14.126 Game Theory Problem Set 4

Due after Class #21

1. Consider the following Hawk-Dove game, where doves also incur some small cost t when they meet each other:

	H	D
H	$(v-w)/2, (v-w)/2$	$v, 0$
D	$0, v$	$v/2-t, v/2-t$

.

   (a) Assuming that $w > v$ and $t > 0$ find all evolutionarily stable strategies.

   (b) Now consider a third type, B, who plays H if he is the first to come to a territory and plays D (without incurring the cost t) if he is the second:

	H	D	B
H	$(v-w)/2, (v-w)/2$	$v, 0$	$3v/4 - w/4, (v-w)/4$
D	$0, v$	$v/2-t, v/2-t$	$v/4 - t/2, 3v/4 - t/2$
B	$(v-w)/4, 3v/4 - w/4$	$3v/4 - t/2, v/4 - t/2$	$v/2, v/2$

.

   For which values of v, w, and t, B is an evolutionarily stable strategy?

2. Let $\underline{z} = (\underline{z}_1, \ldots, \underline{z}_n)$ be the smallest rationalizable strategy profile in a given supermodular game. Let also $\underline{y}$ be the smallest Nash equilibrium of the game that is created by fixing Player 1's strategy at $\underline{z}_1$. Show that

$$\underline{z} = \underline{y}.$$

3. Consider the following "ultimatum game," where Player 1 offers some $a \in A = \{0.01, 0.02, \ldots, 1\}$, and Player 2 demands some $b \in A$. If $a \geq b$ (i.e., if Player 2 accepts the offer a), then the payoffs are $(1-a, a)$; otherwise, the payoffs are $(0, 0)$.

   (a) Compute all Nash equilibria.

   (b) Now consider an evolutionary process in which the members of a population are matched in pairs to play this ultimatum game where each agent is equally likely to play the roles of Player 1 and Player 2. Assume that the growth of the strategies in this role-playing game follows the replicator dynamics. Find all asymptotically stable strategies.

4. Consider a linear Cournot duopoly with the true inverse demand function $P = a - Q$ and zero marginal costs, where P is price, $a > 0$, and $Q = q_1 + q_2$ is the total supply of a good. Now imagine that each firm $i \in N = \{1, 2\}$ perceives the inverse demand function as $P = a + b_i - Q$, where $b_i \in R$ is the bias in i's perception. Let $G(b_1, b_2)$ be the game game in which b_1 and b_2 are common-knowledge.

   (a) Show that $G(b_1, b_2)$ has a unique rationalizable strategy profile. Compute the true payoffs $u_1(b_1, b_2)$ and $u_2(b_1, b_2)$ at the rationalizable strategy profile — computed by using $P = a - Q$.

   (b) Consider the meta game $\Gamma = (N, \mathbb{R}, \mathbb{R}, u_1, u_2)$, where the strategies are choices of b_1 and b_2, and u_1 and u_2 are as in (a). Show that Γ is supermodular in a proper order, has a unique Nash equilibrium b^*, and that $b_i^* > 0$ for each $i \in N$. Show that the replicator dynamics for Γ (using the true payoffs) converges to b^*.

   (c) Now consider an evolutionary learning process in which the agents not only develop their perceptions (i.e., b_1 and b_2) but also learn how to play the game $G(b_1, b_2)$ given perceptions. Assume that the learning process is a "two-tiered" replicator dynamics in which they learn how to play $G(b_1, b_2)$ infinitely faster than they change their perceptions, i.e., given any perception-pair (b_1, b_2), the play converges to the limit of the dynamics for fixed (b_1, b_2) before they change their perceptions. What is the limit of this "two-tiered" replicator dynamics?

   (d) Briefly discuss these results.

   [**Hint:** Throughout this exercise, assume that the result of Samuelson and Zhang (i.e., Theorem 3.1 in Weibull) applies.]