14.123
Spring 2001

PROBLEM SET #6

Not handed in

1. Consider an exchange economy with one physical good and two states of nature. Assume that there is a complete set of Arrow-Debreu markets. Assume that all consumers are expected utility maximizers and all consumers have the same subjective probabilities, with p as the probability of state one. Consumer h has utility u^h and endowment e^h. Show that if the aggregate endowment is the same in both states of nature, then the relative price of the commodity in the two states is $p/(1-p)$.

2. Consider the set of date-event pairs depicted in figure 6.7 in Kreps. Recall the following data given before:

   At date zero, a claim paying \$1 at date 1 in event $\{s_3, s_4, s_5\}$ costs \$.50

   At date 1 in event $\{s_3, s_4, s_5\}$, a claim paying \$1 at date 2 in event $\{s_4, s_5\}$ will cost \$.60

   At date 2 in event $\{s_4, s_5\}$, a claim paying \$1 at date 3 in event $\{s_4\}$ will cost \$.40

   Broccoli at date zero costs \$2.00 per unit

   Artichokes at date 3 in event $\{s_4\}$ will cost \$.67 per unit

   Add to this the following addition data:

   At date zero, a claim paying \$1 at date 1 in event $\{s_1, s_2\}$ costs \$.40

   At date 1 in event $\{s_1, s_2\}$, a claim paying \$1 at date 2 in event $\{s_1, s_2\}$ will cost \$.90

   At date 2 in event $\{s_1, s_2\}$, a claim paying \$1 at date 3 in event $\{s_1\}$ will cost \$.40

   At date 3 in event $\{s_1\}$, artichokes will cost \$1.33

   Suppose a consumer wished to sell some date 3-event $\{s_1\}$ artichokes with which she is endowed and use the proceeds to buy date 3-event $\{s_4\}$ artichokes. For every unit of date 3-event $\{s_1\}$ artichokes she sells, how many units of date 3-event $\{s_4\}$ artichokes can she purchase? What is the strategy she follows for affecting such a trade? (This strategy should involve changing her "position" in vegetables at these two date-event pairs only. The key is the first step. She sells date 1-event $\{s_1, s_2\}$ dollars and uses the proceeds from that sale to buy date 1-event $\{s_3, s_4, s_5\}$ dollars.) (Kreps 6.9)

3. Consider an economy with a continuum (of measure 1) of identical households. There are two

states of nature, each with equal probability. Each household is an expected utility maximizer using the known probabilities with u(c) = log(c). Assume that by home production each household can produce 1 unit of output in state 1 or 2 units in state 2. Find competitive equilibrium with a complete set of Arrow-Debreu markets.

4. Consider a competitive stock market economy with 2 consumers, 2 firms and 3 states of nature. Assume that firm 1 has output $y^1 = (1,1,1)$ across the three states, while firm 2 has output $y^2 = (2,0,0)$. The consumers own equal shares in the two firms. Both are expected utility maximizers with u = log(c). However, they have different beliefs about the states of nature with consumer one having subjective probabilities (1/3, 1/3, 1/3) while consumer two has subjective probabilities (1/2, 1/4, 1/4). Derive the relative price of the two firms in stock market equilibrium.

   If the government makes a market for a bond that pays the same amount in all 3 states of nature, what happens to the relative price of the shares of the two firms in stock market equilibrium? Hint: No calculations needed.

5. Consider a competitive economy with two periods and two states of nature. The probability of state 1 is 1/3 and the probability of state 2 is 2/3. There are three goods in this economy, first period consumption, a, and second period consumption in each state of nature, b_1 and b_2. There is a continuum of measure 1 of identical consumers, who are expected utility maximizers, using the correct probabilities. The utility function of the consumers (for expected utility maximization) is

$$u = (1/2)\ln[a] + (1/2)\ln[b],$$

   where a is consumption in the first period and b is consumption in the second period. The endowment of each consumer is 12 units of the first period consumption good and none of the second period consumption good in either state of nature.

   There are two types of firms in this economy. There are large numbers of each type of firm. All firms have constant returns to scale. Firms of type 1 can convert units of the first period consumption good into units of the second period consumption good that only are produced in state of nature 1, with 1.25 units of output for each unit of input. Firms of type 2 can convert units of the first period consumption good into units of the second period consumption good that are produced in fixed proportions in the two states of nature. For a firm of type 2, one-half unit of output in each state is produced for each unit of input. Production decisions are made before the state of nature is known. Each firm is owned in equal shares by all the consumers.

   a. Assume there is a complete set of contingent commodity markets. Determine competitive equilibrium prices and quantities. Hint: Be careful about corner conditions.

Now assume that there are two types of consumers in the economy. One type is as described above. Assume there is a continuum of measure 1 of a second type, called type B. The utility function of a consumer of the second type (for expected utility maximization) is

$$u^B = (1/2)a + (1/2)b,$$

where a is consumption of consumer B in the first period and b is consumption of consumer B in the second period. The endowment of each consumer is 12 units of the first period consumption good and none of the second period consumption good in either state of nature.

b. Assume there is a complete set of contingent commodity markets. Determine competitive equilibrium prices and quantities. Hint: Be careful about corner conditions.

6. Consider a perfect foresight incomplete market economy with 2 goods and 2 states of nature with equal probabilities. Assume there is a continuum of consumers of measure 2. All consumers are expected utility maximizers, using the known probabilities. Half of the consumers have utility function $u^A(c_1, c_2) = (c_1 c_2)^a$, $0 < a < 1$ and are endowed with 2 units of good 1 in both states of nature. Half of the consumers have utility function $u^B(c_1, c_2) = (c_1)^b$, $0 < b < 1$. These consumers can choose between two different (home) production plans. With plan 1 a consumer has 1 unit of good 2 in both states. With plan 2 a consumer has e_1 units of good 2 in state 1 and e_2 units of good 2 in state 2. Assume that there are only markets for trading good 1 against good 2 after the state is realized. Consider only equilibria where all type B's choose the same plan (uniform equilibria).

a. Calculate the price of good 2 in the ex-post market as a function of the endowment of good 2 in the state.

b. Show that the expected utility of each person of type B is equal to 1 if they all follow the same plan.

c. Show that having all type B's follow plan 1 is the unique uniform incomplete market competitive equilibrium if

$$\left(e_1^b + e_2^b\right)/2 \le 1 \le \left(e_1^{-b} + e_2^{-b}\right)/2$$

d. Show that having all type B's follow plan 2 is the unique uniform incomplete market competitive equilibrium if

$$\left(e_1^{-b} + e_2^{-b}\right)/2 \le 1 \le \left(e_1^b + e_2^b\right)/2$$

e. Show that having the government order all the B's to follow plan 1 (followed by competitive markets) Pareto dominates having all the B's follow plan 2 if $1 > \left(e_1^a + e_2^a\right)/2$. Do not confuse e_1 raised to the power a with A's endowment.

f. Show that ordering all the B's to follow plan 2 (followed by competitive markets) Pareto dominates having all the B's follow plan 1 if $\left(e_1^a + e_2^a\right)/2 > 1$.

g. Show that the competitive equilibrium might be Pareto dominated by another allocation if the government can costlessly induce all type B's to change their production plan.