Lectures 12-13

Incomplete Information
Static Case

14.12 Game Theory

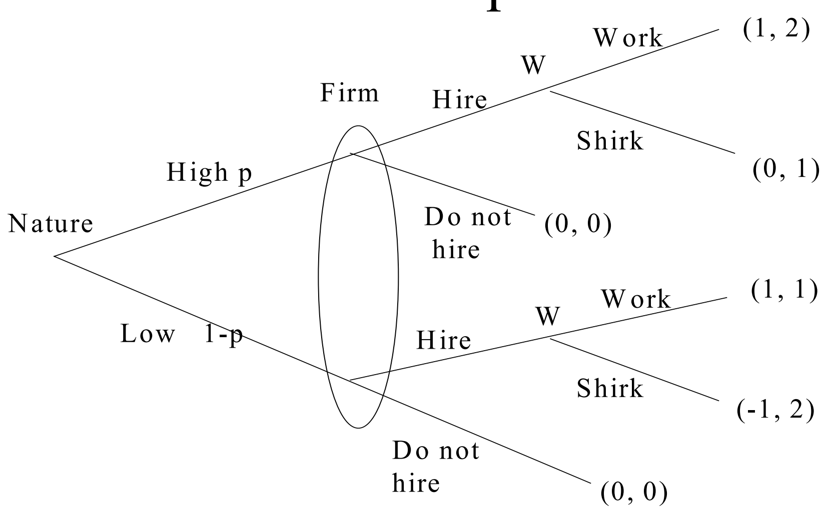
Road Map

1. Examples
2. Bayes' rule
3. Definitions
   1. Bayesian Game
   2. Bayesian Nash Equilibrium
4. Mixed strategies, revisited
5. Economic Applications
   1. Cournot Duopoly
   2. Auctions
   3. Double Auction

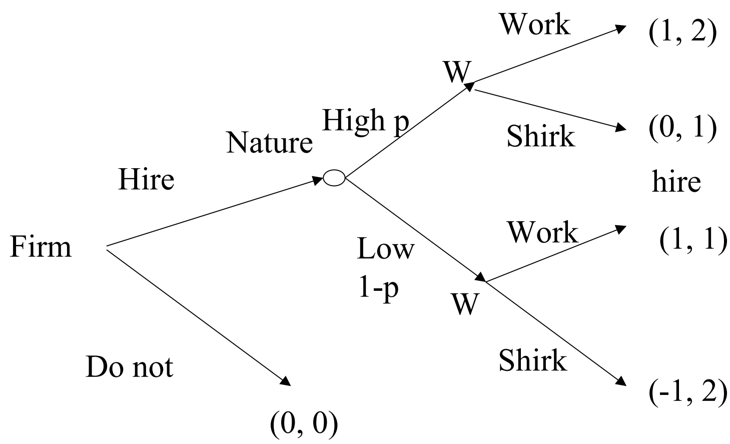
Incomplete information

We have incomplete (or asymmetric) information if one player knows something (relevant) that some other player does not know.

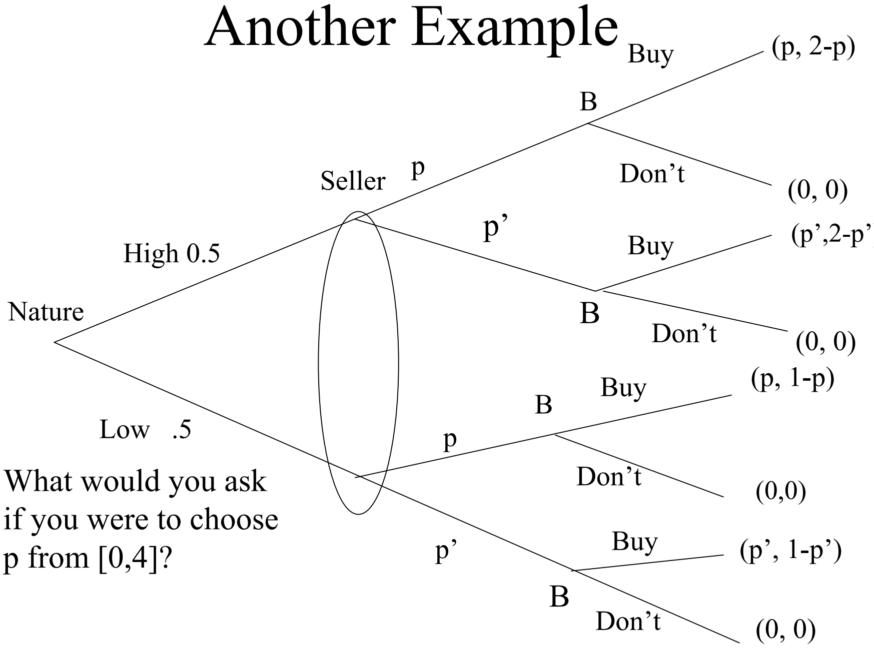
An Example



The same example

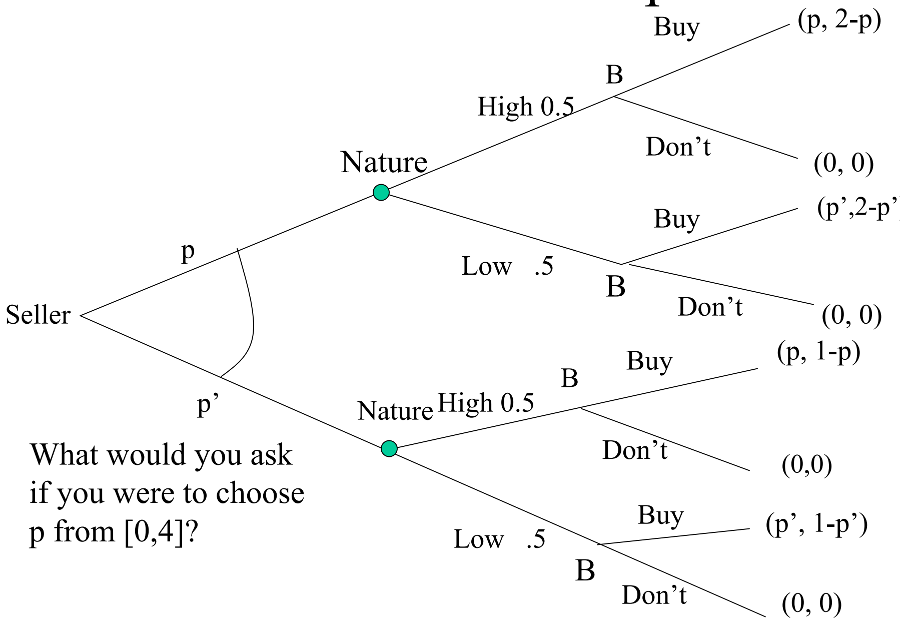


Another Example



What would you ask if you were to choose p from [0,4]?

Same "Another Example"

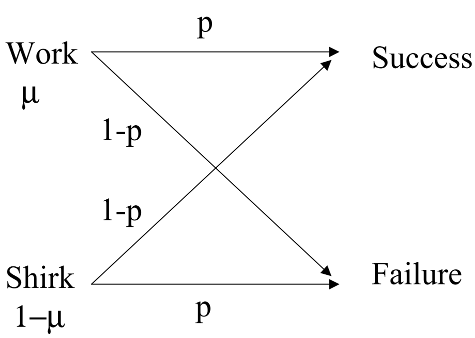


What would you ask if you were to choose p from [0,4]?

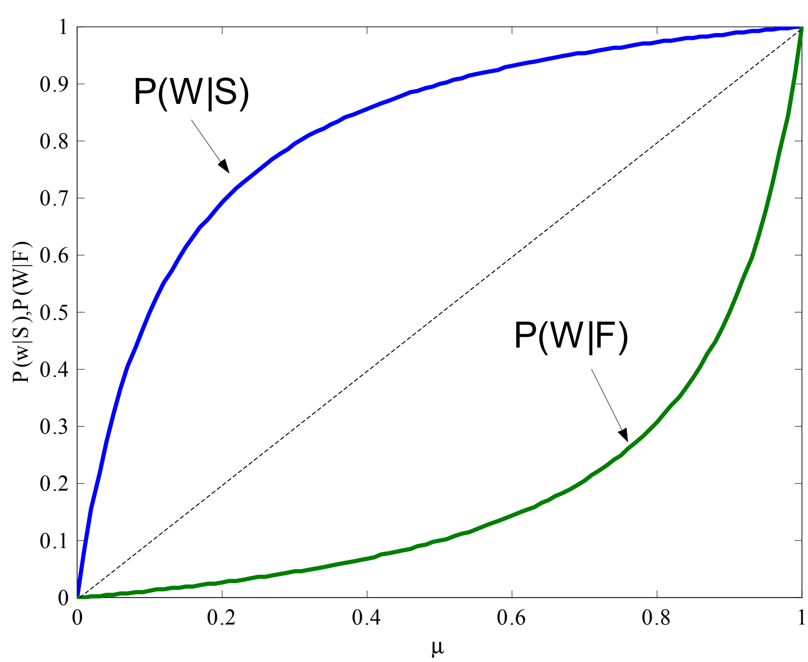
Bayes' Rule

- $\text{Prob}(A|B) = \dfrac{\text{Prob}(A \text{ and } B)}{\text{Prob}(B)}$
- $\text{Prob}(A \text{ and } B) = \text{Prob}(A|B)\text{Prob}(B) = \text{Prob}(B|A)\text{Prob}(A)$
- $\text{Prob}(A|B) = \dfrac{\text{Prob}(B|A)\text{Prob}(A)}{\text{Prob}(B)}$

Example



- Prob(Work|Success) = $\mu p/[\mu p + (1-\mu)(1-p)]$
- Prob(Work|Failure) = $(1-\mu)p/[\mu(1-p) + (1-\mu)p]$



Bayesian Game (Normal Form)

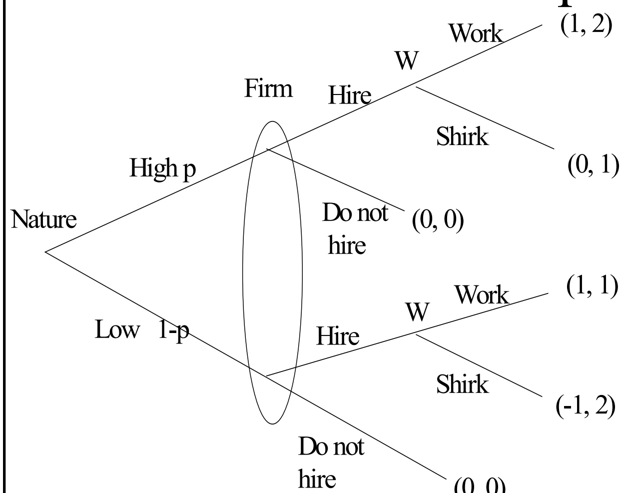
A Bayesian game is a list

$G = \{A_1,\ldots,A_n;T_1,\ldots,T_n;p_1,\ldots,p_n;u_1,\ldots,u_n\}$

where

- A_i is the action space of i (a_i in A_i)
- T_i is the type space of i (t_i)
- $p_i(t_{-i}|t_i)$ is i's belief about the other players
- $u_i(a_1,\ldots,a_n;t_1,\ldots,t_n)$ is i's payoff.

An Example



$T_{Firm}=\{t_f\}$;
$T_W = \{High,Low\}$
$A_{Firm} = \{Hire, Don't\}$
$A_W = \{Work,Shirk\}$
$p_F(High) = p$
$p_F(Low) = 1-p$

Bayesian Nash equilibrium

A Bayesian Nash equilibrium is a Nash equilibrium of a Bayesian game.

Given any Bayesian game G = $\{A_1,\ldots,A_n;T_1,\ldots,T_n;p_1,\ldots,p_n;u_1,\ldots,u_n\}$

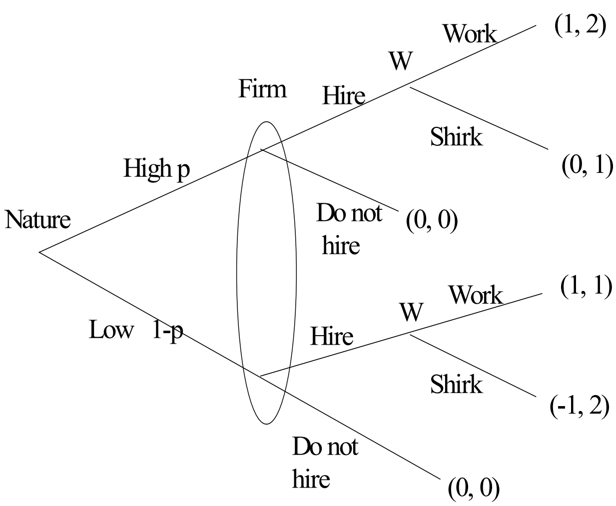
a **strategy** of a player i in a is any function $s_i:T_i \rightarrow A_i$;

A strategy profile $s^* = (s_1^*,\ldots, s_1^*)$ is a **Bayesian Nash equilibrium** iff $s_i^*(t_i)$ solves

$$\max_{a_i\in A_i} \sum_{t_{-i}\in T_{-i}} u_i\left(s_1^*(t_1),\ldots,s_{i-1}^*(t_{i-1}),a_i,s_{i+1}^*(t_{i+1}),\ldots,s_n^*(t_n);t\right)p_i(t_{-i}\mid t_i)$$

i.e., s_i^* is a best response to s_{-i}^*.

An Example



$T_{Firm}=\{t_f\}$;
$T_W = \{High,Low\}$
$A_{Firm} = \{Hire, Don't\}$
$A_W = \{Work,Shirk\}$
$p_F(High) = p > 1/2$
$p_F(Low) = 1-p$

s_F^* = Hire,
s_F^* (High) = Work
s_F^* (Low) = Shirk

Another equilibrium?

Stag Hunt, Mixed Strategy




(2,2)	(4,0)
(0,4)	(6,6)

Mixed Strategies




2+t,2+v	4+t,0
0,4+v	6,6

$U_1(R|t) = t + 2q + 4(1-q) = t + 4 – 2q$
$U_1(S|t) = 6(1-q)$;
$U_1(R|t) > U_1(S|t) \Leftrightarrow t+4–2q > 6(1-q)$
$\Leftrightarrow t > 6-6q+2q-4 = 2 – 4q = 0.$

- t and v are iid with uniform distribution on [–ε,ε].
- t and v are privately known by 1 and 2, respectively, i.e., are types of 1 and 2, respectively.
- Pure strategy:
  - $s_1(t)$ = Rabbit iff $t > 0$;
  - $s_2(v)$ = Rabbit iff $t > 0$.
- $p = \text{Prob}(s_1(t)=\text{Rabbit}|v) = \text{Prob}(t > 0) = 1/2.$
- $q = \text{Prob}(s_2(v)=\text{Rabbit}|t) = 1/2.$