Use single deviation principle:

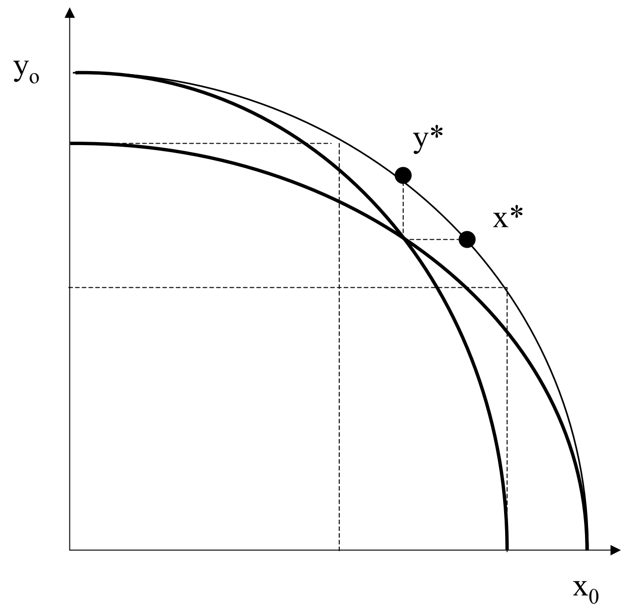
1. If player 2 rejects an offer x at t, she will get y_2* at t+1. Hence, Accept iff $x_2 \geq \delta_2 y^*_2 = x^*_2$ is optimal at t.
2. At t, it is optimal for 1 to offer

x* = argmax $\{x_1 | x_2 \geq x^*_2\}$.

"Extensive-form rationalizability" [FT 4.6]

Definition: In a multistage game with observable actions, action a_i^t is conditionally dominated at stage t given history h^t iff, in the subgame starting at h^t, every strategy for player i that assigns positive probability to a_i^t is strictly dominated.

Theorem: In any perfect-information game, every subgame-perfect equilibrium survives iterated elimination of conditionally dominated strategies.



A usual generalization of Rubinstein's Model

At any t,

1. A player i is recognized with probability p_t^i;
2. Player i offers some x;
3. The other player
    1. either Accepts, bargaining ends with payoffs $\delta^t x$, or
    2. Rejects the offer, when we proceed to t+1.

[There may also be a deadline $\underline{t}$, when the game automatically ends, yielding (0,0).]

SPE

Take X = $\{(x^1,x^2)|x^1+x^2 \leq 1\}$. Iterated elimination of conditionally dominated strategies yields a unique vector V_t of continuation values at each t where

$$V_t^i = (1-\delta)\sum_{s\geq t} p_t^i.$$

Any SPE is payoff equivalent to the following SPE. At any t, the recognized player i gives δV^j_{t+1} to the other player j and keeps $1-\delta V^j_{t+1}$ for himself, and the offer is barely accepted.

Proof

- Write S* for the strategies that survive IECDS. Write $\overline{V}_t^i$ and $\underline{V}_t^i$ for the max and min payoff for i at t over S*, and $\Delta_t = \max\{\overline{V}_t^1 - \underline{V}_t^1, \overline{V}_t^2 - \underline{V}_t^2\}$
- Then,

$$\overline{V}_t^i \leq p_t^i\left(1-\delta\underline{V}_{t+1}^j\right)+\left(1-p_t^i\right)\delta\overline{V}_{t+1}^i$$

$$\underline{V}_t^i \geq p_t^i\left(1-\delta\overline{V}_{t+1}^j\right)+\left(1-p_t^i\right)\delta\underline{V}_{t+1}^i$$

$$\overline{V}_t^i - \underline{V}_t^i \leq p_t^i\delta\left(\overline{V}_{t+1}^j - \underline{V}_{t+1}^j\right)+\left(1-p_t^i\right)\delta\left(\overline{V}_{t+1}^i - \underline{V}_{t+1}^i\right)$$

$$\leq p_t^i\delta\Delta_{t+1} + (1-p_t^i)\delta\Delta_{t+1} = \delta\Delta_{t+1}.$$

$$\Delta_t \leq \delta\Delta_{t+1}$$

Proof, continued

- If there is deadline at $\underline{t}$, then $\Delta_{\underline{t}} = 0$, and thus $\Delta_t = 0$ for all t. If infinite horizon, then $\Delta_t \leq \delta^n$ for all n, hence $\Delta_t = 0$.
- Write $S_t = V_t^1 + V_t^2$.
- $V_t^i = p_t^i\left(1-\delta V_{t+1}^j\right)+\left(1-p_t^i\right)\delta V_{t+1}^i = p_t^i\left(1-\delta S_{t+1}\right)+\delta V_{t+1}^i$
- $S_t = 1-\delta S_{t+1}+\delta S_{t+1} = 1.$
- $V_t^i = p_t^i\left(1-\delta\right)+\delta V_{t+1}^i$
- $$V_t^i = (1-\delta)\sum_{s\geq t} p_t^i.$$

A usual generalization of Rubinstein's Model

At any t,

1. A player i is recognized with probability p_t^i;
2. Player i offers some x;
3. The other player
   1. either Accepts, bargaining ends with payoffs $\delta^t x$, or
   2. Rejects the offer, when we proceed to t+1.

[There may also be a deadline $\underline{t}$, when the game automatically ends, yielding (0,0).]

Components of complete information

	CPA	No CPA
CK	Rubinstein, Stahl	
No CK	Rubinstein 85, Admati-Perry, Gul-Sonnenchein-Wilson and many others	

Without a common prior

At any t,

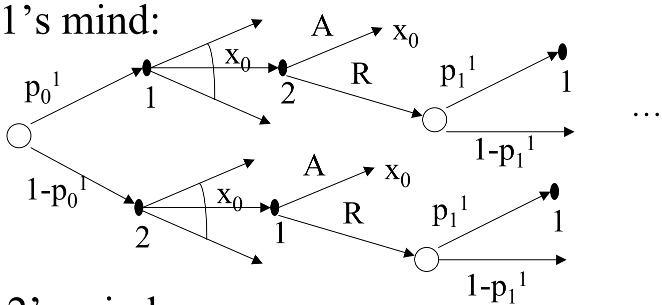
1. A player i is recognized by Nature;
2. Player i offers some x;
3. The other player
    1. either Accepts, bargaining ends with payoffs $\delta^t x$, or
    2. Rejects the offer, when we proceed to t+1.

[There may also be a deadline $\underline{t}$, when the game automatically ends, yielding (0,0).]

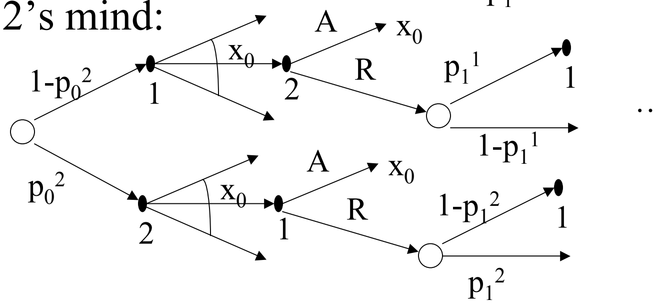
Each i believes that he will be recognized at t with probability p_t^i.

Game Tree

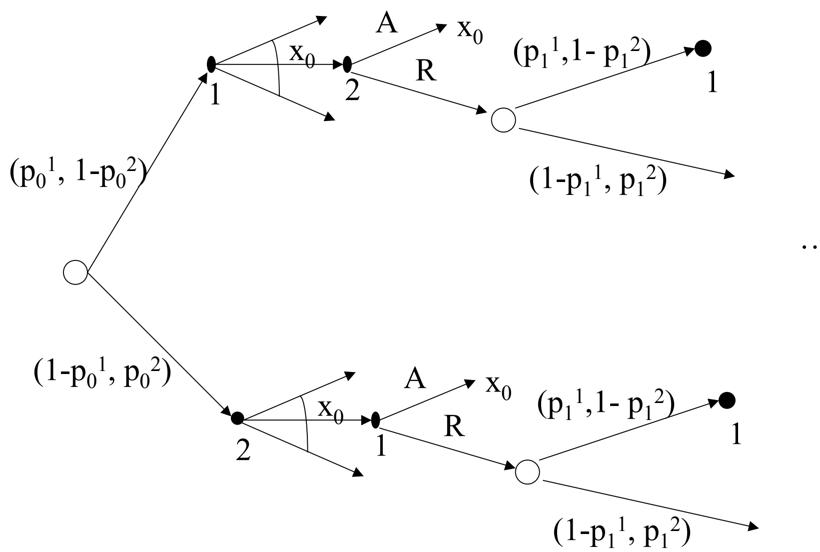
In 1's mind:



In 2's mind:

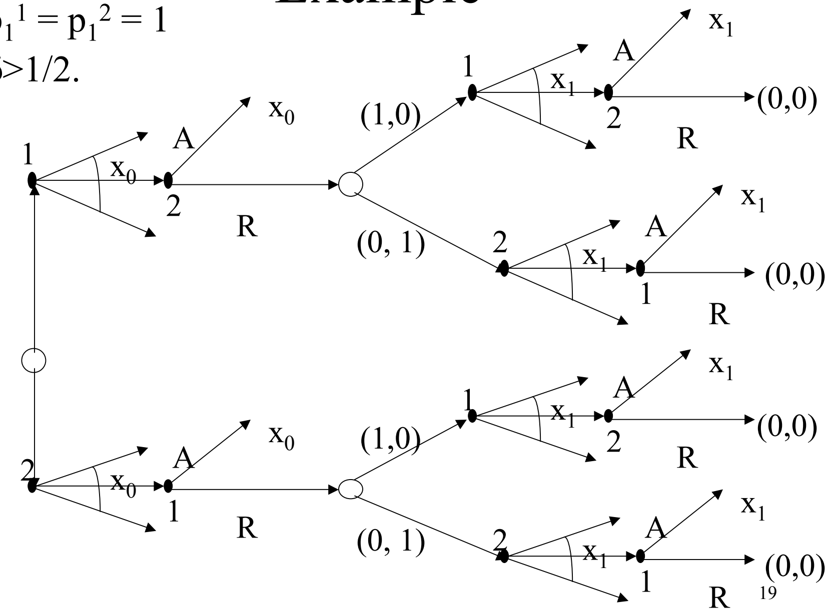


Game Tree



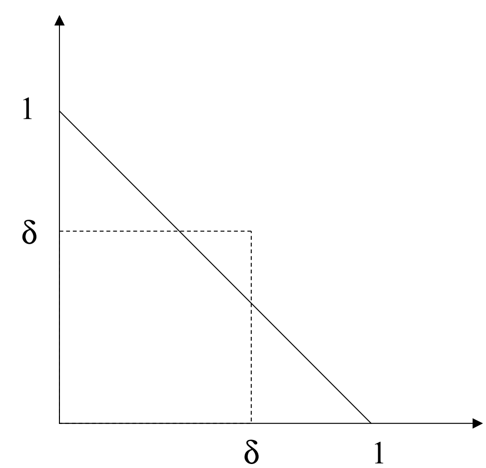
Example

- $\underline{t} = 2$.
- $p_1^{\,1} = p_1^{\,2} = 1$
  $\delta > 1/2$.



Example, continued

- $V_1 = (1,1)$
- $S_1 = 2$.



Optimism, Agreement-Disagreement

- The level of optimism for t: $y_t = p_t^1 + p_t^2 - 1$.
- There is a **disagreement regime** at t iff $\delta S_{t+1} > 1$.
  - No agreement; $V_t = \delta V_{t+1}$; $S_t = \delta S_{t+1}$.
- There is an **agreement regime** at t iff $\delta S_{t+1} \leq 1$.
  - They agree;
  - Recognized i gets $1- \delta V^j_{t+1}$, the other j gets δV^j_{t+1};
  - $V_t^i = p_t^i(1- \delta V^j_{t+1}) + (1- p_t^i)\delta V^i_{t+1} = p_t^i(1- \delta S_{t+1}) + \delta V^i_{t+1}$
  - $S_t = (1+y_t)(1- \delta S_{t+1}) + \delta S_{t+1} = 1 + y_t(1- \delta S_{t+1})$.

Lemma

If $S_{t+1} \in [1,1/\delta]$, then $S_t \in [0,2-\delta] \subset [1,1/\delta]$.

Proof:

- $1- \delta S_{t+1} \in [0,1-\delta]$ and $y_t \in [0,1]$.
- $S_t = 1 + y_t(1- \delta S_{t+1}) \in [0,1+ 1-\delta] = [0,2-\delta]$.

Immediate agreement

Definition: $1 < 2^{L(\delta)} \leq 1/\delta$.

Theorem: Given any t*, assume that $y_t \geq 0$ for each $t \leq t^*$. Then, there is an agreement regime at each $t < t^* - L(\delta) - 1$.