Forward Induction, Signaling and Reputation

14.126 Game Theory

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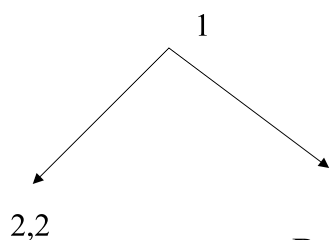
Road Map

1. Forward Induction
2. Signaling games
   1. Sequential Equilibria
   2. Intuitive Criteria
3. Reputation
   1. Chain-store paradox, finitely repeated games
   2. Centipede game with incomplete information
   3. Finitely repeated entry-deterrence game with incomplete information.

Forward Induction

The Battle of the Sexes with outside options



	B	S
B	3,1	0,0
S	0,0	1,3

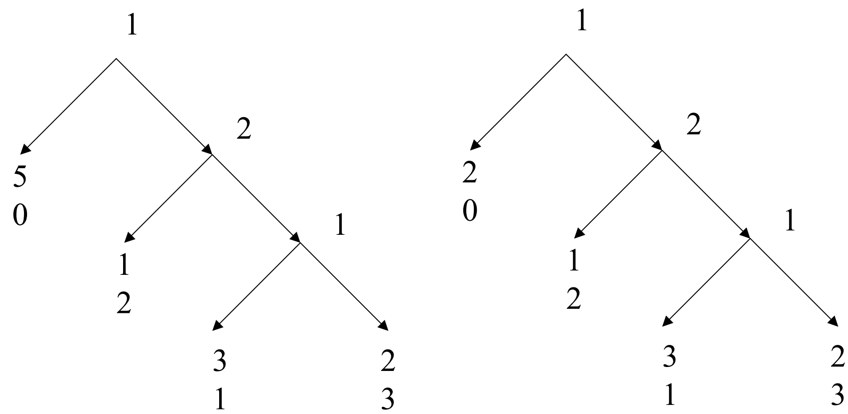
Forward Induction

- One ought to interpret the actions as outcomes of conscious choice even off the path.
- Intuitive criterion
- Mistaken theories

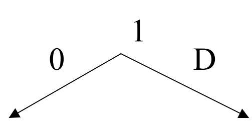
Strong belief in rationality

At any history of the game, each agent is assumed to be rational if possible. (That is, if there are two strategies s and s’ of a player i that are consistent with a history of play, and if s is strictly dominated but s’ is not, at this history no player j believes that i plays s.)

Examples



Burning Money



	B	S
B	3,1	0,0
S	0,0	1,3

	B	S
B	2,1	-1,0
S	-1,0	0,3

	BB	BS	SB	SS
0B				
0S				
DB				
DS				
O	T	H	E	R

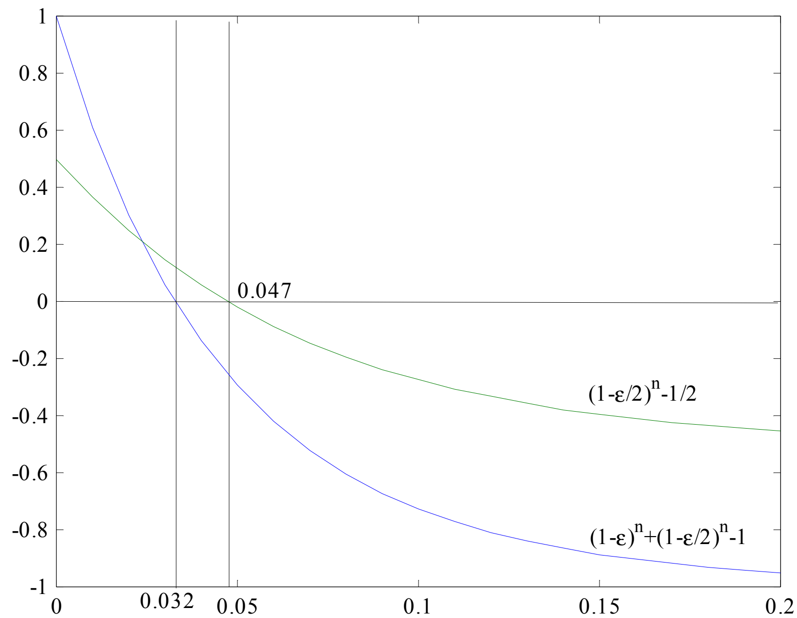
Table for the bidding game

$U_i = 20(2+2\min_j bid_j - bid_i)$

bid \ min	1	2	3
1	60	-	-
2	40	80	-
3	20	60	100

Nash equilibria of bidding game

- 3 equilibria: s^1 = everybody plays 1; s^2 = everybody plays 2; s^3 = everybody plays 3.
- Assume each player trembles with probability $\varepsilon < 1/2$, and plays each unintended strategy w.p. $\varepsilon/2$, e.g., w.p. $\varepsilon/2$, he thinks that such other equilibrium is to be played.
  - s^3 is an equilibrium iff
  - s^2 is an equilibrium iff
  - s^1 is an equilibrium iff



Bidding game with entry fee

Each player is first to decide whether to play the bidding game (E or X); if he plays, he is to pay a fee p > 60.

Bid \ min	1	2	3
1	60	-	-
2	40	80	-
3	20	60	100

For each m =1,2,3, ∃SPE: (m,m,m) is played in the bidding game, and players play the game iff 20(2+m) ≥ p.

Forward induction: when 20(2+m) < p, (Em) is strictly dominated by (Xk). After E, no player will assign positive probability to min bid ≤ m. FI-Equilibria: (Em,Em,Em) where 20(2+m) ≥ p.

What if an auction before the bidding game?

Signaling

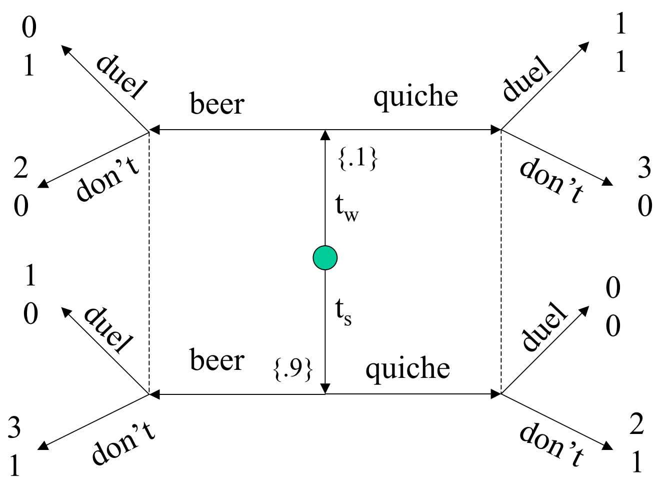
Model

- Players: (S)ender, (R)eceiver

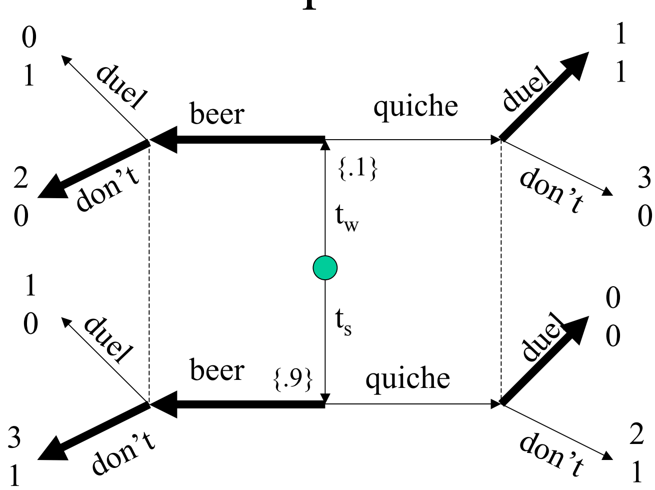
1. Nature selects t from T – the probability distribution is π;
2. S observes t, and sends message m from a set M;
3. R observes m – but not t – and takes an action *a*;
4. S gets $U^S(t,m,a)$ and R gets $U^R(t,m,a)$.

This is common knowledge.

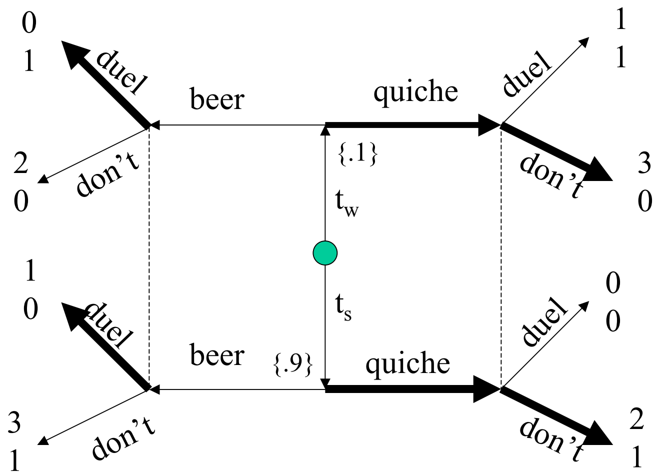
Beer – Quiche



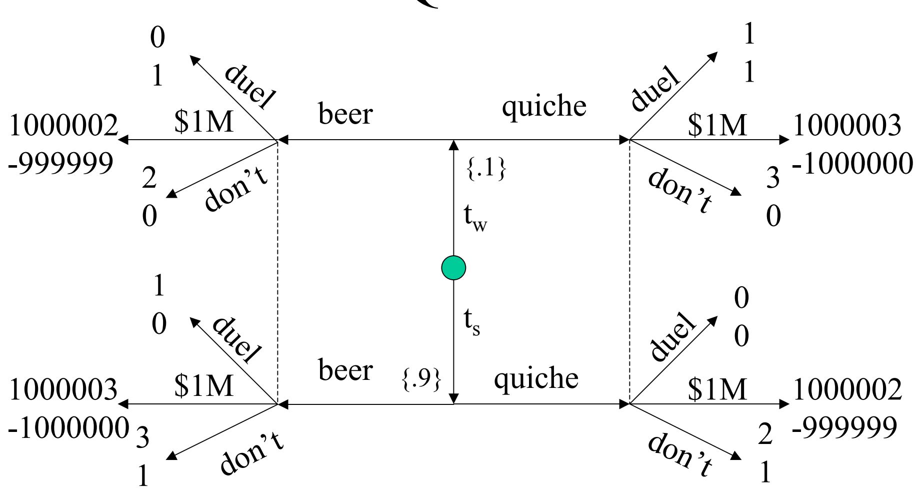
Good equilibrium



Bad equilibrium



Beer – Quiche – M



Cho -- Kreps

- T(m); M(t); A(m)
- Action set is finite;
- ρ(m;t) = the probability that t sends m;
- ϕ(a;m) = the probability that R chooses a, receiving m;
- $BR(\mu,m) = \arg\max_{a\in A(m)} \sum_{t\in T(m)} U^R(t,m,a)\mu(t)$
- For subset I of T,

$$BR(I,m) = \bigcup_{\{\mu:\mu(I)=1\}} BR(\mu,m)$$

- MBR

Sequential equilibrium

- Beliefs:

$$\mu(t\mid m) = \begin{cases} \dfrac{\pi(t)\rho(m;t)}{\sum_{t'\in T(m)}\pi(t')\rho(m;t')} & \text{if } \sum_{t'\in T(m)}\pi(t')\rho(m;t') > 0 \\ \text{something} & \text{otherwise} \end{cases}$$

- ρ(m,t) > 0 => $\Sigma_a U^S(t,m,a)\phi(a;m)$ is maximized at m;
- ϕ(.;m) is in MBR(μ(.|m),m)

Testing an equilibrium

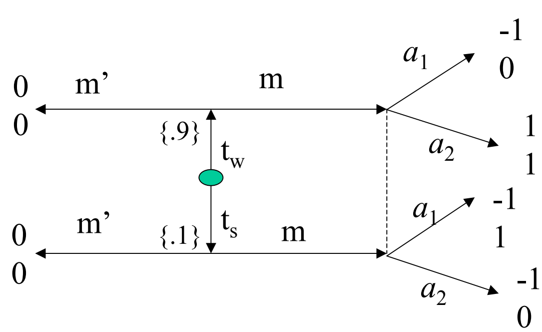
- U*(t) = expected utility of type t in equilibrium;

1. Pick a criterion, saying that particular out-of equilibrium message (OEM) cannot be sent by some type t. Also, say that a will not bet taken in response to m if a is not in BR(T(m),m). Iterate. [T^s(m)]
2. For each OEM m, consider all sequential equilibrium responses of R to m in the original game. Are all of of these are sequentially rational, given T^s(m). If not, FAIL.

Dominance

- For any OEM m, eliminate t if ∃m’ s.t.

$$\min_{a\in A(m')} U^S(t,m',a) > \max_{a\in A(m)} U^S(t,m,a)$$



Equilibrium Dominance & Intuitive Criterion

Equilibrium Domination: ∀OEM m, eliminate (t,m) if

$$U^*(t) > \max_{a\in A(m)} U^S(t,m,a).$$

Intuitive Criterion: ∀OEM m, define

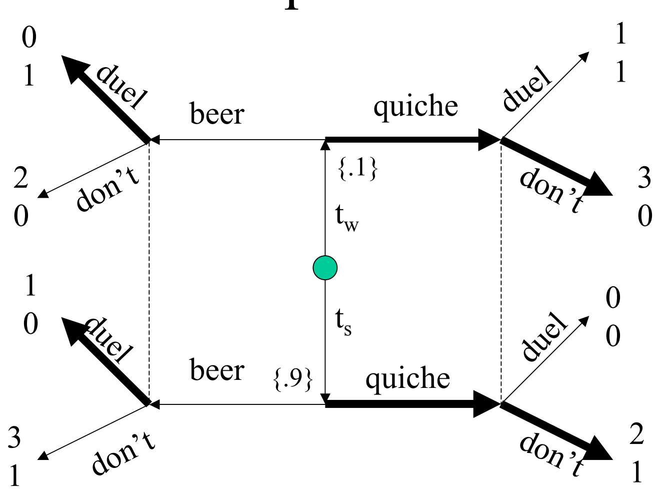
$$\tilde{T}(m) = \{t \mid U^*(t) > \max_{a\in BR(T(m),m)} U^S(t,m,a)\}.$$

If ∃(t',m) s.t.

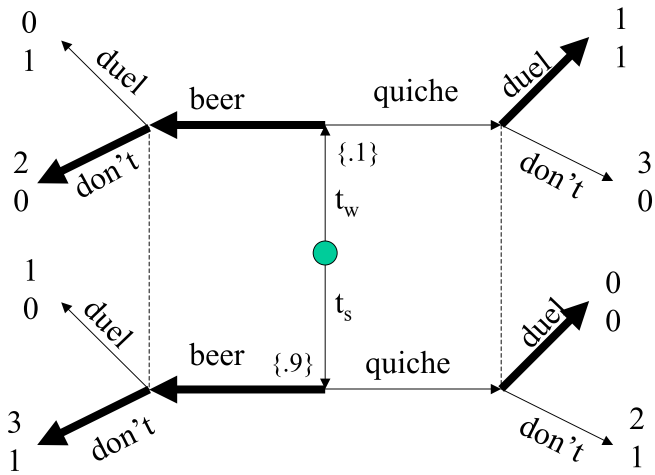
$$U^*(t') < \min_{a\in BR(T(m)\setminus\tilde{T}(m),m)} U^S(t',m,a),$$

then the equilibrium fails the Intuitive Criterion.

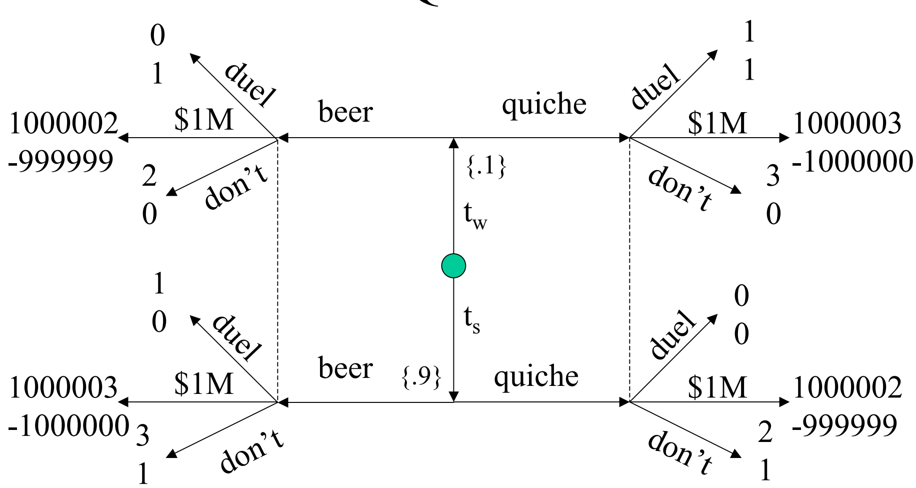
Bad equilibrium



Good equilibrium

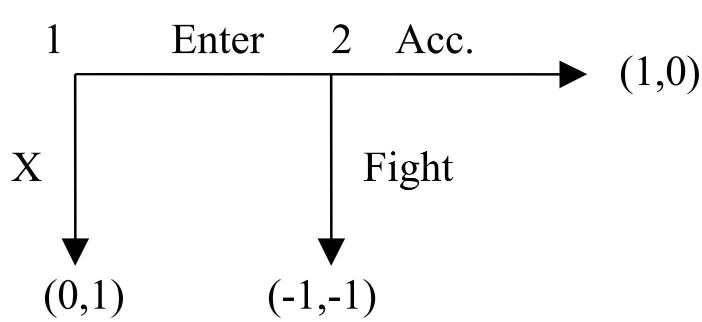


Beer – Quiche – M

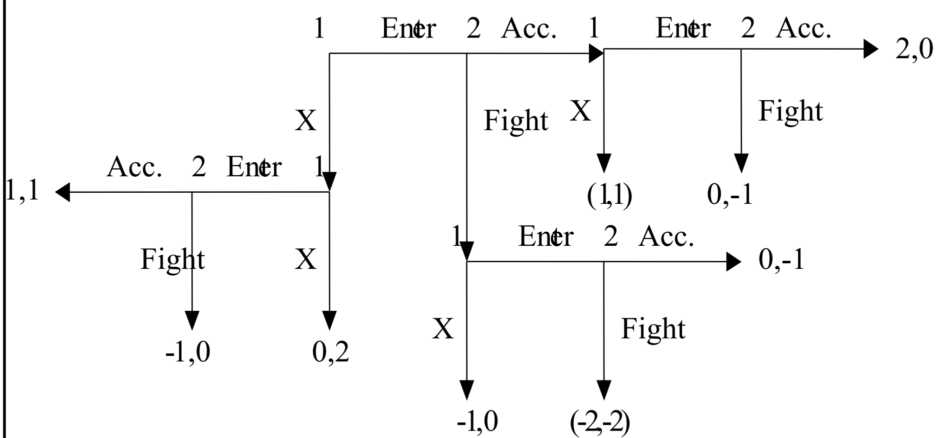


Reputation

Entry deterrence

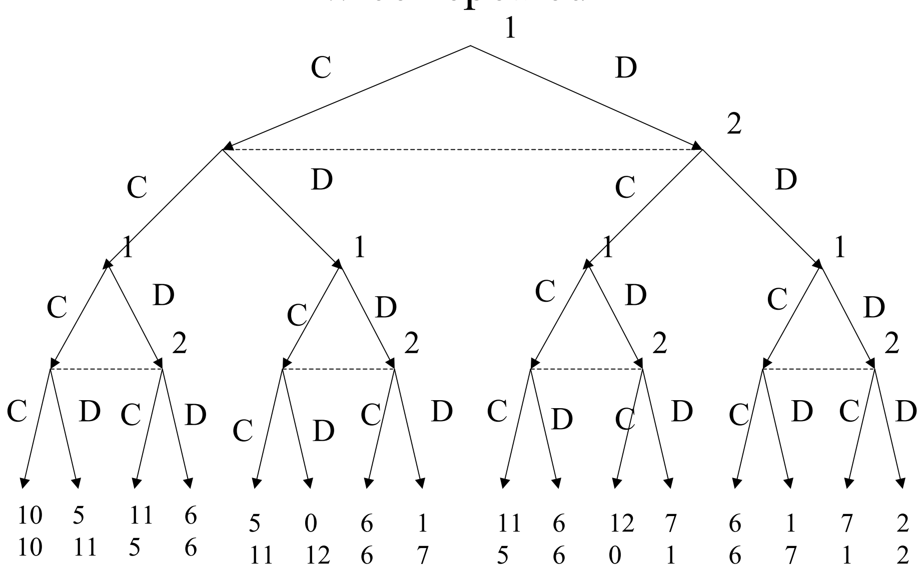


Entry deterrence, repeated twice, many times



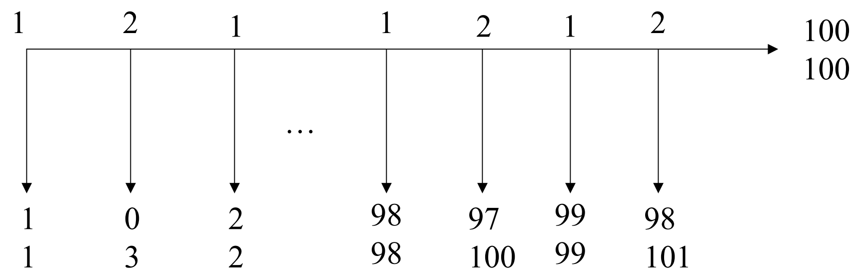
What would happen if repeated n times?

Twice-repeated PD

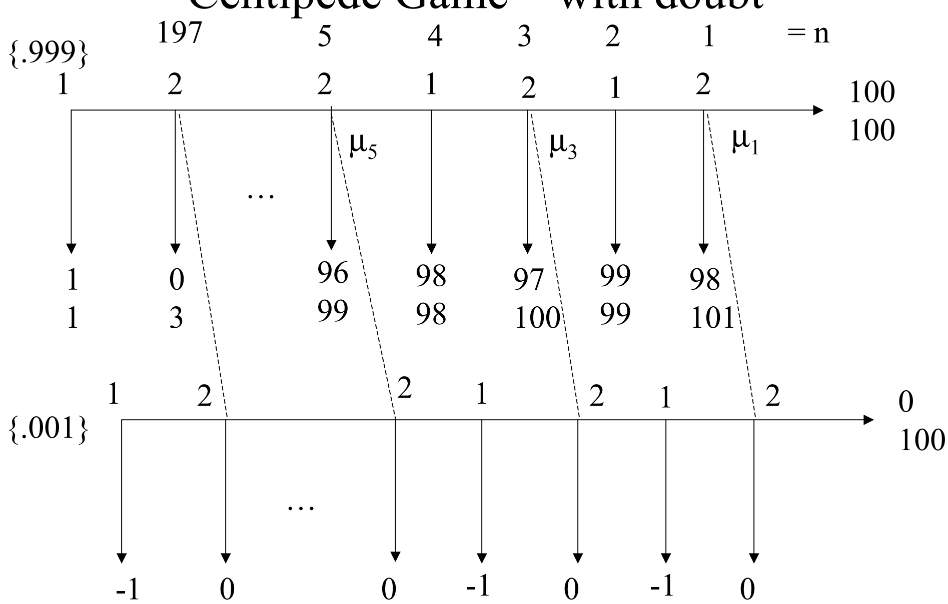


What would happen if T = {0,1,2,…,n}?

Centipede Game



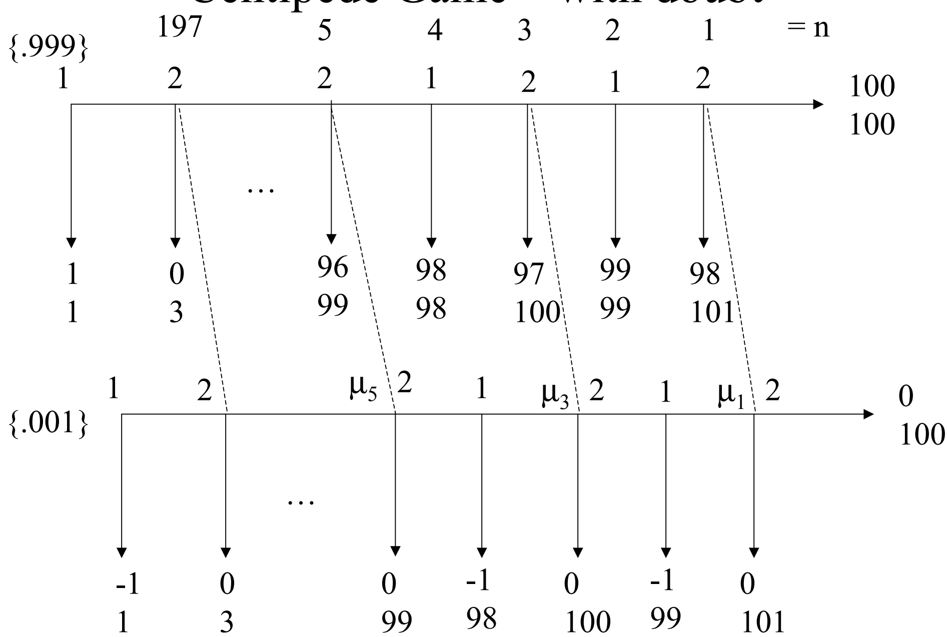
Centipede Game – with doubt



Facts about the Centipede

- Every information set of 2 is reached with positive probability.
- 2 always goes across with positive probability.
- If 2 strictly prefers to go across at n, then
  - 1 must strictly prefer to go across at n+1,
  - 2 must strictly prefer to go across at n+2,
  - her posterior at n is her prior.
- For any n > 2, 1 goes across with positive probability. If 1 goes across w/p 1 at n, then 2's posterior at n-1 is her prior.

Centipede Game – with doubt

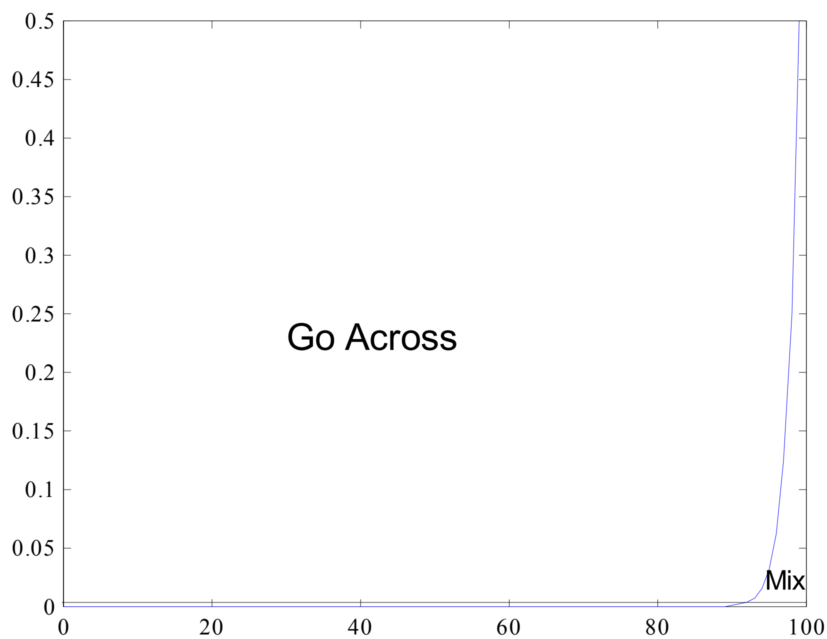


If 2's payoff at any n is x and 2 is mixing, then

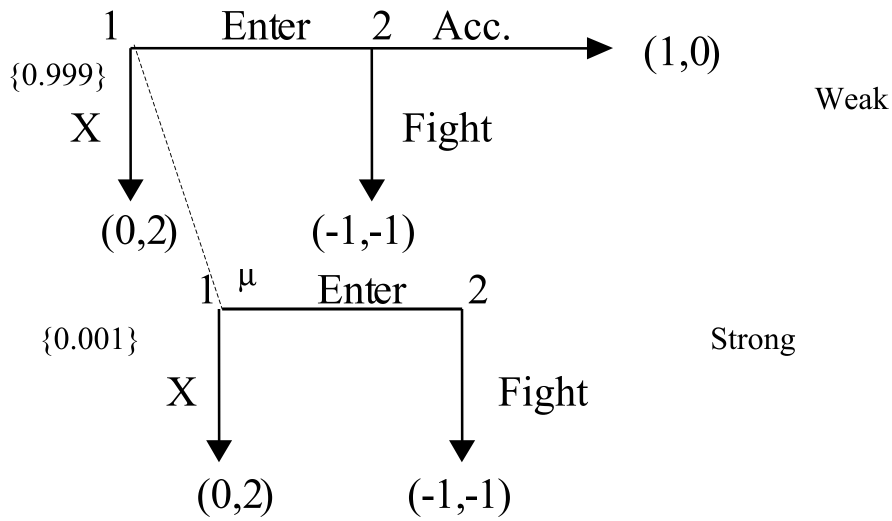
$$x = \mu_n(x+1) + (1-\mu_n)[(x-1)p_n + (1-p_n)(x+1)]$$
$$= \mu_n(x+1) + (1-\mu_n)[(x+1) - 2p_n]$$
$$= x+1 - 2p_n(1-\mu_n)$$
$$\Leftrightarrow (1-\mu_n)\, p_n = 1/2$$

$$\mu_{n-1} = \frac{\mu_n}{\mu_n + (1-\mu_n)(1-p_n)} = \frac{\mu_n}{\mu_n + (1-\mu_n) - p_n(1-\mu_n)} = 2\mu_n$$

$$\mu_n = \frac{\mu_{n-1}}{2}$$



Entry-deterrence with doubt



Entry-deterrence with doubt

- Incumbent is as before.
- Each day there is a new entrant.
- Two types of entrants:
  - W/p q < 1/2, tough, u(enter, fight) = 1;
  - W/p 1-q, weak, u(enter, fight) = 1.

Sequential Equilibrium

- Tough entrant (tE) always enters; strong incumbent (sI) always fights.
- If any entrant (E) is accommodated; it becomes common knowledge that incumbent (I) is weak …

- In the last period,
  - wI accommodates;
  - wE enters iff

$$-1\mu_0 + (1-\mu_0) \geq 0 \Leftrightarrow \mu_0 \leq 1/2$$

- if $\mu_0 < 1/2$, at n=1, wI accommodates => $\mu_0 = 1$.
- if $\mu_0 > 1/2$, at n=1, wI fights => $\mu_0 = 0.001$.
- $\mu_0 = 1/2$.

- $\mu_0 = \mu_1/(\mu_1 + \phi_1(1-\mu_1)) = 1/2$

$\Leftrightarrow \mu_1 = \phi_1(1-\mu_1)$

=>Prob(fight|μ_1) = $2\mu_1$.

=>at n=1, wE enters iff $2\mu_1 < ½$ i.e., $\mu_1 < ¼$.

- Similarly, $\mu_1 = ¼$.
- Proceed as before.