Lecture 3
Game Theory

14.12 Game Theory

Road Map

1. **Quiz**
2. Representation of games in strategic and extensive forms
3. **Quiz?**

Multi-person Decision Theory

- Who are the players?
- Who has which options?
- Who knows what?
- Who gets how much?

Knowledge

1. If I know something, it must be true.
2. If I know x, then I know that I know x.
3. If I don't know x, then I know that I don't know x.
4. If I know something, I know all its logical implications.

Common Knowledge: x is common knowledge iff

- Each player knows x

- Each player knows that each player knows x

- Each player knows that each player knows that each player knows x

- Each player knows that each player knows that each player knows that each player knows x

- … ad infinitum

Representations of games

Normal-form representation

Definition (Normal form): A game is any list

$$G = (S_1, \ldots, S_n; u_1, \ldots, u_n)$$

where, for each $i \in N = \{1, 2, \ldots, n\}$,

- S_i is the set of all strategies available to i,
- $u_i : S_1 \times \cdots \times S_n \to \Re$ is the VNM utility function of player i.

Assumption: G is common knowledge.

Definition: A player i is rational iff he tries to maximize the expected value of u_i given his beliefs.
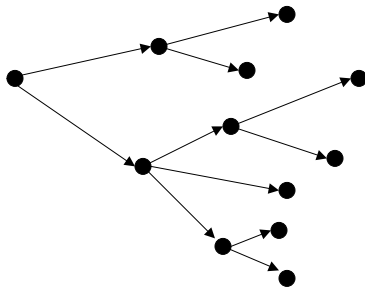
Chicken

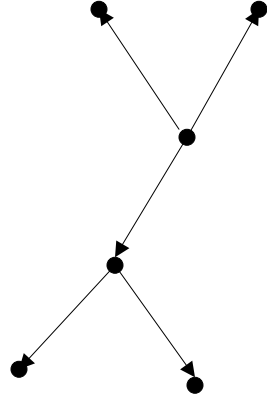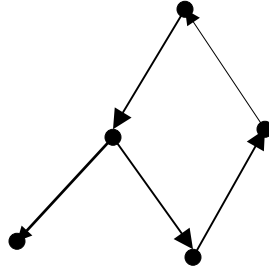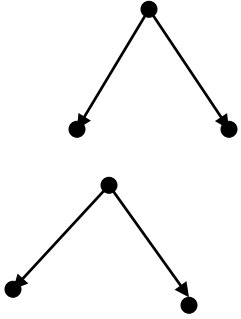|	Lion	Chicken
Lion	(-1,-1)	(1,0)
Chicken	(0,1)	(1/2,1/2)

Extensive-form representation

Definition: A **tree** is a set of nodes connected with directed arcs such that

1. For each node, there is at most one incoming arc;
2. each node can be reached through a unique path;

A tree?

A tree??

A tree

Non-terminal nodes

Terminal Nodes

Extensive form – definition

Definition: A game consists of
– a set of players
– a tree
– an allocation of each non-terminal node to a player
– an informational partition (to be made precise)
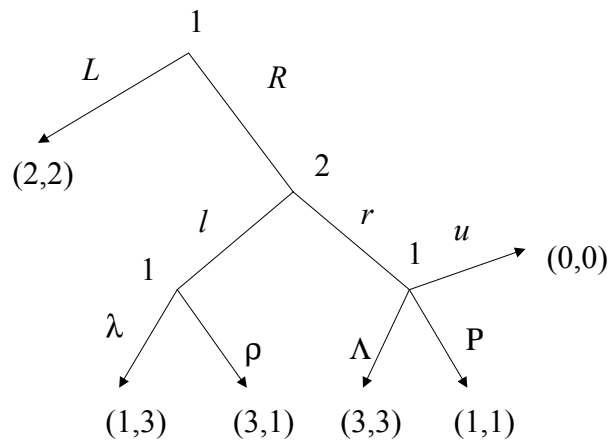– a payoff for each player at each terminal node.
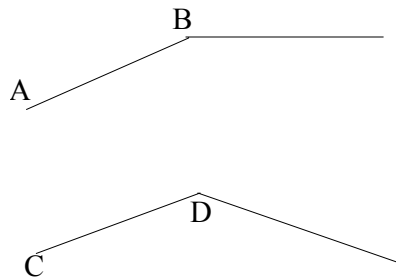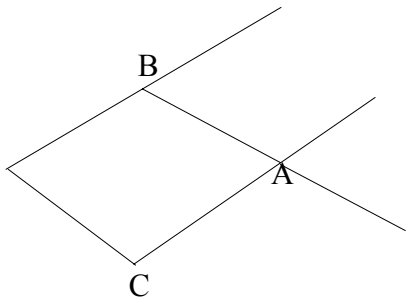
Information set

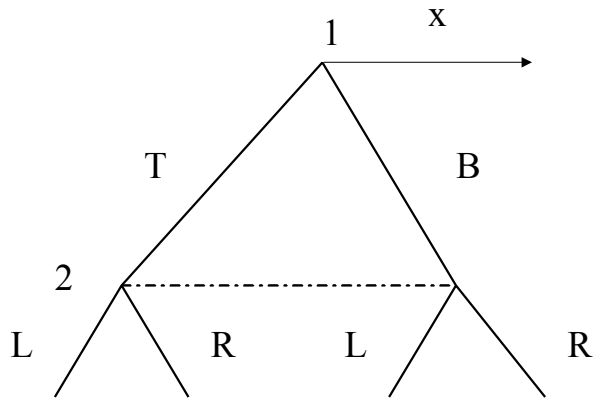An **information set** is a collection of nodes such that

1. The same player is to move at each of these nodes;
2. The same moves are available at each of these nodes.

An **informational partition** is an allocation of each non-terminal node of the tree to an information set.

A game

Another Game

The same game
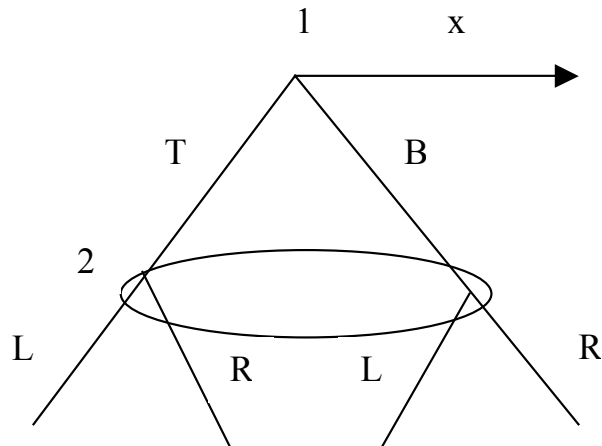
Strategy

A **strategy** of a player **is a complete contingent-plan**, determining which action he will take at each information set he is to move (including the information sets that will not be reached according to this strategy).

Matching pennies with perfect information
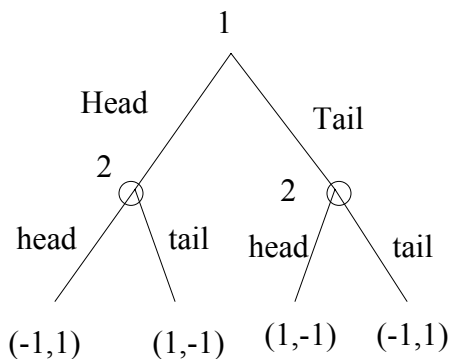
2's Strategies:

HH = Head if 1 plays Head, Head if 1 plays Tail;

HT = Head if 1 plays Head, Tail if 1 plays Tail;

TH = Tail if 1 plays Head, Head if 1 plays Tail;

TT = Tail if 1 plays Head, Tail if 1 plays Tail.

Matching pennies with perfect information

	HH	HT	TH	TT
Head				
Tail				

(tree: 1 / 2)

Matching pennies with Imperfect information

1

Head — Tail

2

head tail head tail
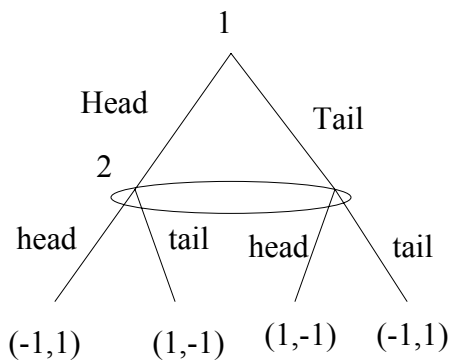
(-1,1) (1,-1) (1,-1) (-1,1)

1 \ 2	Head	Tail
Head	(-1,1)	(1,-1)
Tail	(1,-1)	(-1,1)

A game with nature

```
                                      Left      (5, 0)
                                   1
                          Head
                          1/2          Right
                                                 (2, 2)
Nature  ○
                          1/2                    (3, 3)
                                       Left
                          Tail     2
                                       Right
                                                 (0, -5)
```

A centipede game

```
   1     A     2    α    1    a
   │───────────│─────────│──────────▶  (1,-5)
   │           │         │
   │           │         │
 D │         δ │       d │
   ▼           ▼         ▼
 (4,4)       (5,2)     (3,3)
```