14.12 Game Theory
Fall 2000
Final Exam Solutions

1. (a) Given perfect competition among firms, wages will be $w_1 = 1$,$w_2 = 2$ and $w_3 = 3$ for types $t = 1, 2, 3$ respectively. The incentive compatibility constraints give us the conditions that the education levels must fulfill in equilibrium: $e_2^* \geq 1 + e_1^*$ and $e_3^* \geq 2 + e_1^*$ for $t = 1$, $e_2^* \leq 2 + e_1^*$ and $e_3^* \geq 2 + e_2^*$ for $t = 2$, and $e_3^* \leq 3 + e_2^*$ and $e_3^* \leq 6 + e_1^*$ for $t = 3$.

   If we let, for example, $e_1^* = 0$, then $e_2^* = 1$ and $e_3^* = 3$. Beliefs are $\mu(t = 1|e') = 1$, where e' is in $[0, \inf)$, $\mu(t = 2|e = 1) = 1$ and $\mu(t = 3|e = 3) = 1$.

   (b) Let $w_1 = 1$ as before, since type 1 is separating. Given that types 2 and 3 are pooling, their wage is the expected marginal productivity or $1/2(2) + 1/2(3) = 5/2$. Letting $e_1^* = 0$, the compatibility constraints provide the conditions that the education levels must fulfill in equilibrium: $e^* \geq 3/2$ and $e^* \leq 3$. For instance, we choose $e^* = 2$. Beliefs are $\mu(t = 1|e \neq e^*) = 1$, $\mu(t = 2|e^*) = 1/2$ and $\mu(t = 3|e^*) = 1/2$.

2. (a) Let the separating equilibrium be one where top plays R, bottom plays L. The beliefs are $\mu(top|R) = 1$ and $\mu(bottom|R) = 1$. Player 2, the receiver, plays up in the information set on the right-hand side, and down, on the information set on the left-hand side. Now it is easy to check that player 1's types do not want to deviate: if top type plays L instead, she gets 0 as opposed to 3, while bottom type, if she plays R instead, gets 1 as opposed to 2.

   (b) Pooling on R: player 2 plays down when sees R since $EU(up|R) = 0.4(1) + 0.6(0) = 0.4 < EU(down|R) = 0.4(0) + 0.6(1) = 0.6$. Any beliefs about types given L actually support this equilibrium.

   (c) Mixed strategies equilibrium: Bottom mixes $\alpha R + (1 - \alpha)L$. Player 2 mixes $1/2up+1/2down$ when sees R. By applying Bayes' rule and forcing it to be equal to 1/2, because this is the value that would make the bottom type want to mix, we have $\mu(top|R) = 0.4(1)/(0.4+0.6(\alpha)) = 1/2$. In particular, for player 2, $EU(up|R) = P(top|R)$ and $EU(down|R) = 1 - P(top|R)$. Setting these two equal, because we need player 2 to mix after observing R in order for bottom type to want to mix, we obtain that $P(top|R) = 1/2$. From the Bayes' rule equation we solve for α above, such that bottom type's strategy is $2/3R + 1/3L$. Top type always plays R. Player 2 plays $1/2up + 1/2down$ when sees R and plays down when sees L. Beliefs are $\mu(bottom|L) = 1$ and $\mu(top|R) = 1/2$.

3. Given no discounting, and the fact that the plaintiff gets to make the first offer, the game ends in the first day, with the plaintiff making an offer w and the defendant accepting the offer. In particular, taking into account the legal fees both parties must pay until court day, and the amount the defendant must pay then, which is certain and common knowledge, the plaintiff will offer $1,110K$ and the defendant will accept all offers less than or equal to this amount, and reject all others.

4. One pooling equilibrium of this game would be for the firm to reject all offers $w_0 > 5$ and accept all others, for both types of workers to offer $w_0 = 5$, and to offer $w_1 = 5$ in the following period if reached. Beliefs: $\mu(type = 0|w \neq 5) = 1$. Workers do not want to deviate because that would lead to a rejection and a wage of 5 in the following period, which is what they can get now (so they are indifferent).