Lecture 4
Rationalizability &
Nash Equilibrium

14.12 Game Theory

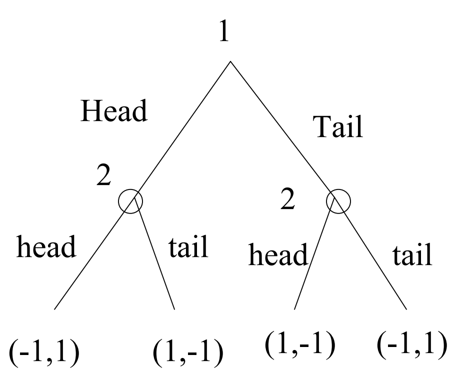
Road Map

1. Strategies – completed
2. Quiz
3. Dominance
4. Dominant-strategy equilibrium
5. Rationalizability
6. Nash Equilibrium

Strategy

A **strategy** of a player **is a complete contingent-plan**, determining which action he will take at each information set he is to move (including the information sets that will not be reached according to this strategy).

Matching pennies with perfect information



2's Strategies:

HH = Head if 1 plays Head,
Head if 1 plays Tail;

HT = Head if 1 plays Head,
Tail if 1 plays Tail;

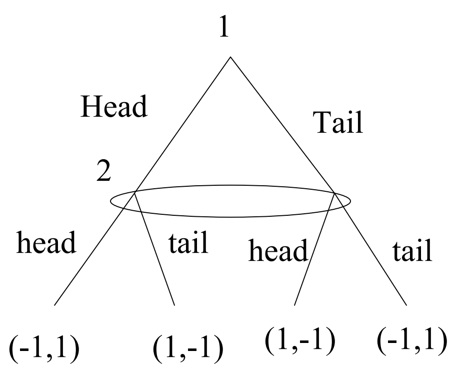
TH = Tail if 1 plays Head,
Head if 1 plays Tail;

TT = Tail if 1 plays Head,
Tail if 1 plays Tail.

Matching pennies with perfect information

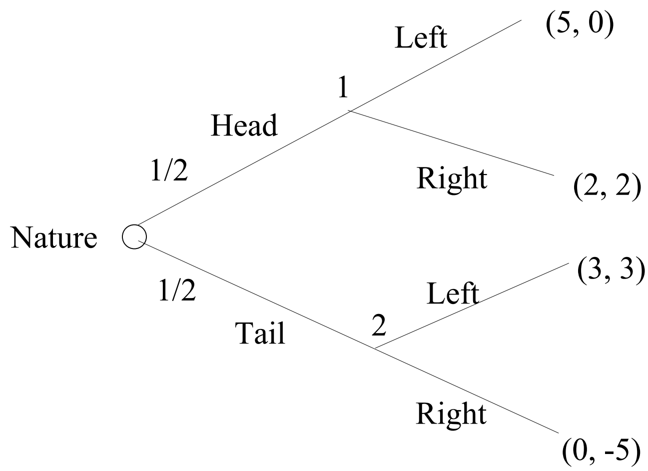
1 \ 2	HH	HT	TH	TT
Head				
Tail				

Matching pennies with Imperfect information



1 \ 2	Head	Tail
Head	(-1,1)	(1,-1)
Tail	(1,-1)	(-1,1)

A game with nature



Mixed Strategy

Definition: A **mixed strategy** of a player is a probability distribution over the set of his strategies.

Pure strategies: $S_i = \{s_{i1}, s_{i2}, \ldots, s_{ik}\}$

A mixed strategy: σ_i: $S \rightarrow [0,1]$ s.t.

$$\sigma_i(s_{i1}) + \sigma_i(s_{i2}) + \ldots + \sigma_i(s_{ik}) = 1.$$

If the other players play $s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$, then the expected utility of playing σ_i is

$$\sigma_i(s_{i1})u_i(s_{i1}, s_{-i}) + \sigma_i(s_{i2})\, u_i(s_{i2}, s_{-i}) + \ldots + \sigma_i(s_{ik})\, u_i(s_{ik}, s_{-i}).$$

How to play

Dominance

$s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$

Definition: A pure strategy s_i* **strictly dominates** s_i if and only if

$$u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_{-i}.$$

A mixed strategy σ_i* **strictly dominates** s_i iff

$$\sigma_i(s_{i1})u_i(s_{i1}, s_{-i}) + \cdots + \sigma_i(s_{ik})u_i(s_{ik}, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_i$$

A rational player never plays a strictly dominated strategy.

Prisoners' Dilemma

1 \ 2	Cooperate	Defect
Cooperate	(5,5)	(0,6)
Defect	(6,0)	(1,1)

A game

1 \ 2	L	m	R
T	(3,0)	(1,1)	(0,3)
M	(1,0)	(0,10)	(1,0)
B	(0,3)	(1,1)	(3,0)

Weak Dominance

Definition: A pure strategy s_i* weakly **dominates** s_i if and only if

$$u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \quad \forall s_{-i}.$$

and at least one of the inequalities is strict. A mixed strategy σ_i* **weakly dominates** s_i iff

$$\sigma_i(s_{i1})u_i(s_{i1}, s_{-i}) + \cdots + \sigma_i(s_{ik})u_i(s_{ik}, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_i$$

and at least one of the inequalities is strict.

If a player is rational and cautious (i.e., he assigns positive probability to each of his opponents' strategies), then he will not play a weakly dominated strategy.

Dominant-strategy equilibrium

Definition: A strategy s_i* is a **dominant strategy** iff s_i* **weakly dominates** every other strategy s_i.

Definition: A strategy profile s* is a dominant-strategy equilibrium iff s_i* is a dominant strategy for each player i.

If there is a dominant strategy, then it will be played, so long as the players are …

Prisoners' Dilemma

1 \ 2	Cooperate	Defect
Cooperate	(5,5)	(0,6)
Defect	(6,0)	(1,1)

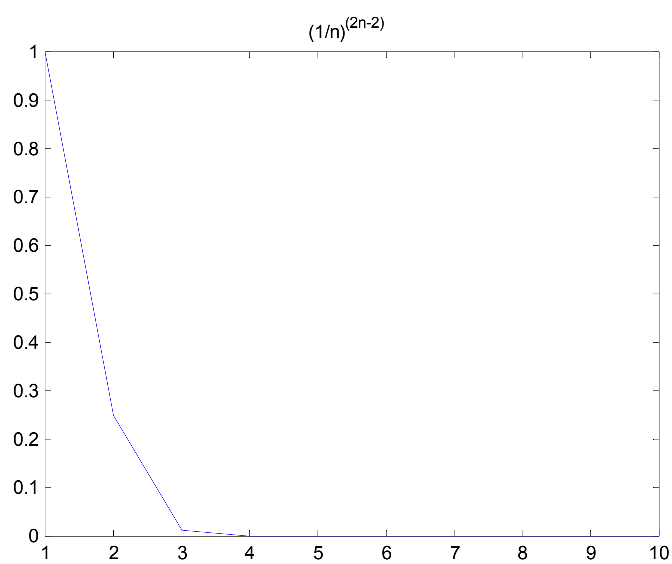
Second-price auction

- N = {1,2} buyers;
- The value of the house for buyer i is v_i;
- Each buyer i simultaneously bids b_i;
- i* with $b_{i*} = \max b_i$ gets the house and pays the second highest bid

  $p = \max_{j \neq i} b_j$.

Question

What is the probability that an nxn game has a dominant strategy equilibrium given that the payoffs are independently drawn from the same (continuous) distribution on [0,1]?



A game

1 \ 2	L	m	R
T	(3,0)	(1,1)	(0,3)
M	(1,0)	(0,10)	(1,0)
B	(0,3)	(1,1)	(3,0)

Assume: Players are rational and player 2 knows that 1 is rational.

1 is rational and 2 knows this:

	L	m	R
T	(3,0)	(1,1)	(0,3)
B	(0,3)	(1,1)	(3,0)

And 2 is rational:

	L	R
T	(3,0)	(0,3)
B	(0,3)	(3,0)

Rationalizability

The play is rationalizable, provided that …

Simplified price-competition

Firm 1 \ Firm 2	High	Medium	Low
High	6,6	0,10	0,8
Medium	10,0	5,5	0,8
Low	8,0	8,0	4,4

Dutta

A strategy profile is rationalizable when …

- Each player's strategy is consistent with his rationality, i.e., maximizes his payoff with respect to a conjecture about other players' strategies;
- These conjectures are consistent with the other players' rationality, i.e., if i conjectures that j will play s_j with positive probability, then s_j maximizes j's payoff with respect to a conjecture of j about other players' strategies;
- These conjectures are also consistent with the other players' rationality, i.e., …
- Ad infinitum

Stag Hunt




(2,2)	(4,0)
(0,4)	(6,6)

A summary

- If players are rational (and cautious), then they play the dominant-strategy equilibrium whenever it exists
  - But, typically, it does not exist
- If it is common knowledge that players are rational, then they will play a rationalizable strategy-profile
  - Typically, there are too many rationalizable strategies
- Now, a stronger assumption: The players are rational and their conjectures are mutually known.

Nash Equilibrium

Definition: A strategy-profile s* =(s_1*,…,s_n*) is a **Nash Equilibrium** iff, for each player i, and for each strategy s_i, we have

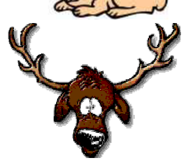
$$u_i(s_1^*, \ldots, s_{i-1}^*, s_i^*, s_{i+1}^*, \ldots, s_n^*)$$
$$\geq u_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*),$$

i.e., no player has any incentive to deviate if he knows what the others play.

If players' rationality and their conjectures about what the others play are mutually known, then their conjectures must form a Nash equilibrium.

Stag Hunt



(2,2)	(4,0)
(0,4)	(6,6)