Lectures 7

Backward Induction

14.12 Game Theory

Road Map

1. Bertrand competition with costly search
2. Backward Induction
3. Stackelberg Competition
4. Sequential Bargaining
5. Quiz

Bertrand Competition with costly search

- N = {F1,F2,B}; F1, F2 are firms; B is buyer
- B needs 1 unit of good, worth 6;
- Firms sell the good; Marginal cost = 0.
- Possible prices P = {3,5}.
- Buyer can check the prices with a small cost c > 0.

Game:

1. Each firm i chooses price p_i;
2. B decides whether to check the prices;
3. (Given) If he checks the prices, and $p_1 \neq p_2$, he buys the cheaper one; otherwise, he buys from any of the firm with probability ½.

Bertrand Competition with costly search

Check

F1 \ F2	High	Low
High	5/2 5/2 1-c	0 1 3-c
Low	3 0 3-c	3/2 3/2 3-c

Don't Check

F1 \ F2	High	Low
High	5/2 5/2 1	5/2 3/2 2
Low	3/2 5/2 2	3/2 3/2 3

Mixed-strategy equilibrium

- Symmetric equilibrium: Each firm charges "High" with probability q;
- Buyer Checks with probability r.
- $U(\text{check};q) = q^2 1 + (1-q^2)3 – c = 3 - 2q^2 – c$;
- $U(\text{Don't};q) = q1 + (1-q)3 = 3 - 2q$;
- Indifference: $2q(1-q) = c$; i.e.,
- $U(\text{high};q,r) = 0.5(1-r(1-q))5$;
- $U(\text{low};q,r) = qr3 + 0.5(1-qr)3$
- Indifference: $r = 2/(5-2q)$.

Dynamic Games of Perfect Information
&
Backward Induction

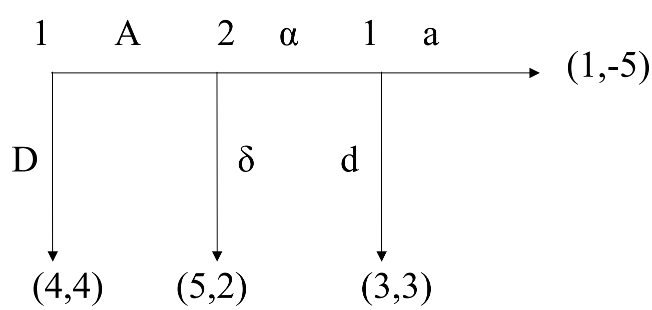
Definitions

Perfect-Information game is a game in which all the information sets are singleton.

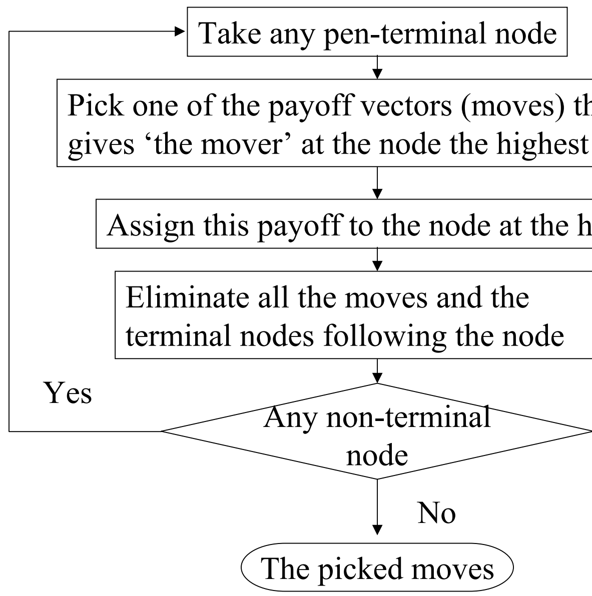
Sequential Rationality: A player is sequentially rational iff, at each node he is to move, he maximizes his expected utility conditional on that he is at the node – even if this node is precluded by his own strategy.

In a finite game of perfect information, the "common knowledge" of sequential rationality gives **"Backward Induction"** outcome.

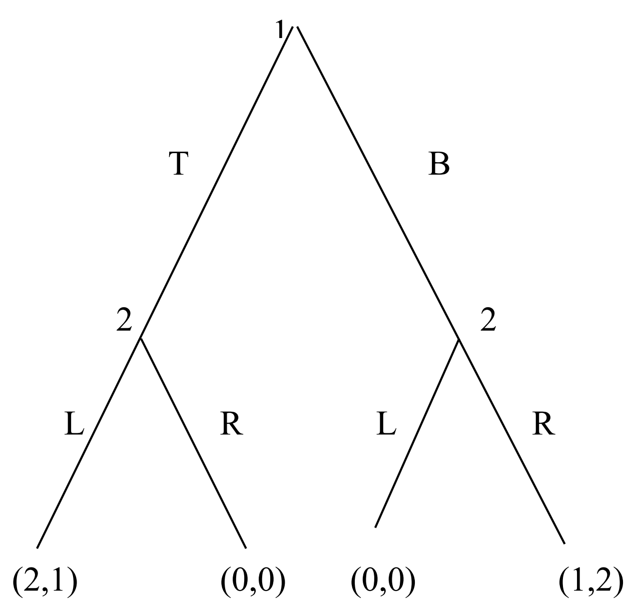
A centipede game



Backward Induction



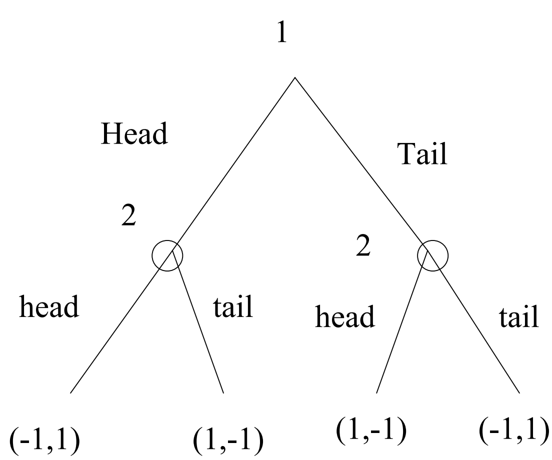
Battle of The Sexes with perfect information



Note

- There are Nash equilibria that are different from the Backward Induction outcome.
- Backward Induction always yields a Nash Equilibrium.
- That is, Sequential rationality is stronger than rationality.

Matching Pennies (wpi)

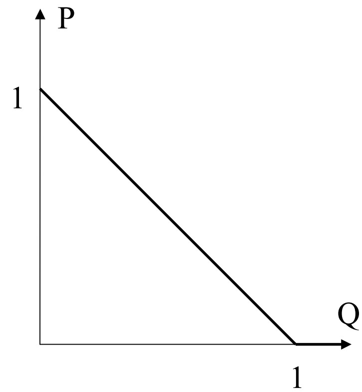


Stackelberg Duopoly

Game:

N = {1,2} firms w MC = 0;

1. Firm 1 produces q_1 units
2. Observing q_1, Firm 2 produces q_2 units
3. Each sells the good at price $P = \max\{0, 1-(q_1+q_2)\}$.



$\pi_i(q_1, q_2) = q_i[1-(q_1+q_2)]$ if $q_1 + q_2 < 1$,
0 otherwise.

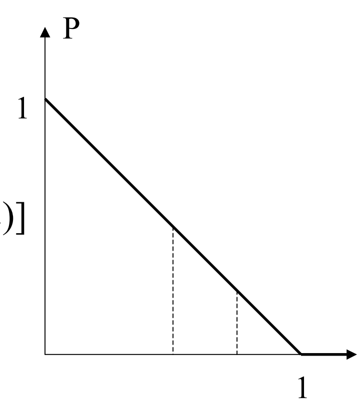
"Stackelberg equilibrium"

- If $q_1 > 1$, $q_2^*(q_1) = 0$.
- If $q_1 \leq 1$, $q_2^*(q_1) = (1-q_1)/2$.
- Given the function q_2^*, if $q_1 \leq 1$

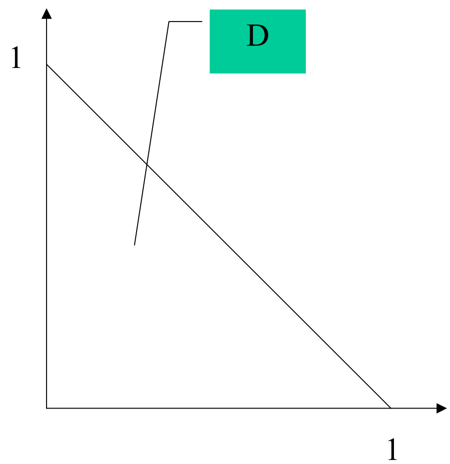
  $\pi_1(q_1;q_2^*(q_1)) = q_1[1-(q_1+ (1-q_1)/2)]$

  $= q_1 (1-q_1)/2$;

  0 otherwise.
- $q_1^* = ½$.
- $q_2^*(q_1^*) = ¼$.



Sequential Bargaining



- $N = \{1,2\}$
- X = feasible expected-utility pairs $(x,y \in X)$
- $U_i(x,t) = \delta_i^t x_i$
- $d = (0,0) \in D$ disagreement payoffs

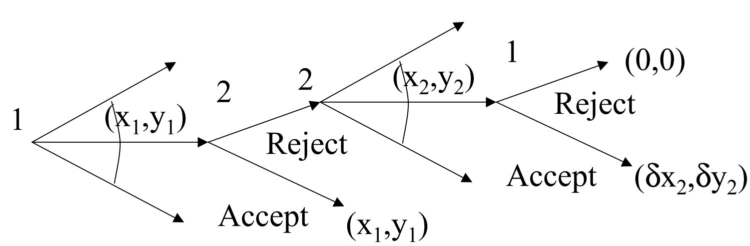
Timeline – 2 period

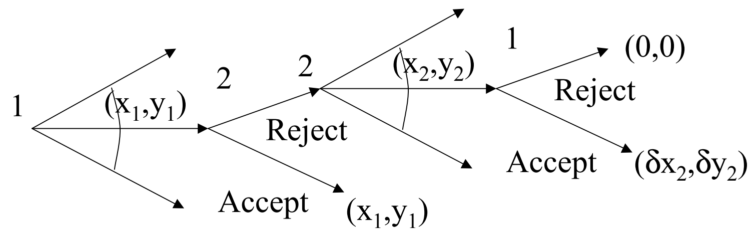
At t = 1,

- Player 1 offers some (x_1,y_1),
- Player 2 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding (x_1,y_1),
- Otherwise, we proceed to date 2.

At t = 2,

- Player 2 offers some (x_2,y_2),
- Player 1 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding payoff $\delta(x_2,y_2)$.
- Otherwise, the game end yielding d = (0,0).





At t = 2,

•Accept iff $y_2 \geq 0$.

•Offer (0,1).

At t = 1,

•Accept iff $x_2 \geq \delta$.

•Offer $(1-\delta,\delta)$.

Timeline – 2n period

T = {1,2,…,2n-1,2n}

If t is odd,

– Player 1 offers some (x_t,y_t),
– Player 2 Accept or Rejects the offer
– If the offer is Accepted, the game ends yielding $\delta^t(x_t,y_t)$,
– Otherwise, we proceed to date t+1.

If t is even

– Player 2 offers some (x_t,y_t),
– Player 1 Accept or Rejects the offer
– If the offer is Accepted, the game ends yielding payoff (x_t,y_t),
– Otherwise, we proceed to date t+1, except at t = 2n, when the game end yielding d = (0,0).